Core Ideas of Unit 11 – Gravity: Curved Spacetime in Action

This chapter acts as the bridge the introduction to general relativity in **Exploring Black Holes: Introduction to General Relativity** by the same authors as the text for this course on special relativity. By necessity, this chapter is less quantitative than the previous chapters. Don't let that lull you into a false sense of confidence.

The qualitative ideas presented are designed to help you grasp the difference between the gravity of Newton and gravity as conceived by Einstein. Before Einstein, space and time were seen as separate and distinct entities. Einstein merged them into a single concept, spacetime. Before Einstein, gravity was a mysterious force that acted instantaneously over vast distances. And coincidently, inertial mass, for no apparent *a priori* reason, seemed to be equivalent to gravitational mass. Einstein banished the force of gravity into the waste bin of history, and instead postulated that mass/energy curves spacetime. A test particle moving through spacetime travels in a straight line in a local free-float reference frame. The effect of gravity can only be discerned as the particle passes from one local frame to the next. General relativity establishes the connection between these contiguous free-float frames because it is <u>impossible</u> to establish a global free-float frame in the vicinity of massive bodies.

When you finish this unit, you are ready for a more serious venture into the physics around and inside a black hole. Congratulations.

Assignment for Unit 11

1) There is no force of gravity in "local" free-float reference frames. Be able to explain how Einstein made the force of gravity disappear!
2) One of the last statements in Section 9.3, **Local Moving Orders for Mass**, is "Does a proof mass have to know the location of Earth and Moon and Sun before it knows how to move?" Be able to answer that question with a coherent explanation.
3) This exercise is designed to supplement the discussion in Section 9.4, **Spacetime Curvature**.
   Imagine a cube formed by eight test masses with sides of length "s" arranged so that four of the masses are a distance r from the center of Earth and the other four are a distance $r – s$ from the center. The volume enclosed by those eight masses is s^3. Those eight masses are in a free-float frame that is initially at rest but then falls towards Earth. When the four masses that started a distance r fall to a distance of $r – h$ from the center of Earth, what is the

horizontal separation between those four masses? At that same instant, what is the location of the four masses that began at r – s? In this problem, r >> h and h >> s. After the cube has fallen a distance h, what is the volume enclosed to <u>*first order in s*</u>*?*

4) If you did the calculation correctly, you discovered that to first order is s the volume enclosed by the eight masses did not change. This is a general property of "tidal forces," namely they conserve volume.
5) Section 9.5, **Parable of the Two Travelers**, shows the qualitative connection between gravity and curvature of space. The way curvature of spacetime leads to gravity is more accurately described in the next section, **Gravitation as Curvature of Spacetime**.
6) Box 9.1 describes how the volume conserving tidal forces work when there is no local source of mass and/or energy. This property of spacetime does not carry over into regions where there is mass and/or energy. Imagine the cube formed by the eight masses in the earlier example magically falling unimpeded through Earth. What would the enclosed volume be at the center of Earth?
7) A black hole is an object from which light cannot escape. The "size" of a black hole is typically described by the radius of the event horizon. Light emitted a little further from the black hole than the event horizon can escape while nothing emitted from inside the event horizon can pass back out through the event horizon. For a given mass, M, general relativity establishes the following equation for the radius of the event horizon,

$$r = \frac{2GM}{c^2}$$

8) Now imagine light escaping from a gravitating body in Newtonian physics where it is assumed that a "particle" of light has a kinetic energy of $\frac{1}{2}mc^2$. Use conservation of energy to calculate the radius of an object of mass M that would just keep these particles of light from escaping. (You ought to get the same equation written above.)
9) What is the value of this radius for an object with the same mass as Earth?
10) Surprise, there are no exercises at the end of this chapter but come by prepared to answer questions about the material.