[1]See, for example, E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1970), 2nd ed.
[2]The potential energy is $V=0$ for $0\leqslant x \leqslant L$ and infinite elsewhere.
[3]The dot represents differentiation with respect to time.
[4]See, for example, S. L. Ross, *Introduction to Ordinary Differential Equations* (Ginn, Boston, 1966).
[5]Other approximation methods also exist, such as the Picard method of successive aproximations, for instance.
[6]See, for example, D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951).
[7]It is possible, however, to avoid using the derivative of the system Hamiltonian by using Eq. (31), for example, to calculate $C_m$.

# Potential energy of interaction for two magnetic pucks

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An air table experiment is described in which the interaction force and potential energy for a pair of magnetic pucks are measured as functions of the separation of the pucks. The relation between the force and potential energy is demonstrated by graphical integration. The experiment is also well suited to an air track.

For a number of years we have employed laboratory experiments on air tables as an integral part of our introductory physics courses. In addition to the usual dynamics experiments illustrating $F=ma$ and the conservation of momentum for pucks moving without friction on the surface of an air table, we have also used an experiment that illustrates the relation between a conservative force and the associated potential energy. In this experiment one first measures the magnetic force between two magnetic pucks as a function of their separation and then one measures the potential energy of the interaction as a function of separation. From plots of these two functions one may then demonstrate $U(x) = -\int F(x')dx'$ by graphical integration. Below we will describe the apparatus and the methods employed in this experiment.

The apparatus is depicted in Fig. 1. Aside from the Ealing air table, the required apparatus is very simple and inexpensive. The magnetic pucks are ordinary hollow plastic pucks into which have been inserted two disk-shaped ceramic magnets poled on their top and bottom faces. (The cylindrical symmetry of the pucks and magnets is important when the experiment is done on an air table since the pucks are free to rotate if there is a net torque.) One puck is fixed to the table with double sided tape, while the other is free to move. Before afixing the first puck, the table is accurately leveled by adjusting its leveling feet until an ordinary puck does not drift when it is levitated. Thereafter, the table may be inclined by a precisely determined angle by inserting a spacer block under the single foot that is centered at one end of the table. Distances along the incline are measured by means of a meter stick that is taped to the table alongside the fixed magnetic puck and aligned parallel to the long side of the table.

The magnetic force is measured by balancing the magnetic force of repulsion between the pucks against the component of the weight of the upper puck acting down the incline of the table. The floating puck balanced at the unstable equilibrium position may be kept from sideslipping away by delicately restraining it with the tip of a fingernail. The puck is then grounded by opening the air bypass and the position of the puck is measured. The angle of incline is set by the height $h$ of the spacer block or stack of blocks inserted under the air table leg as shown in Fig. 1. A series of heights $h$ (e.g., 3, 5, 8, 10, 15, 18, and 20 cm) may be obtained using just two rectangular laminated oak blocks having appropriate dimensions. For each height $h$ one measures the distance $x$ between the pucks, averaging several measurements if required. The magnetic force at the separation $x$ is given by the expression

$$F(x) = (mg/H)h. \quad (1)$$

The second part of the experiment involves measurement of the magnetic potential energy. The movable magnetic puck is held at a distance $x$ from the fixed puck. When it is released, or "launched," it will be accelerated by the magnetic force, rise up the incline to the maximum dis-

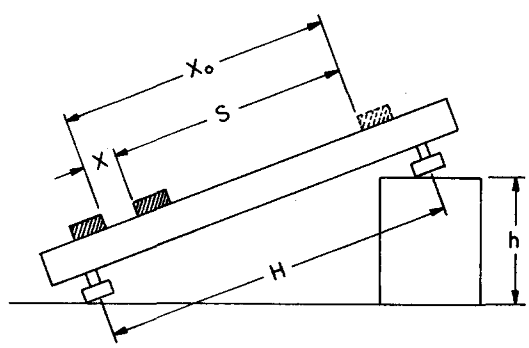


Fig. 1. Diagram of the apparatus, showing the air table with a pair of magnetic pucks resting on its surface, tilted up by a block of height $h$ inserted under one foot. The equilibrium separation of the pucks in the force study is $x$, while in the energy study the initial and final separations are $x$ and $x_0$, respectively.

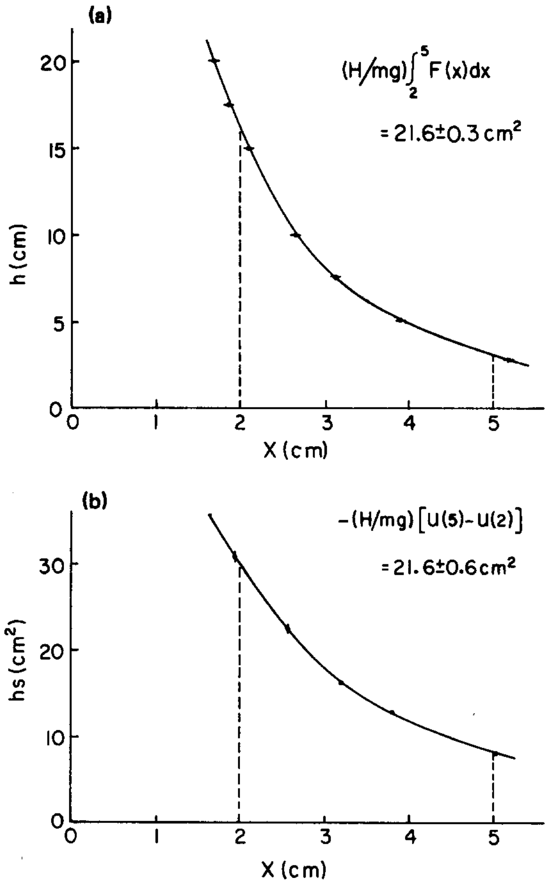


Fig. 2. Experimental measurements of the (a) force and (b) energy of interaction as functions of the separation $x$ of two magnetic pucks. Note that $F = (mg/H)h$ and $U = (mg/H)hs$.

tance $x_0$, and then fall back. The distance $x_0$ may be measured if the puck is grounded just as it comes to a stop at the top of its trajectory. Since the initial and final kinetic energies are both zero, the decrease in magnetic potential energy for the puck traveling from $x$ to $x_0$ is just equal to the increase in its gravitational potential energy. The magnetic potential energy is therefore given by the expression:

$$U(x) - U(x_0) = (mg/H)hs, \qquad (2)$$

where $s = (x_0 - x)$ is the distance traveled up the incline. The air table is inclined at a much smaller angle than before, so the puck rebounds to a final position $x_0$ large enough that the magnetic force at $x_0$ is very small. The initial separations $x$ of the pucks are chosen to fall in the same range as the $x$ values in the force study and are set by aluminum spacer bars (19, 26, 32, 38, and 50 mm wide), which may also serve as set squares to assist in reading the puck position with the meter stick. The angle of the incline is set using matched and color-coded aluminum spacer disks (23, 19, 15, 13, and 9 mm high) to determine the height $h$. The disk heights and bar widths are matched such that the final position $x_0$ will be nearly the same for each pair. The residual energy $U(x_0)$ at $x_0 \sim 14$ cm is so small and slowly varying that it may be taken as a constant if the final position $x_0$ is constant within about 1 cm. The average of $s$ for several "good" launches is measured, for each of the five initial separations $x$.

The analysis proceeds by plotting $h$ vs $x$ for the force data and $hs$ vs $x$ for the energy data. Note that Eqs. (1) and (2) have the factor $(mg/H)$ in common, so it is sufficient to plot $h$ rather than $F(x)$ and $hs$ rather than $U(x) - U(x_0)$. The "force" and "energy" functions obtained by drawing a smooth curve through the datapoints from a typical experiment are shown in Figs. 2(a) and 2(b), respectively. The relation between the potential energy and force functions is given by:

$$U(x_B) - U(x_A) = -\int_{x_A}^{x_B} F(x)dx. \qquad (3)$$

This relation may be tested by graphically integrating the $h$ vs $x$ curve from $x_A$ to $x_B$ and comparing the resulting area with the change in the height of the $hs$ curve over the same interval. The integral may be done by applying the trapezoidal rule with intervals 0.2 cm wide, for example. The force integral and potential energy change evaluated from the data of our typical experiment are shown in Fig. 2 for a particular choice of endpoints $x_A$ and $x_B$. The quantitative agreement is very good.

This experiment nicely illustrates some key concepts and relations in mechanics. In particular, force and two types of potential energy enter the theory of the experiment, the relations between force, work, and potential energy in a conservative system are illustrated, and the idea of the definite integral of a function as the area under its graph is reinforced. The procedure of the experiment is simple enough that it may be completed in a single laboratory session. While we have implemented this experiment on an air table, it is perhaps even better suited to the air track.


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