

Core Ideas of Unit 2 – Floating Free

The primary goals of this unit are to show the value of viewing events from inertial reference frames, ones “floating free,” and to develop an operational definition of such reference frames. In a reference frame that is floating free there is “locally” no “force of gravity.”

After finishing this unit, you ought to have a solid understanding of the meaning of inertial, free fall, and local as used in the preceding paragraph.

To record the coordinates of an event, each inertial reference frame has observers located at fixed points with properly “synchronized” clocks. The standard procedure for synchronizing clocks is described in section 2.6, **Locating Events with a Lattice of Clocks**. Coordinates of an event are always recorded by the nearest observer in order to minimize the light travel time between the event and the observer.

Any number of inertial reference frames, each moving with a constant velocity, can be used to denote the coordinates of an event in spacetime. And the observers in each of those different inertial frames can with equal legitimacy declare that their frame is at rest and all the other inertial frames are moving with respect to it!

Assignment for Unit 2

- 1) Keeping the core ideas in mind, carefully read through **Chapter 2: Floating Free** in its entirety.
- 2) Now start re-reading the chapter with pencil and paper in hand.
- 3) The goal of the following steps is to quantify the situation described by figure 2.4. Namely, that the path of the stone depends on the state of motion of the reference frame from which the stone’s motion is being viewed. Keep in mind that the stone’s path can be imagined as a series of events, for example ticks of the stone’s wristwatch.
 - a) A reference frame has to be defined in order to describe the path in terms of coordinates. In figure in 2.4 of the text, call the left-hand reference frame that is fixed to earth A and the free-fall frame on the right B. Frame B is at rest until the moment the stone is launched. At that instant, frame B is allowed to fall freely. Let the launching point be the origin of both frames, $t_A = x_A = y_A = 0$ and $t_B = x_B = y_B = 0$.

- b) Explain why during the motion of the stone $t_A = t_B = t$.
 - c) The motion of the stone is determined by its initial velocity. Since both frames were at rest at the instant of the launch, observers in frames A and B agree that the initial velocity has components v_{x0} and v_{y0} .
 - d) In terms of v_{x0} and v_{y0} , what are the coordinates of the stone's path when viewed from frame A, X_A and Y_A ?
 - e) Show that when time is eliminated the path of stone in reference frame A is described by $Y_A = X_A \left(\frac{v_{y0}}{v_{x0}} - \frac{g}{2v_{x0}^2} X_A \right)$. Convince yourself that this is the equation for a parabola.
 - f) An observer in frame B would describe the path of the stone as the motion of the origin of frame A plus the coordinates of the stone's path, X_A and Y_A .
 - g) Show that the path of the stone in frame B is $X_B = X_A$ and $Y_B = \frac{g}{2} t^2 + Y_A$.
 - h) Now use your results from d) and eliminate time to get the equation that describes the stone's path in reference frame B, $Y_B = \frac{v_{y0}}{v_{x0}} X_B$, the equation for a straight line.
- 4) In section 2.3, **Local Character of Free-Float Frame**, the authors on page 31 say "In the 42 minutes it takes for our small room to fall through a tunnel North Pole to South Pole, ..." Show that a stone dropped through that tunnel will move with simple harmonic motion from one Pole to the other. Use the frequency of that motion to show that the time to travel from one Pole to other is in fact 42 minutes.
 - 5) Section 2.4, **Regions of Spacetime**, describes how any particular free-fall reference frame remains inertial for a restricted region of space and time because of the action of tidal forces and the accuracy with which the motion they cause can be measured.
 - 6) Newton's equations $F = m_{inertial}a$ and $F_G = \frac{GMm_{gravity}}{r^2}$ show that, in principle, there are two different kinds of mass. There is no *a priori* reason that these two kinds of mass ought to be the same. The "inertial" mass describes the way a mass responds to a general force. The "gravitational" mass determines the magnitude of the gravitational force acting on the mass. For example, if a lump of coal and a bar of gold each have identical accelerations when acted upon by equal forces, a spring for example, the lump and bar have identical inertial masses. Now if the lump and bar are placed on opposite sides of a balance, which just compares the gravitational force of the Earth on each of the objects, it is possible that their weights would differ. If their weights did differ, that would mean that their gravitational masses were

different even though their inertial masses were the same! Very careful experiments have not been able to detect any difference between inertial and gravitation mass to less than 1 part in 10^{11} . Consequently all masses moving through a gravitational field follow identical paths regardless of the makeup of the mass because in the equation $F = m_{inertial}a = \frac{GMm_{gravity}}{r^2}$ and $m_{inertial} = m_{gravity}$ leading to $a = \frac{GM}{r^2}$ which has no reference to the makeup of the stone that is moving under the influence of gravity.

- 7) Work through Sample Problem 2.1.
- 8) Do practice exercises 2.3, 2.4, 2.5 2.6, 2.8, and 2.9.
- 9) Do problems 2.11 and 2.13.
- 10) When finished with the practice exercises and problems, bring them by my office. If everything looks okay, you will be given a quiz to test your mastery of the material in Unit 2.