1. **(5 points)** An astronaut visiting Mars communicates with Mission Control on Earth. He requests guided help to perform a necessary task. What is the time difference between when he asks his question and when he received a response? Assume that Mars and the Earth are collinear with and on the same side as the sun, i.e. use the shortest distance between the two planets.

\[
\Delta t = \frac{2 \times (5.46 \times 10^9 \text{ m})}{c} = \frac{2 \times (5.46 \times 10^9 \text{ m})}{3 \times 10^8 \text{ m/s}} = 3.64 \text{ s}
\]

\[
= 3.64 \text{ s} \times \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 0.061 \text{ min}
\]

2. **(5 points)** An electric dipole antenna is placed on the roof of a house to detect TV signals. If the antenna is 1.2 m long in total length, calculate the frequency of electromagnet radiation that it will optimally detect. What is the television band of this optimized/resonant frequency (e.g. VHF, UHF)?

\[
L = \frac{\lambda}{2} \implies \lambda = 2L = 2(1.2 \text{ m}) = 2.4 \text{ m}
\]

\[
\implies f = \frac{c}{\lambda} = \frac{3 \times 10^8}{2.4 \text{ m}} = 1.25 \times 10^8 \text{ Hz}
\]

\[
= 125 \text{ MHz} = f
\]

This is in the **VHF** (Very High Frequency) band (30-300 MHz).
3. **(5 points)** A technician detects the electric field from an isotropic source that is 22 km away and finds that $E_{\text{rms}} = 55 \text{ mV/m}$. What is the average power of the source?

$$\langle U \rangle = \bar{U} = E_{\text{rms}}^2 = (8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}) (55 \times 10^{-3} \frac{V}{m})^2 = 2.68 \times 10^{-14} \frac{J}{m^2}$$

$$\Rightarrow \bar{J} = \bar{U} C = (2.68 \times 10^{-14} \frac{J}{m^2}) (3 \times 10^8 \frac{m}{s}) = 8.04 \times 10^{-6} \frac{W}{m^2}$$

$$\Rightarrow \bar{P} = \bar{J} A = (8.04 \frac{W}{m^2}) (4\pi)(22 \times 10^3 \frac{m}{s})^2 = 4.9 \text{ kW}$$

4. **(5 points)** Wavelengths of light from a distant galaxy are found to be 0.5% longer than the corresponding wavelengths measured in a terrestrial laboratory. Is the galaxy approaching or receding from the Earth? At what speed?

$$\lambda_0 > \lambda_{\text{source}} \Rightarrow \text{Galaxy is Receding (Red Shift)}$$

$$f_0 = f_{\text{source}} (1 - \frac{V_{\text{rel}}}{c}) \text{ where } V_{\text{rel}} = \text{relative velocity}$$

$$\Rightarrow \lambda_0 \lambda_{\text{source}} = \frac{\lambda_0}{\lambda_{\text{source}}} = (1 - \frac{V_{\text{rel}}}{c})$$

$$= 1.005 \Rightarrow \frac{\lambda_0}{\lambda_{\text{source}}} = 1 - \frac{V_{\text{rel}}}{c} \Rightarrow \frac{V_{\text{rel}}}{c} = 0.005 = 5 \times 10^{-3}$$

$$= V_{\text{rel}} = 5 \times 10^{-3} \text{ c} = 0.5 \% \text{ c}$$

$$= 1.5 \times 10^6 \text{ m/s} = V_{\text{rel}}$$