Core Ideas of Unit 9 – Momenergy

In Newtonian mechanics momentum and energy are each conserved during a collision. In special relativity, there is a single conserved quantity, a 4-vector with space components equal to momentum and a time component equal to energy. The magnitude of this 4-vector for a stone moving through spacetime is an invariant, the rest mass of the stone. Different free-float observers measure different values for the momentum and energy of the stone, but when those values are used to calculate the rest mass, all free-float observers agree. Sound familiar!

The idea of momenergy can be quantified by considering the invariant interval between two nearby points on a stone's worldline. The stone's wristwatch ticks off a short amount of proper time while some free-float observer measures the space and time intervals between those nearby points:

$$\Delta\tau^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

If this equation is divided by $\Delta\tau^2$, the result is,

$$1 = \frac{\Delta t^2}{\Delta\tau^2} - \frac{\Delta x^2}{\Delta\tau^2} - \frac{\Delta y^2}{\Delta\tau^2} - \frac{\Delta z^2}{\Delta\tau^2}$$

The 4-velocity of the stone is defined by the components on the right-hand side of the equation. Although different free-float observers measure different space and time components, the magnitude of the 4-velocity is the same for all free-float observers, namely 1!

Multiply the previous equation by the square of the rest mass, m^2

$$m^2 = m^2\frac{\Delta t^2}{\Delta\tau^2} - m^2\frac{\Delta x^2}{\Delta\tau^2} - m^2\frac{\Delta y^2}{\Delta\tau^2} - m^2\frac{\Delta z^2}{\Delta\tau^2} = \mathrm{E}^2 - \mathrm{p}_\mathrm{x}^2 - \mathrm{p}_\mathrm{y}^2 - \mathrm{p}_\mathrm{z}^2 = \mathrm{E}^2 - \mathrm{p}^2$$

The preceding analysis shows how the momentum-energy 4-vector follows naturally from the invariance of the interval.

In a collision between two particles, A and B, the total momenergy 4-vector before the collision is equal to the total momenergy 4-vector after the collision. The conservation of momenergy means that each component of the 4-vector is conserved, four equations:

$E_{A,before} + E_{B,before} = E_{A,after} + E_{B,after}$, and three analogous equations, one for each component of the momentum.

Assignment for Unit 9

1) Keeping the core ideas in mind, carefully read through **Chapter 7: Momenergy** in its entirety.
2) Now start re-reading the chapter with pencil and paper in hand.
3) Read Box 7.1 to learn about 4-vectors. Four vectors have 3 space components and 1 time component. The magnitude of a 4-vector is calculated by subtracting the squares of the 3 space components from the square of the time component. This minus sign distinguishes Lorentzian from Euclidean geometry.
4) Do Sample Problem 7.1.
5) The "invariant" hyperbola in figure 7.4 reinforces the connection between proper time and rest mass. Each is the magnitude of a 4-vector. For the momenergy 4-vector, the magnitude is not only invariant but also independent of the velocity of stone.
6) Do Sample Problems 7.2 and 7.3.
7) Take some time to study figure 7.6. It contains a lot of information.
8) Carefully read the summary of this chapter. Table 7.1 lists the relevant equations from this chapter and compares their relativistic form when v is dimensionless to their conventional form when v is measured in units of length/time. Notice that the relativistic forms are less busy, that is they are not cluttered with c's!
9) Do practice exercises 7.1 through 7.4.
10) Do problems 7.5 through 7.11. The transformation equations connecting energy and momentum in one free-float frame to another that are derived in 7.5 are very important.
11) When finished with the practice exercises and problems, bring them by my office. If everything looks okay, you will be given a quiz to test your mastery of the material in Unit 9.