Core Ideas of Unit 4 – Lorentz Transformation

The Lorentz transformation equations connect the coordinates of two free-float frames moving with respect to one another. Any event in spacetime can be located by its coordinates in a free-float frame. If the event is located at (x, y, z, t) in one frame and (x′, y′, z′, t′) in another frame, the Lorentz transformation equations are the set of relations connecting the unprimed to the primed coordinates.

Though simple to state, using the Lorentz transformation equations successfully and correctly requires practice and careful attention to the details of the events involved in the problem.

Assignment for Unit 4

1) Keeping the core ideas in mind, carefully read through **Special Topic: Lorentz Transformation** in its entirety.
2) Now start re-reading the material with pencil and paper in hand.
3) Note that in table L.1 and the Lorentz equations directly below the table that the origins of the unprimed and primed coordinates are coincident at t = t′ = 0. The Lorentz transformations assume this to be the case. If you attempt to use the equations in situations where the two origins are not coincident at t = t′ = 0, you will get WRONG answers!
4) In section L.3, **Getting Started**, notice two important things. First the rocket clock records time t′ and is moving through the laboratory. Consequently it gets compared to two laboratory clocks, one at the origin x = 0, and other at $x = v_{rel}t$. In order for the time t to be useful, the clocks in the laboratory frame must have been properly synchronized. With this proviso, note that the "moving" clock ticks off less time when compared to two synchronized "stationary" clocks. The second important thing to keep in mind is that equation L.4, $x = v_{rel}\gamma t'$, gives the location of the ORIGIN of the rocket frame, $x' = 0$, at time t'; a laboratory space coordinate, x, in terms of a rocket time, t′!
5) Carefully work through the derivation of equations L.10a and L.10b in section L.5, **Completing the Derivation**. In particular, be able to explain why the transformations are linear.
6) The Lorentz transformations are summarized below for a rocket moving in the positive x direction:

$$x = \frac{1}{\sqrt{1-v^2}}(x' + vt'), y = y', z = z', and\ t = \frac{1}{\sqrt{1-v^2}}(t' + vx')$$

$$x' = \frac{1}{\sqrt{1-v^2}}(x - vt), y' = y, z' = z, and\ t' = \frac{1}{\sqrt{1-v^2}}(t - vx)$$

a) Notice that the equations for x and t in terms of x′ and t′ are symmetrical. The equation for t can be found by interchanging x′s and t′s in the equation for x in terms of x′ and t′. The same is true for the equations for x′ and t′ in terms of x and t.
b) Also the only difference between the equations for x and t in terms of x′ and t′ and the inverse equations, is the sign of the relative velocity v. Therefore if the equations for x and t are known, it is a trivial matter to write the inverse equations for x′ and t′.
c) The equation I always write down first is the one for x′, the origin of the rocket coordinates, in terms of x and t. I know that the rocket equation is described in the laboratory coordinates by the equation, $x = v_{rel}t$. This relationship requires that the equation for x′ in terms of x and t have a MINUS sign between x and vt. Once this equation is written down, the remaining three Lorentz transformation equations fall into place.

7) **Sample Problem L.1** is a straightforward application of the Lorentz equations. Work out the answers for x, y, z, and t and then check them by transforming back to x′, y′, z′, and t′ WITHOUT looking at the answers!
8) Carefully work through the derivation of equation L.13 in section L.7, **Addition of Velocities**. Convince yourself that as long as v′ and v_{rel} are less than 1, v has to be less than 1. Also show that if v′ is the speed of light, either +1 or -1, then v is also +1 or -1. This shows that the speed of light is the same for all free-float observers! Of course this had to be the case because the constancy of the speed of light was integral for deriving the Lorentz transformation equations.
9) **Sample Problem L.2** is another exercise in plugging values into the Lorentz equations. But before trying to do that, think about the problem first. There are three events, Caesar's death, the "present" moment on Earth, and the firecracker going off in the Andromeda galaxy. Which of the two events are simultaneous in the Earth reference frame? Which of the two events are simultaneous in the rocket frame? Also note, as mentioned before, in order for the Lorentz equations to work,

the origins of both frames had to coincide at Caesar's death. Now work through the problem but skip b), the spacetime diagram.

10) Use graph paper to reproduce the two diagrams in Box L.1. Note that origin of the Federation rocket frame is NOT located in the rocket!
11) Work through **Sample Problem L.3**. Note that the pi plus and pi minus particles have the same mass. In the rocket frame, they have to be emitted in manner consistent with conservation of momentum!
12) Read the summary and make sure you understand how those highlighted equations came into existence.
13) Do practice exercises L.1 and L.3.
14) Do problems L.5 through L.11.
15) When finished with the practice exercises and problems, bring them by my office. If everything looks okay, you will be given a quiz to test your mastery of the material in Unit 4.