Chapter Nine – Impulse and Momentum

When Newton formulated has second law, it was in a slightly different form than $\mathbf{F} = m\mathbf{a}$, the actual statement was that $\mathbf{F} = d(m\mathbf{v})/dt = d\mathbf{p}/dt$ where $\mathbf{p} = m\mathbf{v}$ is the momentum. In words, the net force acting on an object causes its momentum to change. If the net force is zero, the momentum does not change, it is <u>conserved</u>.

$$d(m\mathbf{v})/dt = \mathbf{v}\ dm/dt + m\ d\mathbf{v}/dt = \mathbf{v}\ dm/dt + m\ \mathbf{a}$$

If the mass of the object is constant, $dm/dt = 0$ so $d(m\mathbf{v})/dt = m\ \mathbf{a}$ and the two versions of Newton's second law are equivalent. But if the mass of the object is changing, than $\mathbf{F} = d\mathbf{p}/dt$ is the correct version! The most famous example of a situation with changing mass is the rocket engine. The rocket engine produces thrust by pushing mass out of a nozzle with a high velocity causing the engine, and anything attached to the engine, to accelerate in the opposite direction.

In physics, momentum has a precise definition. Notice first that it is a vector quantity with magnitude and direction. It points in the direction of the object's velocity and has units kg m/s. In terms of components $\mathbf{F} = d\mathbf{p}/dt$ can be written as,

$$F_x\ \mathbf{i} + F_y\ \mathbf{j} = dp_x/dt\ \mathbf{i} + dp_y/dt\ \mathbf{j}.$$

The net force in the x-direction causes the x-component of the momentum to change while the net force in the y-direction causes the y-component of the momentum to change. For the remainder of the summary, the discussion will focus on the x-direction but keep in mind that there are completely analogous equations for the y-direction.

An <u>impulsive</u> force is one that lasts for a "short" time. An example is the force exerted by a tennis racket on a tennis ball. That force on the ball only acts while the racket is in contact with the ball. Also the size of the force changes during that short time of interaction. The force is zero at the instant the racket meets the ball, increases to a maximum value when the racket's strings are stretched furthest, and then returns to zero as the ball leaves the racket. The detailed way the force changes with time is complicated but we can learn much about the interaction between the racket and the ball without knowing about the details of the force the racket exerts on the ball. Start with,

$$F_x = dp_x/dt,$$

and multiply both sides of the equation by dt to get,

F_x dt = dp_x.

Now integrate that equation over the short time the ball is in contact with the racket,

$\int_{t_1}^{t_2} F_x \; dt = \int_{p_{x1}}^{p_{x2}} dp_x = p_{x2} - \; p_{x1}$.

P_{x1} is the value of the momentum right as the ball makes contact with the racket while p_{x2} is the value of the momentum right as the ball leaves the rocket. Regardless of the direction of the ball to begin with, since the racket changes the direction of the ball, one of the momenta will be positive and the other negative.

The integral, $\int_{t_1}^{t_2} F_x \; dt$, is called the impulse caused by the force acting from t_1 to t_2. The impulse causes the momentum to change from p_{x1} to p_{x2}. Although we usually cannot do the integral, we can equate the integral, the area under F_x(t), to $F_{x,average}$ times ($t_2 - t_1$), a rectangle with the same area as the integral. Given some estimate for the duration of the interaction, ($t_2 - t_1$), it becomes possible to estimate the average force necessary to cause the observed change in momentum,

$F_{x,average}$ = $(p_{x2} - \; p_{x1})$/($t_2 - t_1$).

More typically, the value of $\mathbf{F}_{net}$ = d**p**/dt will be used to solve problems involving a system of two or more interacting objects when there is no net external force acting on the system. The forces of interaction between the objects inside the system are not important because whatever force object A exerts on object B, there is the equal and opposite force of object B on object A. Therefore all the interaction forces acting between the objects inside the system cancel because of Newton's third law and the only remaining forces are those exerted on the system by external agents. If there are no external agents, $\mathbf{F}_{net}$ = 0 and d**p**/dt = 0, which means **p** = **constant** where **p** is the sum of all the individual momenta of the objects that comprise the system, **p** = $\mathbf{p}_A$ + $\mathbf{p}_B$ + ….
The total momentum of the system is conserved. This is a very powerful and useful conservation law that can be used to solve a variety of problems.

When two or more objects collide, the total momentum before the collision is equal to the total momentum after the collision. In other words, the collision just redistributes the individual momenta amongst the colliding objects in such a way to insure that $\mathbf{P}_{before}$

= $\mathbf{P}_{after}$! Remember momentum is a vector quantity so the conservation of the vector quantity **P** implies conservation in both the x-direction and in the y-direction separately.

The duration of a collision is usually so short that the impulse caused by any external force acting on the system, $\int_{t_1}^{t_2} F\,dt$, is typically too small to effect the total momentum of the system. Therefore the momentum right before the collision can be equated to the momentum right after the collision even if some external force, gravity for example, is acting on the system.

A system that "explodes" is analogous to a collision in the sense that a lot happens in a short time! If the object exploding initially had zero momentum, then immediately after the explosion, the total momentum of all the pieces must still add to zero. Of course if the object exploded into a million pieces, adding together a million separate momenta would be rather tedious! But in any case, the total momentum of the system right before the collision is equal to the total momentum of the system right after the collision.

The best way to develop a better understanding of momentum and the value of the Conservation of Momentum Law is by solving an array of different problems. Also keep in mind that momentum is a different sort of thing than force. Try to be very precise when using worlds like force, momentum, acceleration, velocity, etc because the connection between those quantities is well defined in a physics class!