Core Ideas of Unit 5 – Interlude: The Far Away Observer

This break from the lock-step march through the textbook is designed to introduce the only non-local observer who may be of pedagogical value. This observer is far from the scene of action and stationary with respect to one of the inertial reference frames of interest. The key feature of this observer is that she is so far from the action that she actually "sees" all the clocks in a particular reference frame synchronized and ticking at the same rate. Consequently, she also "sees" all the clocks in a moving frame as being unsynchronized and ticking at a slower rate. Furthermore she "sees" moving meter sticks actually being shorter than stationary meter sticks.

The questions in the assignment outlined below quantify the idea of the faraway observer.

Assignment for Unit 5

Imagine two inertial reference frames, S and S′. S′ is moving along the positive x-axis of S with speed v. When the two origins are coincident, the clocks at their respective origins read 0. Also the clocks in S and S′ have been properly synchronized. The space interval of interest runs from x = -L to x = +L. The faraway observer (FAO) is located a perpendicular distance d from the origin of S and is stationary with respect to S. It takes a time d/c seconds for the image of the clock at x = 0 to reach the FAO, or d meters of time. The FAO sets her clock to zero at the instant she sees the clock at x = 0 read zero.

1) Draw a picture that includes reference frame S and the FAO. Note that the local observers at +x and –x are the same distance from the FAO. Use the Pythagorean Theorem to find this distance?
2) For this problem, $d \gg L$, so the approximation $\sqrt{d^2 + L^2} \approx d + \frac{L^2}{2d}$ is valid. Because light from the clocks at $\pm x$ have to travel a little further than the clock at x = 0, the FAO observer sees the clocks at $\pm x$ read how many meters of time when she sees the clock at the origin read zero?
3) The largest difference in time that the FAO sees is from the clocks at $\pm L$. The difference in time gets smaller and smaller as d is made larger and larger. Consequently, the difference in time between all the clocks in S seen by the FAO between $\pm L$ can be made arbitrarily close to the time she sees on the clock at the

origin of S. To quantify this idea, assume L = 5 meters and S′ moves with speed v = 0.8. How many meters of time elapse as the origin of S′ traverses the 10 meters from x = -5 to x = +5 meters? That is the amount of time that is relevant as the FAO watches S′ move through S. Assume that the clock used by the FAO is accurate to 1/100 of this time. What is the accuracy of her clock in meters of time?

4) To be safe, you want the maximum difference in light travel time between the 11 clocks in S viewed by the FAO to be substantially less than the accuracy of her clock. You choose to make that maximum time difference 1/10 of the accuracy of FAO's clock. What is the minimum distance d for the FAO that meets this?
5) As the FAO watches the clock at the origin of S′ move from x = -5 to x = +5, explain in detail what she sees. For example, what time does she see on the two coincident clocks as the clock at the origin of S′ passes the clock in S at x = -5? What does she see on the two coincident clocks when the moving clock at the origin of S′ passes the clock in S at x = 0? At x = +5? Feel free to use the Lorentz equations to answer this question.
6) Imagine a meter stick in S′ that stretches from x′ = 0 to x′ = 1. Explain what the FAO sees for the length of that meter stick when its left end is at x = x′ = 0?
7) When finished with the assignment, come by my office. If everything looks okay, you will be given a quiz to test your mastery of the material in Unit 5.