since we get zero

if $\Psi_m \neq \Psi_{\hat{l}}$

and we get zero if $\Psi_m = \Psi_{\hat{l}}$

The two states are orthogonal.

So starting from a complete set of single particle states

orthonormal

$$\{ | \Psi_i \rangle \}$$

which could be infinite in number

We can construct for N particles a complete set (no proof here) of

orthonormal we have proven.

$$\{ | \Psi_{\text{sym-izd}} \rangle \}$$

N-particle antisymmetric state.

— In general a lot of work.

Say we had G single particle states → maybe all there is the space or maybe this is all we choose to include. The G-set may be all the lowest energy eigen~~st~~ states and the higher energy states ~~may~~ we assume are never accessed.

And we have N particles.

recall the Pauli exclusion principle forbids repeats.

How many unique product states for N particles can we create?

ANS. G choices for the 1st particle
G−1 for the 2nd particle.

$$\therefore \quad G\cdot(G-1)(G-2)\dots(G-(N-1))$$

$$\cancel{\frac{G!}{N!}} = \frac{G!}{(G-N)!}$$

5-223

But all the particles are identical

— So rearrangements among the make no new ~~states~~ product states

$$\therefore \quad \frac{G!}{(G-N)!} = N!\,C$$

C is the number of actual unique states

Number of product state = Number of symmetrized states since from one product state one can only make one symmetrized state.

That is one antisymmetric state for fermions

$$C = \frac{G!}{N!\,(G-N)!}$$

$$= \binom{G}{N}$$

the binomial coefficient.

If the particles had be all distinct with isomorphic eigenstates

Then $\frac{G!}{(G-N)!}$ would be the number of unique states

This assumes No repeats But no Pauli exclusion principle & so repeats are actually allowed.

$\boxed{5-224}$

which particle you assign to which single particle state matters if the particles are distinct.

Ex: Four single particle ~~states~~ ~~energy~~

G=4

$$\left.\begin{array}{l} \psi_a(\underset{\sim}{r})\chi_+ \\ \psi_a(\underset{\sim}{r})\chi_- \\ \psi_b(\underset{\sim}{r})\chi_+ \\ \psi_b(\underset{\sim}{r})\chi_- \end{array}\right\}$$

spin ½ particles, so $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$

$\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$

The set is orthonormalized

$$\int \psi_a(\underset{\sim}{r})^* \psi_b(\underset{\sim}{r})\, d\underset{\sim}{r} = 0$$

$$\int \psi_a^* \psi_a\, d\underset{\sim}{r} = 1$$

$$\int \psi_b^* \psi_b\, d\underset{\sim}{r} = 1$$

$$\chi_+^\dagger \chi_+ = 1\,, \quad \chi_+^\dagger \chi_- = 0\,. \quad \chi_-^\dagger \chi_- = 1$$

Now we have 2 spin $1/2$ identical particles who have these single-particle states.

Question

How many unique product states and symmetrized states can we create?

ANS $\binom{G}{N} = \binom{4}{2} = \frac{4!}{2!\,2!}$

$= 6$

	product state	symmetrized state — drawn — without being explicit
i)	$\psi_a(\vec{r}_1)\chi_{+1}\,\psi_a(\vec{r}_2)\chi_{-2}$	$\frac{1}{\sqrt{2}}\left[\ldots - \ldots\right]$
ii)	$\psi_a(\vec{r}_1)\chi_{+1}\,\psi_b(\vec{r}_2)\chi_{+2}$	$\frac{1}{\sqrt{2}}\left[\ldots - \ldots\right]$
iii)	$\psi_a(\vec{r}_1)\chi_{+1}\,\psi_b(\vec{r}_2)\chi_{-2}$	$\frac{1}{\sqrt{2}}\left[\ldots - \ldots\right]$
iv)	$\psi_a(\vec{r}_1)\chi_{-1}\,\psi_b(\vec{r}_2)\chi_{+2}$	$\frac{1}{\sqrt{2}}\left[\ldots - \ldots\right]$
v)	$\psi_a(\vec{r}_1)\chi_{-1}\,\psi_b(\vec{r}_2)\chi_{-2}$	
vi)	$\psi_b(\vec{r}_1)\chi_{+1}\,\psi_b(\vec{r}_2)\chi_{-2}$	$\frac{1}{\sqrt{2}}\left[\ldots - \ldots\right]$

5-226

i) & vi) fall into the same class.

i) $\psi_a(\mathbf{r}_1)\,\psi_a(\mathbf{r}_2)\,\frac{1}{\sqrt{2}}\left[\chi_{+1}\chi_{-2} - \chi_{+2}\chi_{-1}\right]$

vi) $\psi_b(\mathbf{r}_1)\,\psi_b(\mathbf{r}_2)\,\frac{1}{\sqrt{2}}\left[\chi_{+1}\chi_{-2} - \chi_{+2}\chi_{-1}\right]$

overall antisymmetry of course.

spatially symmetric

spin antisymmetric

Question These spin states are ?

ANS. Singlets in the jargon.

$S^2 = S_1^2 + S_2^2$

eigenvalue S_i is $s(s+1)\hbar^2 = \frac{3}{4}\hbar^2$

$S^2 \chi_{\text{singlet}} = \frac{1}{\sqrt{2}}\left[\left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2\right)\chi_{+1}\chi_{-2}\right.$

(Gr-186)

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

$S^2 = S_1^2 + S_2^2 + \cancel{2}\,\mathbf{S}_1\cdot\mathbf{S}_2 + \mathbf{S}_2\cdot\mathbf{S}_1$

~~Matrices that don't in general commute.~~

but ~~$S_1\ \sqrt{s(s+1)}\,\hbar$~~

The matrices operate on different particles and so they commute.

i) & vi) fall into the same class

$$\psi_a(\mathbf{r}_1)\psi_a(\mathbf{r}_2)\frac{1}{\sqrt{2}}\left[\chi_{+1}\chi_{-2}-\chi_{+2}\chi_{-1}\right]$$

$$\psi_b(\mathbf{r}_1)\psi_b(\mathbf{r}_2)\frac{1}{\sqrt{2}}\left[\chi_{+1}\chi_{-2}-\chi_{+2}\chi_{-1}\right]$$

— both are symmetric in space & antisymmetric in spin & so overall antisymmetric as they should be for fermions.

[illegible]

$P_{+1,-2}=\frac{1}{2}$

$P_{-1,+2}=\frac{1}{2}$

Just square of coefficients. But since the particles are identical P(one up, one down) = 1/2 + 1/2 = 1

Question:

What kind spin states are they.

ANS. They are both singlet states in QM jargon. (Gr-185)

Ro

$$S^2 = (\mathbf{S}_1+\mathbf{S}_2) = S_1^2+S_2^2+2\mathbf{S}_1\cdot\mathbf{S}_2$$ No proof here.

$$S^2|\text{singlet}\rangle = 0$$

$S_z \equiv S_{z1}+S_{z2}$, $S_z|\text{singlet}\rangle = 0$ too.

5-228

We can't construct triplet states for the $\psi_a\psi_a$ & $\psi_b\psi_b$ spatial combinations since triplet states are symmetric.

(Gr-185)

The other four symmetrized states are interesting though tedious to examine

ii) $$\underbrace{\frac{1}{\sqrt{2}}\left[\psi_a(r_1)\psi_b(r_2) - \psi_a(r_2)\psi_b(r_1)\right]}_{\text{spatially antisymmetric}}\underbrace{\chi_{+1}\chi_{+2}}_{\text{spin symmetric}}$$

$\downarrow$

if $a=b$ this state would be zero and unphysical since it can't be normalized.

Quantum number $s=1$

$$S^2\chi_{+1}\chi_{+2} = 1(1+1)\hbar^2\chi_{+1}\chi_{+2} = 2\hbar^2\chi_{+1}\chi_{+2}$$

$$S_z\chi_{+1}\chi_{+2} = \underbrace{\hbar}_{\text{quantum number } m=1}\chi_{+1}\chi_{+2}$$

No proof here.

5-229

v) $\frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2) - \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\right]\chi_{-1}\chi_{-2}$

S^2 eigenvalue is $2\hbar^2$
with quantum number 1

S_z is $-\hbar$ with quantum number -1

So (ii) & (V) are 2 members of a triplet spin system.

but what of (iii) & (iv)?

iii) $\frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2)\chi_{+1}\chi_{-2} - \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\chi_{+2}\chi_{-1}\right]$

iv) $\frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2)\chi_{-1}\chi_{+2} - \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\chi_{-2}\chi_{+1}\right]$

They are NOT the same state. Look closely.
For example in (iii) you have $\psi_a(\underset{\sim}{r}_1)\chi_{+1}$
and $\psi_b(\underset{\sim}{r}_1)\chi_{-1}$

and in ~~the second~~ (iv) $\psi_a(\underset{\sim}{r}_1)\chi_{-1}$
and $\psi_b(\underset{\sim}{r}_2)\chi_{+1}$

5 - ~~2~~30

But they are NOT ~~singlet~~ triplet - singlet spin ~~states~~.

— they are the states ~~the~~ Slater determinant approach gave us.

We know what we'll need

But what if we ~~add~~/subtract and renormalize

(iii) + (iv) to get (vii) & (viii)

$$\frac{1}{2}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\Big[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2)\left[\chi_{+1}\chi_{-2} \pm \chi_{-1}\chi_{+2}\right]$$

$$-\psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\left[\chi_{+2}\chi_{-1} \pm \chi_{-2}\chi_{+1}\right]\Big]$$

~~(vi)~~

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2) \mp \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\right]\left[\chi_{+1}\chi_{-2} \pm \chi_{+2}\chi_{-1}\right]$$

$$= \underbrace{\frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2) \mp \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\right]}\ \underbrace{\frac{1}{\sqrt{2}}\left[\chi_{+1}\chi_{-2} \pm \chi_{+2}\chi_{-1}\right]} \qquad \begin{matrix}(vii)\\(viii)\end{matrix}$$

— upper case antisymmetric
— lower case symmetric.

upper case is the triplet s = 1, m = 0 state
lower case is the singlet s = 0, m = 0 state.

So now we have found a complete triplet-singlet set of states (ii), (v)

and linear combinations (vii) & (viii)

triplet

singlet.

But you ask which is physically correct ~~(i) (vi)~~ (iii) & (iv)

~~Well~~

or (vii) & (viii)?

~~Well they all~~

~~Well they both~~

Well both pairs are really.

(iii) & (iv) can be regarded as superpositions of (vii) & (viii)

just as (vii) & (viii) are superpositions of (iii) & (iv)

5-232

But say our original single particle states were exact stationary states

single particle Hamiltonian

$$H_i \psi_a \chi_+ = E_{a+}$$

$$H_i \psi_a \chi_- = E_{a-}$$

$$H_i \psi_b \chi_+ = E_{b+}$$

$$H_i \psi_b \chi_- = E_{b-}$$

Now say the two particles didn't interact at all,

$H = H_1 + H_2$ would then be exact.

All 6 Slater Determinant states are stationary states of H, Abbreviating the states to their labels we get

$$H(i) = (E_{a+} + E_{a-})(i)$$

$$H(ii) = (E_{a+} + E_{b+})(ii)$$

$$H(iii) = (E_{a+} + E_{b-})(iii)$$

$$H(iv) = (E_{a-} + E_{b+})(iv)$$

$$H(v) = (E_{a-} + E_{b-})(v)$$

$$H(vi) = (E_{b+} + E_{b-})(vi)$$

But

$$H\,\begin{matrix}(vii)\\(viii)\end{matrix} = (E_{a+} + E_{b-})\frac{1}{2}\frac{1}{\sqrt{2}}(iii)$$

$$\pm (E_{a-} + E_{b+})\frac{1}{2}\frac{1}{\sqrt{2}}(iv)$$

So (vii) + (viii) are NOT stationary states in this case, but just mixed states.

5-234 digression to p. 5-242

Nature somehow favours stationary states for reasons that are "unspeakable" (no textbook explans)

but I can sort of guess maybe.??

For pure unperturbed system both stationary states and mixed states are eternal. Nothing can change them.

e.g., the infinite square well.

But, in fact, all systems are subject to perturbations — For charged particles, there is always the zero-point energy of the electromagnetic field which is the cause of spontaneous emission (Gr-351).

other fluctuating perturbation from the environment exist too. — Thermal motions etc.

5-235

These perturbations can do two things if time dependent (as is the zero-point E-field — it fluctuates — so do thermal motions)

~~① since the & they are~~

① they can cause non-periodic change to the system.

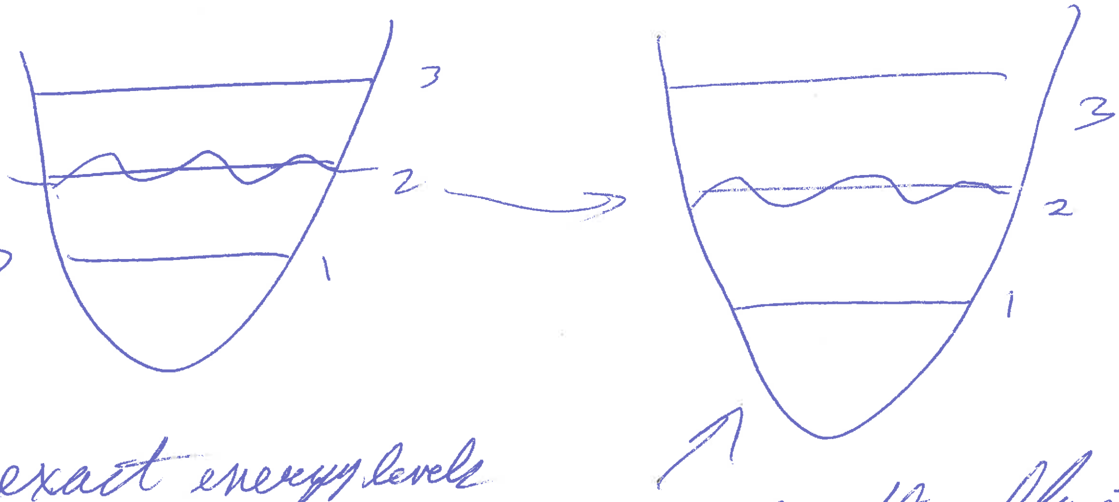
② they broaden stationary states from having exact energies to having an energy spread.

~~[illegible] I think there must be a spread in time? at any instant there is~~

How to explain this is also "unspeakable"!

But here's my best shot

5-236



Ideal potential well with exact energy levels & stationary states.

constantly fluctuating well.

~~Both~~

Say the system is in nearly stationary state j.

— the perturbations constantly change the allowed states both E_j and Ψ_j

Both fluctuating in time

— the system energy E is also always changing, but it's ~~never E~~ virtually never E_j exactly.

So the system isn't exactly in E_j at any instant.

The fluctuating in time set of ~~states~~ stationary states

Ψ_j with E_i

are mythical in a way — since the potential of perturbation is time dependent, one can't formally separate it in the Sch. eqn. and get stationary states.

But at any instant, one imagine it as a constant potential and separate the time of.

Thus at any instant

$$\Psi_{sys} = \sum_i c_i e^{-iE_it/\hbar} \Psi_i$$

where c_j is nearly 1 and all others nearly but not quite zero.

There are perturbed broadened states.

The system energy is being changed by fluctuation, so at any instant it's not in any of the mythical stationary states.

5-238

So at any instant

$$E = \langle \Psi_{sys} | H | \Psi_{sys} \rangle$$

and $\sigma_H = \sqrt{\langle \Psi_{sys} | H^2 | \Psi_s \rangle - \langle \Psi_{sys} | H | \Psi_{sys} \rangle^2}$

$$\geq 0$$

and usually greater than 0.

So the energy state is mixed but only a little.

$$E \hat{=} E_j \cong E_{j\,unperturbed}$$

but broadened to by σ_H.

The time energy uncertainty principle (Gr-115) (which is NOT like the ordinary QM uncertainty relation (Gr-111) — it has a different meaning) in

$$\Delta E \, \Delta t \geq \hbar/2$$ { a pure QM result based on non-commuting observables and Mixed states

or $\Delta t \geq \frac{\hbar}{2} \frac{1}{\Delta E}$

where $\Delta E = \sigma_H$ { standard deviation in energy (or Hamiltonian observable)

Δt is the characteristic time for other observables' expectation values (e.g., position, momentum, ang momentum, perturbation Hamiltonians) to change by 1 standard deviation.

— perturbations set Δt

For a mixed eigenstate (mixed original unperturbed eigenstates)

$\Delta E_{\text{Mixed}} >> \Delta E$ of single broadened state.

5-240

So $\Delta t_{\text{mixed}} << \Delta t_{\text{broadened}}$

Mixed states are eternal without non-periodic perturbations but with them their lifetimes are much shorter than merely broadened stationary states.

Maybe Rabi cycle and similar systems are true driven mixed states — but maybe the ordinary

In mixed states seem rare.

treatment in a semi-classical approximation only (Steinfeld 329) but that is unspeakable

atoms and molecules don't absorb to or emit from mixed states — or not much

if so they'd have a quasi-continuum of emission and absorption

They do at at a low level due to low probability events.

— So mixed states may simply not happen or very much.

— atoms & molecules
and other systems
with quantized states
have primarily <u>line
spectra</u>.

The <u>width of the lines</u> $\Delta E_{\text{line width}}$
is a measure of ΔE
(actual E — i.e. $\langle \Psi_{sys} | H | \Psi_{sys} \rangle$
at any one time is in.
and some sort of collapse
to somewhere in ΔE on
emission.
Of course, the lower
level has a ΔE too, but it
may be small → lower levels
~~are~~ tighter bound & less
perturbed)

The E values differ for different events and can actually stay rather constant. There is coherent scattering where $E_{\text{photon in}} \hat{=} E_{\text{photon out}}$.

For very short lived states,
their ~~[illegible]~~

5-242

~~there~~ mean lifetimes
are of order Δt
and $\Delta t \sim \frac{\hbar}{2} \frac{1}{\Delta E}$ line width.

I'm not sure if
all this back to p.5-234
is exactly right, but
it's my attempt to figure
out ~~what~~ why stationary
states (actually nearly
stationary states due perturbations)
are favored
over mixed states
(of near stationary states)

~~Notes~~ One of the unspeakables
of QM

To return to 2 particles in symmetrized states case.

If our original ~~states~~ single particle states were exact

(vii) + (viii) which complete the triplet-singlet are mixed states and disfavored.

But actually their is usually (always) an interaction between particles with spin.

They have magnetic moments (including the the neutron).

Maybe not neutrinos, but they have the weak interaction.

5-244a

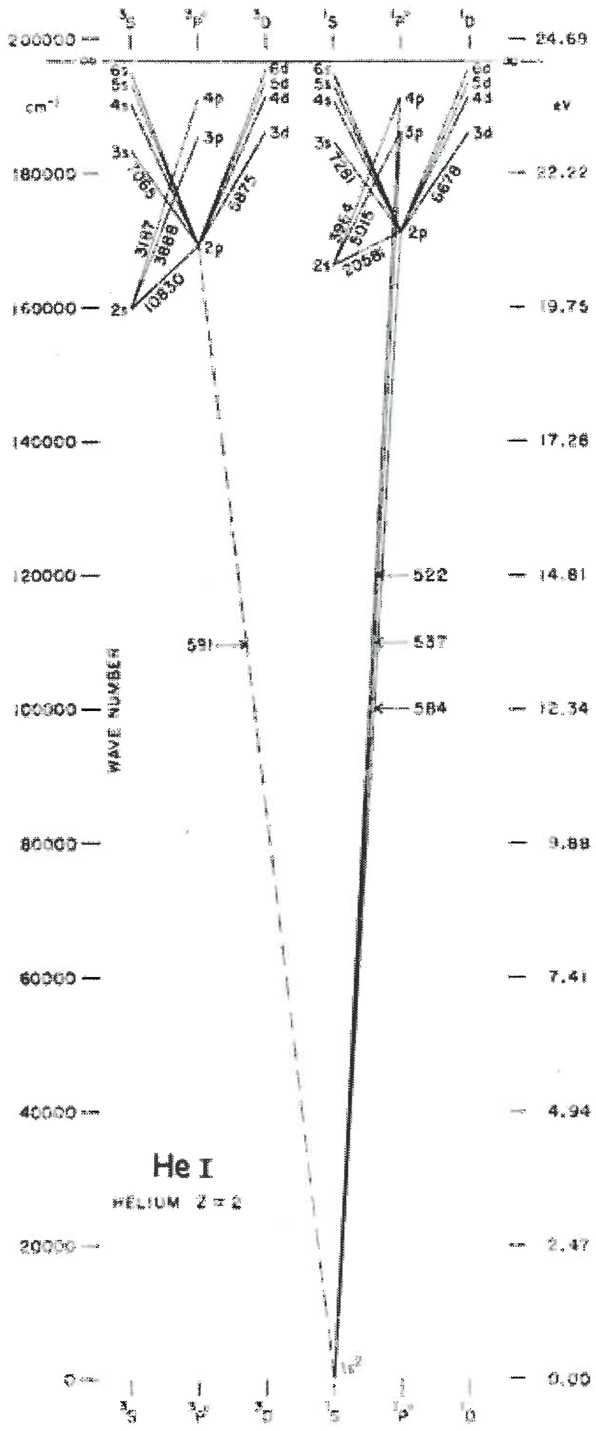
In which case
the two particles
have interactions that
align or ~~disalign~~
anti-align the spins

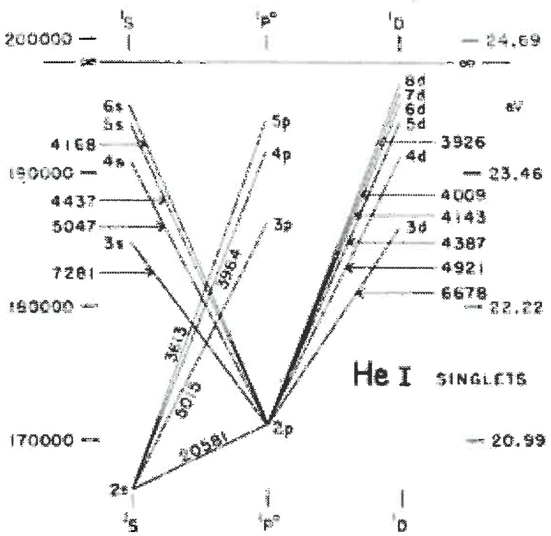
into
aligned
or
anti-align
stationary
states

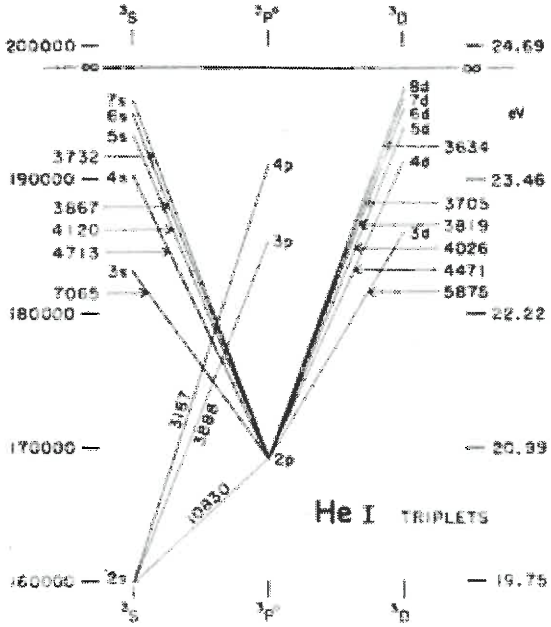
In which case the
states (vii) + (viii)
may be closer to the
trued that states (vii) + (iv)

and so the set (i), (ii), (v) (vi)
+ (vii) + (viii)
may be closer to the
true energy eigenstates.

→ it works out this way
for many atoms







From Moore + Merrill 1968

— still a useful set of Grotrian diagrams

— modern sets are often to messy with a million lines to contemplate.

5-244c

e.g., He I, C III, Mg I,

Ca I

N II, O III

I neutral

II once ionized

III twice ionized

etc.

— it seems like all/most common atomic species with only 2 electrons in the outermost subshell

set by quantum number n, l

"in" in the sense ~~of all the~~ that these states included in symmetrized ground state.

Whether atom is in a ~~[illegible]~~ triplet or singlet state depends on the excitation.

~~B~~- 246

~~Strong~~ Applied external magnetic fields could change energy-eigenstates ~~but~~ back to ~~[illegible]~~ being closer to (i), (ii), (iii), (iv), (v) (vi).

⑨ Symmetrized Boson states for identical bosons

For bosons the overall state has to be symmetric

Example Say we have two single particle states

(stationary states) $\psi_a(\underset{\sim}{r})$ & $\psi_b(\underset{\sim}{r})$ for spin zero bosons,

$\Big\}$ orthonormalized of course.

Product states which are ~~no~~ mathematical solutions of the 2-particle Sch. eqn.

are

$$\psi_a(\underset{\sim}{r}_1)\,\psi_b(\underset{\sim}{r}_2)$$

$$\psi_a(\underset{\sim}{r}_2)\,\psi_b(\underset{\sim}{r}_1)$$

Mathematically distinct treating coordinates $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ as distinct coordinates.

But physically the particles are identical and so the states are.

To avoid exchange degeneracy (where different physical states are superpositions of the same physical state)

5-248

the symmetrization principle is invoked.

$$\therefore \quad \psi_{sym} = \frac{1}{\sqrt{2}}\left[\psi_a(\underset{\sim}{r}_1)\psi_b(\underset{\sim}{r}_2) + \psi_a(\underset{\sim}{r}_2)\psi_b(\underset{\sim}{r}_1)\right]$$

where we assume $\psi_a \neq \psi_b$

— so the states are distinct.

— if not distinct $\psi_a(\underset{\sim}{r}_1)\psi_a(\underset{\sim}{r}_2)$ is already symmetrized.

No Pauli exclusion principle in this case

$$\underset{\sim}{r}_1 = \underset{\sim}{r}_2 = \underset{\sim}{r}_{particular}$$

and ψ is not zero

Actually there is a bunching effect

relative to the product states and fermion antisymmetric states

$$\Psi_{sym}(r_1 = r_{part}, r_2 = r_{part})$$

$$= \sqrt{2}\,\Psi_a(r_{part})\,\Psi_b(r_{part})$$

$\uparrow$ greater by $\sqrt{2}$ at $r_1 = r_2 = r_{part}$ compared to product state $\Psi_a(r_{part})\,\Psi_b(r_{part})$

$P_{sym} = 2P_{product}$ for $r_1 = r_2 = r_{part}$.

Since Ψ_{sym} is still normalized, if it is larger when $r_1 \simeq r_2$ it must tend to be s~~maller~~

5-250

~~when $\mathbf{r}_1$ and $\mathbf{r}_2$ are somehow far apart.~~

smaller probability densities in some places than the product states.

Consider

$$\rho_{\text{product}} = |\psi_a(\mathbf{r}_1)|^2 \, |\psi_b(\mathbf{r}_2)|^2$$

is the simple product probability density

$$\rho_{\text{sym}} = \frac{1}{2}\Big[|\psi_a(\mathbf{r}_1)|^2 |\psi_b(\mathbf{r}_2)|^2 + |\psi_a(\mathbf{r}_2)|^2 |\psi_b(\mathbf{r}_1)|^2 + 2\,\mathrm{Re}\big(\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)\psi_a(\mathbf{r}_2)^*\psi_b(\mathbf{r}_1)\big)\Big]$$

say
there
were
$\sim \rho_{\text{product}}$

this term could be positive or negative

OK, the argument wanders off indecisively.

What if you had N bosons and N distinct single-particle states?

Well a modified Slater determinant will give a symmetrized state } see p. 5-216

$$\Psi_{sym} = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(1') & \psi_2(1') & \cdots & \psi_N(1') \\ \psi_1(2') & & & \\ \psi_1(3') & & & \\ \vdots & & & \\ \psi_1(N') & \cdots & & \psi_N(N') \end{vmatrix}$$

— but all minus signs suppressed.

What if some of states are

B-252

the same?

Well the construction of a fully symmetrized state is trickier and I draw the line at going into that.

Nothing forbids all the <u>bosons</u> from being in 1 single particle state

$$\Psi_{sym} = \underbrace{\psi_a(\underset{\sim}{r}_1)\,\psi_a(\underset{\sim}{r}_2)\ldots\psi_a(\underset{\sim}{r}_N)}_{\text{already symmetrized}}$$

— so No ~~state exc~~ Pauli exclusion principle either.

All bosons can go into one single-particle state.

But this is unlikely except at very low temperature where all or a large fraction of bosons can go into the ground state or a low excitation state.

↳ a Bose-Einstein condensate which I'm not going to discuss much

But real Bose-Einstein condensate can't be simple things } They can be made

→ The particles are composite particles

and always interact

Consider He-4 { He-4 superfluid is certainly a non-simple manifestation of Bose-Einstein condensation.

The overall state must be symmetric for exchange of He-4's

— 2p, 2n, 2e

So an even number of fermions

But for exchange of any component ~~particle~~ the wave function must be antisymmetric

→ ~~So~~ the probability density of finding all components of He-4 in same place with same spin is zero — just from symmetrization principle.

Any He-4's repel each other ~~if you~~ by real forces if

try to segregate ~~them together~~
so ~~they do~~ their electron
clouds together.

They are interacting particles.

<u>But</u> because they are overall
Bosons, somehow a macroscopic
amount of them at very
low temperature can
go into <u>something like</u> the same
center-of-mass state.

something like because
up close they repel each other
& their <u>fermion components</u>
~~are~~ must be antisymmetrized

but ~~they~~ at some larger
distance must attract
each other to be a
fluid at all.

— discuss~~ion~~ breaks off indecisively again.

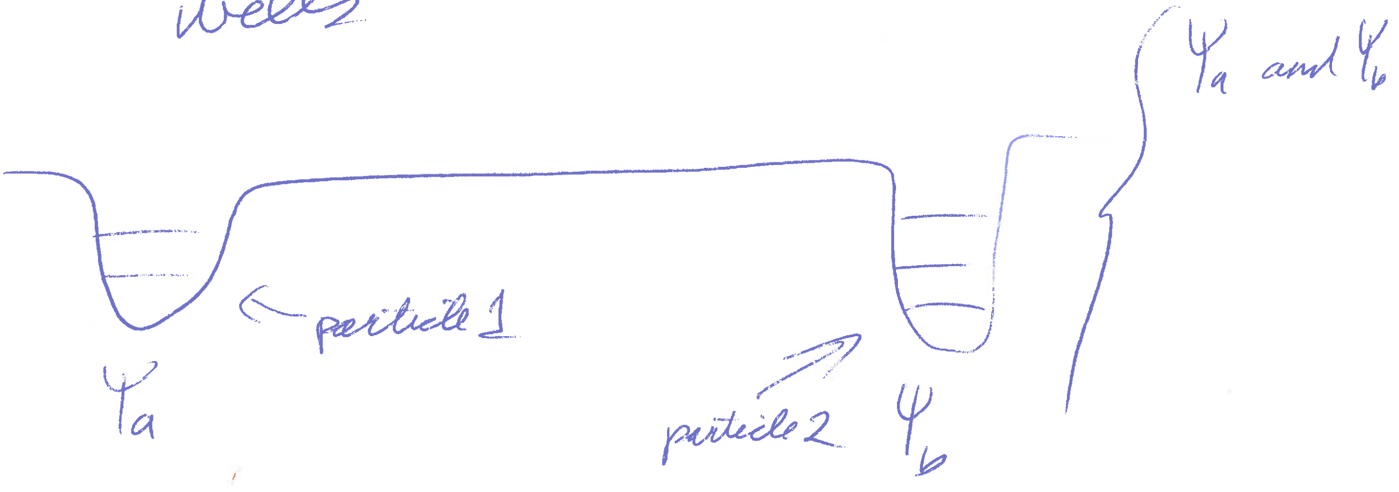
[illegible]-256

(10)

Do we Have to symmetrise the wave function of Identical Particles for whole Universe?

No, but we won't do a full proof, just a special case proof. (I'd like to see a simple general proof but that's unspeakable)

Say we have 2 identical particles in well separated wells



We initially solve them as separate systems.

But then we say, we've got to symmetrize.

~~$\Psi_{sym} = \frac{1}{\sqrt{2}}[\psi_a(1)\,\psi_1$~~

First a general proof of an interesting point

Say $\Psi(1, 2, \ldots, N)$ is completely symmetrized

$\uparrow \quad \uparrow$ stands for all coordinates of particle 1, 2, etc space and spin.

a stands for 1st slot

$$\rho_a(1) = \int |\Psi(1, 2, \ldots, N)|^2 d2\, d3, \ldots dN$$

using symmetries

$$= \int |\Psi(2, 1, \ldots N)|^2 d2\, d3 \ldots dN$$

by relabeling

$$= \int |\Psi(1', 1, \ldots N)|^2 d1'\, d3 \ldots dN$$

$$= \rho_b(2)$$

~~2 is just~~ a ~~label~~ ~~for the coordinate~~ in the 2 slot.

There's nothing special about ~~1 and 2~~ slots a and b ~~particles 1 and 2~~.

So we conclude

ρ_i ~~$\rho(1)$~~ is the same for all particles i (or all slots i)

Now to return to our two well separated well system.

$$\psi(1,2) = A\left[\psi_a(1)\,\psi_b(2) \pm \psi_a(2)\,\psi_b(1)\right]$$

States ψ_a and ψ_b are NOT necessarily orthogonal but in the limit the states do NOT overlap at all,

we consider both fermions and boson cases

$$A \rightarrow \frac{1}{\sqrt{2}}$$

Now

$$P(1,2) = A^2 \Big[|\psi_a(1)|^2 |\psi_b(2)|^2 + |\psi_a(2)|^2 |\psi_b(1)|^2 \pm 2\,\mathrm{Re}\big[\psi_a(1)\,\psi_b(2)\,\psi_a(2)^*\,\psi_b(1)^*\big] \Big]$$

$$P(1) = A^2 \Big[|\psi_a(1)|^2 + |\psi_b(1)|^2 \pm 2\,\mathrm{Re}\big[\psi_a(1)\,\psi_b(1)^* \int \psi_a(2)^*\,\psi_b(2)\, d2\big] \Big]$$

= [illegible] by general proof on p. 5-257]

$P(2) =$ ...... same thing with labels 1 and 2 interchanged.

B-260

If go to the limit where ψ_a and ψ_b have zero overlap

$$A^2 \Rightarrow \frac{1}{2}$$

The cross term vanishes

$$\rho(1) = \frac{1}{2}\left[\left|\psi_a(r_1 m_1)\right|^2 + \left|\psi_b(r_1 m_1)\right|^2\right]$$

$$\rho(2) = \frac{1}{2}\left[\left|\psi_a(r_2 m_2)\right|^2 + \left|\psi_b(r_2 m_2)\right|^2\right]$$

If evaluate r_1 and r_2 in the vicinity of well a then $\left|\psi_b(r_1 m_1)\right|^2$ and $\left|\psi_b(r_2 m_2)\right|^2$ are zero.

Not probability density of finding one particle.

The average particle density r in ~~state~~ spin state m in vicinity of well a is

$$P(1) + P(2)$$

$$= \left| \Psi_a(\underset{\sim}{v}, m) \right|^2$$

which is just what you would've gotten without <u>symmetrization</u>.

~~So of th~~

This generalizes (though how the proof goes I don't know).

If system states have no overlap, then ~~system~~ symmetrization is <u>not needed</u>.

But it's not all or nothing.

The smaller the overlap of separately solved states, the less symmetrization is needed.

So we don't have to symmetrization a solution for a particular system with all systems in universe.

— On the other hand nature has hid from us if the universal wavefunction one kind of identical particles is symmetrized

Y-263

on if symmetrization
magically turns
on when systems
get close to each
other.

11) Hartree-Fock Method

(ER-319)

For their ground states, Not excited states

This is how atoms
were solved traditionally
and it still used (I think)
but for higher accuracy
advanced many-body techniques
are used (Lindmilla).

— It uses the mean potential
approximation at all iteration
steps and that is never
exactly right.

5-264

The HF method is an iteration to a self-consistent solution.

There is actually no ~~gu~~ mathematical guarantee of convergence I guess, but, in fact, convergence is obtained without difficulty

For atom atomic number Z, you assume an initial ~~mean~~ mean potential

$$V = \begin{cases} -\dfrac{Ze^2}{4\pi\varepsilon_0 r} & \text{for } r \text{ small near nucleus assumed to be a point.} \\ \\ -\dfrac{e^2}{4\pi\varepsilon_0 r} & \text{for larger } r \text{ near assumed surface} \end{cases}$$

— the inner electron screen the nucleus.

The (Z-1) other electrons screen (Z-1) of the positive charges of nucleus