1. (5 points) Consider a spacecraft engine that ejects exhaust gases at a rate of 25 kg/s and with an exhaust gas velocity of 3,100 m/s. If the initial mass of the rocket is 40,000 kg, what is the change in velocity experienced by the rocket for 2 minutes of burn?

\[ \Delta v = (3100 \text{m/s}) \ln \left( \frac{M_i}{M_f} \right) \]

\[ M_i = 40,000 \text{kg} \]

\[ M_f = M_i - \left( \frac{25 \text{kg}}{s} \right) \left( 3600 \text{s} \right) = \frac{40,000 - 3000}{37,000} \text{kg} = 37,000 \text{kg} = M_f \]

\[ \Rightarrow \Delta v = (3100 \text{m/s}) \ln \left( \frac{40,000}{37,000} \right) = 242 \text{m/s} \]

2. (5 points) A spacecraft's dry (empty) mass is 75,000 kg and the effective exhaust gas velocity of its main engine is 3,100 m/s. How much propellant must be carried if the propulsion system is to produce a total \( v \) of 700 m/s (initially from rest) – neglect gravity?

\[ \Delta v = 700 \text{m/s} = (3100 \text{m/s}) \ln \left( \frac{M_i}{M_f} \right) \Rightarrow M_f = 75,000 \text{kg} \]

\[ \Rightarrow \ln \left( \frac{M_i}{75,000 \text{kg}} \right) = \frac{700}{3100} = 0.226 \]

\[ e \Rightarrow e \]

\[ \Rightarrow \frac{M_i}{75,000} = 1.253 \Rightarrow M_i = 94,000 \text{kg} \]

\[ \Rightarrow M_i - M_f = \Delta m = M_{\text{propellant}} = 19,000 \text{kg} \]

Over→
c. (5 points) Suppose that 2.0 moles of liquid hydrogen and 1.0 moles of liquid oxygen are combined in a rocket engine at atmospheric pressure. How many total liters of both liquids are initially combined?

\[2H_2 + O_2 \rightarrow 2H_2O\]

\[\rho_{H_2} = 0.07 \text{ g/cc} \quad 1 \text{ mole } H_2 = 2 \text{ g}\]

\[\Rightarrow 2 \text{ moles } H_2 = 4 \text{ g}\]

\[\rho_{O_2} = 1.14 \text{ g/cc} \quad 1 \text{ mole } O_2 = 32 \text{ g}\]

\[\Rightarrow 1 \text{ cc } = 1 \text{ ml} = \frac{4 \text{ g}}{0.07 \text{ g/ml}} = 57.1 \text{ ml } H_2, \frac{32 \text{ g}}{1.148 \text{ cc}} = 28.07 \text{ ml}\]

\[57.1 \text{ ml} + 28.07 \text{ ml} = 85 \text{ ml} = 0.085 \text{ L}\]

d. (5 points) How many liters of water vapor would be produced at atmospheric pressure via the following chemical reaction: \(2H_2 + O_2 \rightarrow 2H_2O\) neglecting the atmosphere?

2 moles of \(H_2O\) are produced \(\Rightarrow\) Assume \(RTP = 24.44 \text{ L/mole}\)

\(\Rightarrow 48.8 \text{ L} \text{ of } H_2O\) are produced.

e. (5 points) Calculate the average speed of water molecules being exhausted from a rocket engine where the combustion temperature is 3080 K.

\(\text{Done in class: } \frac{3}{2} k_B T = \frac{1}{2} m v_{\text{rms}}^2 \Rightarrow m = m_{H_2O} \text{ molecule}\)

\(\Rightarrow m_{H_2O} = (18 \text{ g/mole}) \cdot \left(\frac{6.022 \times 10^{23}}{2.99 \times 10^{-23} \text{ g}}\right) = 2.99 \times 10^{-26} \text{ kg}\)

\(\Rightarrow v_{\text{rms}} = \sqrt{\frac{3 k_B T}{m_{H_2O}}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})}{2.99 \times 10^{-26} \text{ kg}}} (3080 \text{ K}) = 2.065 \text{ m/s} = 2.1 \text{ km/s}\)