INTRODUCTION TO INTRODUCTORY PHYSICS

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ABSTRACT

Oh, the poetical-tragical-comical-historical-pastoral introduction to physics. A word on systems, environments, models, and idealization. Some stuff on quantities, base units, and other units. Two words on dimensional analysis. A bit on significant figures. Unit conversions, order-of-magnitude calculations, 1-digit calculations, and no more.

Subject headings: keywords — units — base units — dimensional analysis — conversions — order of magnitude estimation — significant figures

1. INTRODUCTION TO THE INTRODUCTION

To be brief, **PHYSICS** is the science of matter and motion.

To be slightly less brief, **PHYSICS** is the science of matter and motion, time and space.

It's an empirical science: it studies things in the physical world that can be known through observation and experience. Maybe all sciences are really empirical—but usually one says pure math isn't since it studies abstract relationships—but those relationships exist to be discovered and are discovered by experience.

PHYSICS in its modern form is a mathematical science. This was not always true. From the Greek Pre-Socratic philosophers (circa 600–400 BCE) until circa 1600, physics was considered by most scholars to be a qualitative science that gave a philosophically reasonable account of matter and motion, time and space. In the Medieval Islamic and European contexts, the physics of Aristotle (384–322 BCE)—Aristotelian physics—was considered the dominant version. But even in ancient times, some mathematical physics existed starting with Archimedes (circa 287–212 BCE). Thereafter, only small additions were made to mathematical physics until the time of Galileo (1564–1642) whose life coincided with the Scientific Revolution. The work of Galileo, others, and Newton made physics deeply mathematical science.

Nowadays, much of **PHYSICS** is embodied in mathematical laws. These laws relate the physical quantities—which are things that can be measured or calculated. The laws allow us to understand systems: i.e., predict their nature and their past and future evolution.

A system is any particular set of objects which we are studying at the moment—but we'll elaborate more on systems in § 4.

To be a physical law, a formula must apply to a wide variety of cases. In fact, one usually says it must apply nearly exactly in some well defined realm of physics. From physical laws, many general formulae are derivable and infinitely many special case formulae.

There is no physical law or general formula about this chair—the one in this room—sitting on the floor. But there are physical laws to describe the forces on it and why it is at rest and how it would move if various forces were applied to it.

Understanding particular systems involves using general formulae and the peculiar features of the systems. These peculiar features are often called boundary conditions or initial conditions (which are actually boundary conditions in the time dimension).

The ultimate goal of physics—well one of them anyway—is to achieve a true—a truly true—exact—exactly exact—theory of everything—TOE as we call it sometimes. We don't have TOE yet—maybe tomorrow, maybe next year, maybe never.

TOE would be the fundamental physical law. From TOE all known physical laws and general formulae could be derived. The faith is that it will in a simple theory in that it will have very few basic fundamental laws or axioms and few or even no free parameters. It's not likely to be simple in an everyday sense—you'd likely need three years of study to understand it.

Physical laws which can be derived are not fundamental physical laws—in the most fundamental sense of fundamental.

But the term "fundamental" is used in various senses. In a first sense, only TOE is fundamental—or whatever supercedes TOE. In second sense, the currently known most basic laws are fundamental and they will cease to be fundamental if superceded—the standard model of particle physics is an example of current fundamental law. In a third sense, fundamental can be used to describe laws that fully describe some limited realm of physics—Newton laws of motion are fundamental inside Newtonian physics. Newtonian physics was once considered fundamental in the second sense and was a candidate for being fundamental in the first sense—but those days are no more.

Context must decide what meaning of "fundamental."

Physics which is not fundamental physics can be called applied physics.

But the term applied physics is used variously too. Some would only call physics directly used in some technology applied physics.

2. EMERGENT PRINCIPLES

In my view, TOE is a misnomer.

It is not a theory of everything, it's not even a theory of all physics.

It's a theory about the most basic features of matter and motion.

When I was a naive boy physicist, I used to think that all things could be reduced to fundamental physics or TOE.

But philosophically I've moved away from **REDUCTIONISM** to embrace **EMERGENCE**—of which I've only got a vague understanding too—I really should read the Wikipedia article and not just look at the picture of the termite cathedral.

The notion—as I think of it—is that there are principles that transcend physics—these are emergent principles.

For a trivial example, the game of chess. The board and pieces could be made out of anything at all or nothing and yet the game can be played. Similarly, things like biology, evolution, consciousness, and economics could play out in worlds with quite different fundamental physics—at least that is possible as far as we know.

As I hinted above, even some of what is usually considered physics actually involves principles that are not uniquely embedded in TOE.

The entropy concept and the 2nd law of thermodynamics (which makes use of the entropy concept) can exist in worlds with different fundamental physics. Virtually all physicists believe this I believe. Also you can even demonstrate it with toy worlds created on a computer.

I don't think there is any hard line as one builds up from systems only needing fundamental physics—which are ideal systems anyway—to more complicated systems that also need emergent principles. As systems get more complicated more emergent principles are needed to understand them.

After some level in the building up process–high up in the hierarchy of science—one simply stops calling what one does physics for reasons of tradition or because emergent principles have become too dominant.

For example solid state physics is traditionally physics and would usually be called a branch of applied physics. But solid state physics deeply involves chemistry which it not traditionally considered physics. But chemistry is so close to traditional physics that some would say it is applied physics. But since it's practitioners don't call it that neither should we.

Once one gets to biology and above, clearly physics has been left behind. And yet, of course, biology involves physics and there is a field of biophysics which studies the physics of biological systems—biology is too important to be left to biologists. But emergent principles—like evolution—become so important in biology that biology clearly cannot be subsumed under physics.

The same true for other sciences higher in the hierarchy than biology. They need physics, but clearly are not reducible to physics.

3. CLASSICAL PHYSICS AND MODERN PHYSICS

Classical physics is essentially the well-established physics current circa 1900 or a little before.

Modern physics in one sense is all the physics that developed since that time.

In a second sense modern physics is the physics that developed only in the time frame from circa 1900 to circa 1960—it's modern in the sense that Picasso is a modern painter and Hemingway, a modern writer. The second sense is largely used in pedagogy—there are courses and textbooks that are modern in the second sense.

Loosely speaking classical physics consists of Newtonian physics (which is the primary topic of a 1st semester of intro physics), classical or Maxwellian electrodynamics, and classical thermodynamics.

Introductory physics courses are largely about classical physics with some bits of modern physics added.

One would have to be an obscurantist purist not to include bits of modern physics even when teaching essentially classical physics. We know there are electrons, protons, neutrons, atoms, and molecules, and that many things can only be adequately explained in terms of them and other bits of modern physics. So we do introduce bits of modern physics as helpful explanations even in topics that are mostly classical. And, of course, modern physics topics at some level can be included in introductory physics and some introductory physics textbooks include them—at least in extended versions.

Loosely speaking modern physics consists of quantum mechanics (including quantum field theory), special relativity general relativity, and more esoteric realms we will not mention.

Quantum mechanics is needed to understand small systems: molecular size and smaller. Such small systems are called microscopic in physics jargon—even though they are much smaller than can be observed with a traditional optical microscope. Above the microscopic realm is the macroscopic realm. Sometimes people insert a mesoscopic realm in between. There are no hard boundaries between these realms. People use the terms microscopic, mesoscopic, and macroscopic loosely. They are not intended to be strictly defined. Every field needs some elastic terminology

Special relativity is needed to understand systems with relative speeds approaching the vacuum speed of light. The vacuum speed of light is the ultimate physical speed—the highest speed at which information can be transferred. Actually, some qualification is needed to this description of the vacuum speed of light because of quantum mechanical effects—but we won't go into all that. Special relativity upsets some of our classical physics notions. Most notably time flow and length become dependent on the frame of reference in which they are measured. This leads to some very striking violations of the everyday sense about how reality works. But those violations are experimentally verified. We just don't observe them in everyday life because in everyday life relative speeds are usually much smaller than the vacuum speed of light—except for the speed of light itself—but that's a special case.

General relativity is essentially a theory of gravity and is needed to understand systems with strong gravity (like black holes) and the cosmos as a whole.

The realm where modern physics is not needed for an adequate understanding is the realm of classical physics. The center of this realm is can be called the classical limit in which all classical physics is exactly true. The classical limit can't actually be reached in nature and one can't come arbitrarily close to in all senses: getting larger and larger takes you further from the quantum mechanical realm, but brings you closer to the cosmological realm. But between all the modern physics realms, one can still still imagine the classical limit as an ideal point where all of classical physics holds exactly.

There are no hard lines to the realms.

As you move away from the classical limit—going too small, too fast, too strong a gravity, too big—classical physics progressively fails.

But within a broad region of behavior classical physics is entirely adequate—which means that errors due to the failure of classical physics are negligible compared to the errors in measurements. Classical physics can be called a true approximate theory. Within it's realm of validity it's never wrong and no one believes it will be proven wrong in any circumstance. Perhaps, we will never know the true fundamental theory, but we can know true approximate theories.

Since classical physics is usually much easier to deal with than modern physics, one uses classical physics in the realm of classical physics.

Because it's eminently useful and much simpler than modern physics, one teaches mostly classical physics first. For many people, particularly many engineering specialities, classical physics is all one needs in one's professional work.

Many concepts of classical physics get used or generalized in modern physics: this is another reason for studying classical physics first.

The classical limit of modern physics is classical physics although I believe most people say this has not been completely proven. There is still controversy about how classical behavior emerges from quantum mechanics.

Modern physics is, of course, not complete: i.e., it's not the fundamental theory—at least we have strong reasons for believing that. One of the strongest reasons is that general relativity is not consistent quantum mechanics. Quantum mechanics is such a powerful, well verified theory that is difficult to believe that it is fundamentally wrong. So the belief is that there is a quantum theory of gravity of which general relativity is the macroscopic limit or at least an approximation to that limit. Quantum theories of gravity exist, but none have yet become well established.

3.1. Physics at the End of the 19th Century: Optional

Toward the end of the 19th century thought that physics was nearly complete. One prize witness is Lord Kelvin (1824–1907), who in 1900 said:

> "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."

Kelvin was one of the great developers of classical thermodynamics—but in 1900, he was no longer in his prime.

How general the end-of-physics thinking was I don't know.

But I find it hard to believe that it could have been general among the brightest, active scientists.

Classical physics does not explain the properties of materials: e.g., all chemical properties. There are jillions of material properties and in a classical physics perspective (as we now understand it) they are all just givens: fundamental properties. But the plethora of fundamental properties conflicts with idea current in physics from the time of the ancient Greeks that fundamental physics should somehow be simple with a limited number of axioms required to explain everything. So I think that many 1900 physicists must have believed that there was a lot more physics to come.

There are other reasons for thinking this. It was appreciated—as it always should be—that outside of the realm in which theories were experimentally verified, they could fail. There were also particular problems with classical physics. A striking one is that Maxwellian electrodynamics is inconsistent with Newtonian physics. Some may have thought this was a minor problem that had a simple solution. But it turned out that special relativity was needed to replace Newtonian physics: special relativity and Maxwellian electrodynamics are consistent.

If we ever found the fundamental physical theory that would be the end of fundamental physics. But how would we know it when we found it?

4. SYSTEMS, ENVIRONMENTS, MODELS, IDEALIZATION

As discussed in § 1, a **SYSTEM** is any set of objects that one is studying.

Everything else is the **ENVIRONMENT** in physics jargon.

Really in science jargon in general since the system-environment concept is everywhere including in everyday life.

Since the system is the subject of study, one only needs to understand the environment insofar as it affects the system. This is a great simplification—one doesn't have to know everything in the **UNIVERSE** to know something about a small part.

If the environment cannot affect the system at all, the system is a closed system: otherwise it is an open system.

Between the system and the environment is the boundary. The boundary may be a real physical surface or change of conditions or may just be an arbitrary surface we define in space.

Actual real systems are often immensely complex at least in detail.

But often much of that complexity is beyond what you want to know about the system.

So the system can be approximated by a model that is simpler than the actual reality, but accurate enough—realistic enough—to yield what you what to know about the system. The environment can likewise be modeled.

In fact, it's true to say that you must always model real systems and environments. The model can be very simple or it can be very complex and, one hopes, very realistic if very complex. You can always approach complex reality more and more closely in principle. Usually, the model for the environment can be simpler than for the system. For a closed system, you don't need to model the environment at all.

You hope that at some point your model is realistic enough to give you the understanding of the accuracy you desire.

The process of modeling is often one of idealization. You study an ideal system that lacks many of the complications of the real system.

For example, in intro physics, we often neglect resistive forces like friction and air resistance. Those forces are often hard to deal with.

For another example, the completely closed system is an idealization. No system is ever really completely detached from the rest of the universe.

In intro physics, the level of idealization is often very high. This is to make problems simple enough that they are tractable for intro physics students—and instructors too.

The idealized problems illustrate physical law which is one of their main functions. In some cases, the calculated solutions will be quite accurate. In other cases, the calculated solutions will not be all that accurate.

But usually you can't solve more realistic problems, until you can solve the ideal ones.

One particular idealization we will often make is to model finite objects as particles. The term "particle" does not mean small in this usage: it means that we can neglect the internal structure of the object. So a car, a human, and the ever-popular nondescript block can all be regarded as particles.

The particle idealization is actually very realistic for certain, but not all, purposes. The justification for the realism of the particle idealization is the use of the concept of center of mass: this turns up in the lecture **SYSTEMS OF PARTICLES AND MOMENTUM**.

5. QUANTITIES AND UNITS

Much of physics is in the relationships between between physical quantities. The relationships may be physical laws or formulae derived from physical laws.

Physical quantities are measurable or calculable things of relevance in physics.

Now some quantities are discrete: i.e. they come in discrete amounts. Those quantities can be measured exactly.

For a non-physical example, consider sheep. If there are three sheep in a field, then there are exactly three sheep in the field.

For an example from physics, the number of atoms in a container is a discrete quantity. It may be hard to measure how many atoms are there, but there is an exact number of them. Counts of this nature can actually be done in some cases.

But many quantities are continuous at least at the macroscopic level. By continuous, we mean that the quantity can have any real number value: e.g., length and time.

For discrete quantities, the discrete amounts themselves form natural standard units for measuring the quantity. So there is no problem in principle.

But there is a problem with atoms, molecules, and other microscopic entitites at the macroscopic level. These entities are discrete, but we usually can't count them exactly at the macroscopic level. The problem is partially dealt with the concepts of amount of substance and moles. The problem is also partially dealt with by making the continuum approximation for microscopically discrete entites. We assume that they do come in a continuous quantity.

But then how do we deal with continuous quantities that can't be counted exactly in practice in the real world.

Nature has not been altogether kind. It has not provided us with standard objects at the macroscopic level that are absolutely identical. We now this empirically and in modern physics we theoretically understand why it must be true. Therefore there is no macroscopic object that can be used as an ideal standard unit for any continuous quantity.

In the past people agreed on some object or thing that provided a standard unit. In the long ago past that standard object was only very approximately standard.

Human body parts have long been used to define standard units. For example, the human foot has probably been used as measure of length since long ago in prehistory. Since most humans have a foot that is not very different in length from the typical human foot length, using an actual foot as measuring device is not so bad—we still sometimes use it that way. One person's ten feet is near enough to another's for many practical purposes.

But as society became more complex demands for precision increased and more standardized standard objects than any random person's foot was needed. Eventually, a standard foot became defined. The modern American foot is defined to be exactly 1200/3937 meters (Wikipedia: Foot (length)).

The need for standard units leads us to consider the Système International (SI) or the metric system of units. This was first arose in Revolutionary France in the 1790s and has been much developed since then (Wikipedia: International System of Units).

The SI system is the standard unit system for all of modern science, much of modern engineering, and for standard use in all but three countries: Liberia, Myanmar, and the United States of America (Wikipedia: International System of Units).

There are SI units for all standard physical quantities. But only seven of these units are base units. Base units are those for which we require a standard procedure to experimentally determine them. The procedure is in fact the definition of these base units. We will consider discuss the base units below—but some of them only optionally.

The program of modern metrology is to find base unit definitions that in theory are exact so that the base unit never varies in size in theory. Except for mass (as we discuss below in § 5.2), the modern program has been fulfilled. Actual experimental determinations of base units can never be exact at least at the macroscopic level even if the base units are exact in theory, but there is no macroscopic theoretical limit to how accurately they can be determined if they are exact in theory.

You can always try to determine them more accurately in practice than has been done before.

Of course, most measurements with the base units (and all other units) do not use the exact base unit definition directly, but use some measuring device that is calibrated at some remove by the exact base unit definition.

The SI base units are given in Table 1.

Table 1. SI Base Units

Name	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	Kelvin or thermodynamic temperature
mole	mol	amount of substance
candela	cd	luminous intensity

Note. — The entries are from Wikipedia's article "SI Base Unit" which also gives the base unit definitions.

What of the fiducial (i.e., reference) units for quantities that do not have base units?

Exact physical formulae relate those quantities to the quantities that have base units.

So the fiducial units for those quantities without base units can be related exactly to the base units.

The relationships give the non-base fiducial units in terms of the base units. We say that the non-base fiducial units are derived units.

For example, speed is the ratio of length to the time it takes an object to traverse that length. So the SI unit of speed is the meter per second or m/s.

Some quantities without base units have special names (and also special symbols, of course) for their fiducial units. Speed is an example of quantity which does **NOT** have a special name for it's fiducial unit. The speed unit is just the m/s.

An example quantity whose fiducial unit does have a special name is energy. The fiducial unit is the joule with symbol J. The joule is a $\mathrm{kg\,m^2/s^2}$. We get to energy and joules in the lecture **ENERGY**.

The fiducial units (both base and derived) are actually called the MKS units where MKS stands for meters kilograms seconds. The MKS units are a subset of all SI units as we'll discuss below.

Why do we have just 7 base units?

Because all standard physical quantities can be measured in those base units or in the units derived from them.

Are there quantities that can't be measured in the units of the standard physical quantities?

Sure. For a super-trivial example, the amount of oranges in pile. One would naturally measure this amount in the unit of an orange. But the units for such special quantities have no general utility. For example, the amount of oranges in most systems is obviously zero.

Are the 7 base units unique choices?

No. Some of the fiducial derived units could be accepted by convention as base units replacing some of the current base units. The replaced base units could then be derived from the new set of base units. For example, the unit of current the ampere is a base unit. A unit derived from it is the unit of charge, coulomb (C), which is an ampere times a second (A s). One could define the coulomb as a base unit and then the ampere would be a derived unit: the Coulomb per second (C/s).

Why do we use the 7 base units that we do if their choice is not dictated by physical principle?

The current 7 base units have definitions that are judged in some sense to be convenient for accurate and precise measurement of the base units. It's possible that the choices may change in the future.

Who governs the SI system? The organizations created by the Metre Convention (Wikipedia: Metre Convention)—and it's metre, not meter. The US is a signer of the convention and so are most other large countries (and some minor ones too). I imagine the national bureaus of standards of these countries are involved in these organizations. The US national bureau of standards is National Institute of Standards and Technology (NIST)—which until 1988 was conveniently called the National Bureau of Standards (NBS).

As well as the MKS units, SI includes units that are standard multiples of powers of ten of these units. The standard multiples are denoted by standard prefixes given to the fiducial unit. The standard multiples and prefixes are given in Table 2. Units created by standard multiples are also derived units.

Table 2. SI Prefixes for Powers of Ten

Power of Ten	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	a
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{0}		
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

Note. — There are plans a foot to extend the standard powers of ten to include 10^{-27} (prefix: harpo abbreviated ha) and 10^{27} (prefix: groucho abbreviated Gr). But this is still controversial.

One anomaly with the powers-of-ten units is that the prefixed kilogram is the fiducial unit of mass and not the unprefixed gram. Another anomaly is that 10^3 kg is usually called a tonne (or metric ton) and not a megagram (Mg).

Not all the prefixed units are in common use. A prefixed unit tends to be used in fields where it is a convenient size for the quantities of that field. For example, in nanotechnology the nanometer (nm) is commonly used.

In fact, the prefixed units are often only used for mental convenience in thinking about quantities, not in calculations. In most calculations, one uses only MKS units. The reason is that one doesn't have to do any unit conversions (see § 7) or keep track of units. If only MKS units are used with the inputs, then all the results of the calculations will be in MKS units.

If you have a problem where the given values are not in MKS units, the usual procedure is to convert them to MKS units before doing anything else and then if necessary convert any final results to any units one wants to see them in.

In some fields one actually uses another subset of SI units in calculations instead of MKS. The other subset is the CGS units where CGS stands for centimeters grams seconds. CGS units are commonly used in astronomy—and so I'm pretty familiar with them.

As well as official SI units, some other units are in common scientific use. The first example that comes to mind is the angstrom (Å) which is 10^{-10} m. It has a continuing vogue because the atoms have a size scale that is of order 1 Å. Another example is that many fields the Celsius temperature scale is used rather than the Kelvin temperature scale which is the official SI temperature scale. The Celsius and Kelvin degrees have the same size, but the zero temperature is different for the two scales: in the Celsius scale, it is approximately the freezing point of water and in the Kelvin scale, absolute zero.

Of course, special units are used all the time for special purposes. For example, in astronomy, the Sun's mass is a convenient unit for measuring stellar masses. This use is mainly for thinking about stellar masses. For calculations, one almost always uses an SI mass unit, the kilogram or (if you are astronomer) the gram. In fact, many special units are used primarily for convenience in thinking about quantities.

There are non-SI conventional unit systems, but mostly they have fallen out of general or primary use. The great exception is that for everyday and many commercial purposes, the US uses United States Customary System which in the US we usually call British units since they are approximately the units the British used to use (Wikipedia: United States customary units). You know about these—feet, furlongs, hogsheads and so on. Almost no one else in the world uses British (or British-like) units anymore, except for minor purposes. However, Liberia and Myanmar apparently still use them as their primary units (Wikipedia: International System of Units).

Actually, the US is supposedly abandoning British units for SI—a process called metrication. The Omnibus Trade and Competitiveness Act of 1988, says that the United States government designates the metric system of measurement as "the preferred system of weights and measures for U.S. trade and commerce". But the metrication process is proceeding so slowly that to most folks it seems a whole lot like no progress at all.

We could at least get rid of Fahrenheit and use the Celsius scale—or better yet the Kelvin scale.

In the following subsections, we discuss the quantities length, mass, and time and their base units. Optional subsections on the other quantities with base units and their base units also follow.

5.1. Length

Length is such a basic category of our existence that defies easy definition.

One can say that length is extension along a curve in space. Maybe that helps.

We usually think of space as being the 3-dimensional Euclidean space of Euclidean geometry. The space of universe over short distances and far from strong sources of gravity approximates Euclidean space to high accuracy.

The SI base unit of length is the meter.

Originally the meter was defined to be $1/10^7$ of the north-pole-to-equator distance along a meridian. But this definition isn't so good since it turned out to be difficult to measure the meter this way to the desired accuracy. From 1889 to 1960, the meter was defined to be the distance between two marks on platinum-iridium bar that was and is kept in France at Sèvres near Paris.

But the bar is artifact. Its can change length with environmental conditions and with cleaning and caretaking. And you have to go to France to check your meter stick against it.

After some intermediate stages, we arrived at the modern meter definition in 1983. The modern meter is defined to be exactly the distance traveled by light in a vacuum in 1/299792458 s.

This means that the vacuum speed of light is exactly 299792458 m/s by definition.

The meter is thus based on the vacuum speed of light and the definition of the second which we consider in § 5.3.

Why is this a good definition?

Aside from corrections to general relativity which can be made negligible, the vacuum speed of light is in the theory of special relativity an exactly constant value. Special relativity is an extremely well verified theory. Aside for corrections to general relativity, it has never been found to be wrong to any degree. Thus, we believe the constancy of the vacuum speed of light (aside from general relativity corrections) to be exact to a higher degree than we can measure. Perhaps, it is truly exactly constant (aside from general relativity corrections). Thus, people determining the meter from the modern definition are limited in accuracy only by their experimental technique and the accuracy of their second determination.

And they don't have to go to Paris to make their determination.

As we'll discuss in § 5.3, the second is defined using a clock that is in theory exact.

Thus, the modern meter definition accords with the program of modern metrology to find base unit definitions that in theory are exact.

5.2. Mass

Mass as we'll discuss in the lecture **NEWTONIAN PHYSICS**, is the resistance to acceleration of a net force.

Mass is also sometimes defined as the quantity of matter. This definition is useful for many purposes since mass (in the sense of the first definition) scales nearly exactly with the number of atoms in an object of one element or the number of molecules in an object consisting of one kind of molecule. But the first definition is the primary definition and the one we mean usually in intro physics.

We won't go into procedures for measuring mass here. Simple ones turn up in the lectures **NEWTONIAN PHYSICS I** and **NEWTONIAN PHYSICS II**.

The base unit of mass is the kilogram.

It is defined to the mass of platinum-iridium cylinder kept in Sévres along with the old meter bar (Wikipedia: Kilogram). This cylinder is the prototype kilogram. It's pretty small: the height and diameter are 3.917 cm (i.e., about 1.5 inches).

The trouble with using an artifact for kilogram definition is that it is subject to change. The environment (which includes cleaning and caretaking) can add and subtract minute amounts of material. And, of course, the object could be damaged or lost—say in war.

So metrologists would like replace the prototype kilogram cylinder definition with a definition based on some exact feature of nature. There are ideas for doing this, but so far they havn't been good enough.

There is, however, an exact definition for mass on the microscopic scale.

The atomic mass unit or AMU (symbol u) is defined to be the mass of an unperturbed carbon-12 atom. Unperturbed means the atoms are unbound and in their ground state (i.e., their lowest energy state). Carbon-12 is the isotope of carbon with 6 protons and 6 neutrons. Note when the AMU is used as physical constant rather than as as unit, it is often given the symbol a.

In quantum mechanical theory, all unperturbed carbon-12 atoms are absolutely identical. Actually, all unperturbed microscopic particles of a given type are absolutely identical. Quantum mechanics is such a well verified theory that this identically property is believed to more exact than any measurement we can do to disprove it. In fact, it makes sense to believe that the identicality property is truly exact.

The carbon-12 atom is chosen to define the AMU for some reason of experimental convenience I imagine.

The AMU is used in the measurements of the masses of microscopic particles.

Why can't we use the carbon-12 atom to define the kilogram?

At present, the accuracy of measuring macroscopic objects on the scale of carbon-12 masses is not sufficiently accurate. Maybe this will change one day. Maybe one day the kilogram will be defined as so many AMUs (see also § 5.6). But at the moment, we are stuck with the cylinder in Sèvres.

5.3. Time

Time is actually a pretty hard thing to define.

Objects occupy different places in a sequence—time passes.

Often there is a continuum of positions occupied in a sequence—time passes.

Some systems go through repeated motions which we call periodic motions—time passes.

Some periodic motions seem so exact we call them clocks and measure time by them—how many periods of the clock does it take for such and such to happen.

There are many historical clocks.

The sequence of days, lunar months, solar years, and other astronomical repeating events.

In fact, these historical astronomical clocks seemed to repeat so exactly that they were taken to repeat regularly and to measure time itself. Of course, many of the astronomical clocks were known to vary when compared to other astronomical clocks: daylight and night vary in length compared to the day, etc. But as time passed, those variations it seemed could always be found to be part of a larger, more regular cycle.

Other clocks like the passing of human and other life cycles and the beating of the heart seemed too irregular and individual in behavior and only approximately kept pace with the astronomical clocks, and so were not taken as measurers of time, except approximately.

Artificial clocks (water, sand, and mechanical clocks) were invented that were synchronizable with the best astronomical clocks. Various irregularities in their periods compared to those of the astronomical clocks could be removed by refinements. Of course, such clocks had to be maintained and supplied with some source of energy—although that it was energy that was needed was not well understood until the 19th century.

In Newtonian physics when it came along in the 17th century, there is a time parameter which we just call "time". And in Newtonian physics there are ideal periodic systems that should repeat in equal periods of time. Newton and everyone else were not at all surprised—it was built into their preconceptions—that real artificial clocks should approximately keep time (i.e., measure time) according to Newtonian physics and that ideal artificial clocks should keep time exactly.

But no artificial clock is ideal. Even the best astronomical clocks could not keep time exactly in principle—Newtonian physics showed this—but the deviations were not measurable until well after Newton. Newton and probably many of his contemporaries thought that God kept time exactly.

But Newton himself did wonder if time flowed equally in all places and times: i.e., did even ideal clocks in different places and times keep the same time if they could somehow be compared. There was no absolute proof that they did, but it was the simplest hypothesis that they did and nothing then contradicted that.

In the 20th century, relativity and confirming experiments proved that time is reference frame dependent. Even ideal clocks in frames moving with respect to each other and in different gravity field desynchronize in a predictable way. The effects were too small to detect in laboratory situations before the 20th century.

In modern cosmology there is, in fact, a theoretical universal time which is consistent with our observations. This universal time is the time in frames of reference that participate in the mean expansion of the universe since the big bang. We can to some accuracy measure this universal time. In fact, the ordinary time of our reference frame on Earth is not very different from universal time. The time since the big bang in both universal time and Earth frame time is according to best modern theory and measurements is 13.7 billion years (or 13.7 gigayears). This value is may change actually.

But how do measure time to highest accuracy in the modern world.

We use atomic clocks.

The practical aspects of them, we won't discuss. If you want some details, see the Wikipedia article "Atomic clock".

In quantum mechanically theory, such clocks when unperturbed keep exactly regular time. One can't practically reach exact unperturbedness, but there is no limit on how closely one can approach it. The best atomic clocks are those deemed to be the most exact and convenient.

The base unit of time is the second.

It is defined (and therefore measured) to be exactly 9192631770 periods of oscillation of the electromagnetic radiation from a particular transition (emission channel) of the caesium-133 atom. (The 133 indicates the isotope of caesium with 55 protons and 78 neutrons).

Why this particular atom and transition? Some reason of experimental convenience that is beyond me.

Why 9192631770 periods?

To keep the modern definition of the second roughly consistent with the historical definition of the second which was 1/86400 of the mean solar day. Actually, the mean solar day increases in time as people knew by comparison to other more exact astronomical clocks, I believe, even before atomic clocks came along.

5.4. Electrical Current: Optional

This is best left to when electromagnetism is being studied.

5.5. Temperature: Optional

This is best left to when thermodynamics is being studied.

5.6. Amount of Substance: Optional

Amount of substance is a somewhat weird thing.

Formally, it's a quantity that is proportional to the number of of elementary entities in a sample (Wikipedia: Amount of substance). The elementary entities could be electrons, atoms, molecules, neutrons, or anything customarily regarded as a microscopic particle.

The SI base unit is the amount of substance in exactly 12 g of unperturbed carbon-12 atoms. It's impossible to really have 12 g of carbon-12 in this state, but it's an ideal limit one can approach. The unit is called a mole (abbreviation mol).

There is, of course, a number of atoms in 12 g of unperturbed carbon-12 atoms. That number is Avogadro's number

$$N_{\rm A} = 6.02214179(30) \times 10^{23}\,\text{particles/mol}\ , \tag{1}$$

which is customary defined to have units of particles per mole. Contrary to popular believe, the mole is not defined to be Avogadro's number. At present Avogadro's number is an experimental determined value.

But the day may come when Avogadro's number is defined to be an exact value and the gram will be defined to be exactly the mass of 1/12 of a mole of unperturbed carbon-12 atoms. The kilogram would then be exactly 1000/12 moles of unperturbed carbon-12 atoms (Wikipedia: Mole (unit)). Then we could dispense with that prototype kilogram cylinder in Sèvres, France.

At present though, it seems that determining the kilogram experimentally by this definition is not as accurate as determining it from the prototype kilogram cylinder.

But how does one use the mole at our level without going to refinements about unperturbed and perturbed atoms?

Say you have a sample of mass m of some substance made up of a single kind of microscopic entity. The number of entities N in the sample is

$$N = \frac{m}{Aa} = \frac{m}{A \times 1\,\mathrm{g/mol}} \frac{1\,\mathrm{g/mol}}{a} \ , \tag{2}$$

where A is the mass of the entity in terms of AMUs, a is the AMU in grams as constant, and

$$\frac{1\,\mathrm{g/mol}}{a} = \frac{12\,\mathrm{g/mol}}{12a} = N_{\mathrm{A}} \tag{3}$$

is Avogadro's number to the best accuracy it's known to. So the number of moles to good accuracy in the sample is

$$N_{\mathrm{mol}} = \frac{N}{N_{\mathrm{A}}} = \frac{m}{A \times 1\,\mathrm{g/mol}} \ . \tag{4}$$

The quantity $A \times 1\,\mathrm{g/mol}$ is called the molar mass or in older terminology the gram atomic weight for atoms and the gram molecular weight for molecules, but in both cases its actually a mass, not a weight.

5.7. Luminous Intensity: Optional

Luminous intensity is the quantity of human sensitivity to light.

The formula definition is given by

$$I_V = 683 \int_0^\infty I_\lambda V(\lambda)\, d\lambda \ , \tag{5}$$

where 683 (which has units of candela (s nm sr)/watts) is an exact traditional scaling constant, I_λ is the specific intensity which is energy per unit time per unit wavelength per unit steradian (with units of watts/(s nm sr)), λ is wavelength, the integral is over all wavelength as indicated by the limits, and $V(\lambda)$ is CIE official vision curve as adopted in 1924.

The units of I_V work out to be in candelas—and this is by design. We won't go into the details of the procedure for officially determining (i.e., defining) the candela: see Greene (2003, e.g.,).

CIE is the Commission Internationale de l'Éclairage. Their official vision curve $V(\lambda)$ is fiducial average of the normal human sensitivity to light under bright or photopic conditions. It's a fiducial average since it was determined from a limited number of human specimens long ago and has never been updated it seems. So it's not an average for any particular human population nor does it account for any evolution of human population. Still its pretty close to the vision of just about anyone with normal vision.

What $V(\lambda)$ looks like is a curve that rises from zero at about 400 nm (the violet end of human sensitivity), reaches a smooth peak of 1 at 555 nm (which is yellow light), and then falls to zero at about 700 nm.

What does $V(\lambda)$ mean? Well to obtain the same subjective human response to light of arbitrary λ as at 555 nm to $I_{\lambda=555\,\mathrm{nm}}$, the specific intensity of the light must $1/V(\lambda)$ times the $I_{\lambda=555\,\mathrm{nm}}$. That's what it means.

Note that at the ends of the curve $1/V(\lambda)$ goes to infinity and the description of its meaning fails.

Of course, subjective human response is a tricky concept and special procedures are used to define how to measure that. Since subjective human response varies from person to person and even for a person depending on many things, subjective human response is probably not all that constant. The official vision curve $V(\lambda)$ only gives the fiducial human response as established in 1924.

There has been a debate as to whether or not the candela should be a base unit. It is after all unit defined for the very special quantity of human sensitivity to light, and does not have wide applicability like other base units. But traditionally its a base unit and a base unit it remains.

6. DIMENSIONAL ANALYSIS: ASSIGNED READING

The word "dimension" in the context of dimensional analysis is rather tricky to define. At least it seems most textbooks and even Wikipedia do a rather poor job of it.

So I'll offer my own definition.

Dimension in the context of dimensional analysis means a quantity that is not the ratio of like quantities in physics.

For example, the dimension of a length is length and so is the dimension of a velocity times a time.

Quantities like angle or the trigonometric function of an angle are dimensionless because such quantities in physics turn up as ratios of like quantities. Note angle has units (i.e., degrees or radians), but angles and trigonometric functions always turn up in ratios of like quantities. Those like quantities are not always length. But usually we do think of angles as the ratio of circle arc length to circle radius. If both lengths are in the same units, we say that the angle is in the units of radians.

Do we need the word "dimension"? Couldn't we just say quantity? Well yes. But we have to deal with dimensionless quantities and saying quantityless quantities is confusing. Also it trips off the tongue to say the dimension of a variable and doesn't to say the quantity of a variable.

Now what about dimensional analysis?

First, note that in physics, meaningful results only occur when adding values or variables of like quantities and the sum of adding values or variables like quantities is always also of that quantity.

In the jargon of dimensional analysis, one says that meaningful results occur only when adding values or variables of like dimension and the sum of values or variables of like dimension also has that dimension.

If an equation violates these rules, it is physically meaningless and it is dimensionally incorrect which means it's just plain incorrect if it is supposed to mean something in physics. But dimensionally correct equations may be incorrect for other reasons and frequently are.

Being dimensionally correct is a necessary, but not a sufficient condition for an equation to be physically correct.

Dimensional analysis is just the procedure of checking if equations are dimensionally correct. There is a second meaning for dimensional analysis which is discussed in the optional § 6.1 below.

To carry out dimensional analysis, the dimensions of quantities with base units are given certain symbols. (One could also say the quantities are given certain symbols.) The conventional ones for length, mass, and time are, respectively L, M, and T. Note these symbols are roman letters. Conventional symbols probably exist for the other quantities for base units and maybe for other quantities as well. But for one's own purposes, one can use any dimension symbols one likes actually.

There is a conventional function to evaluate the dimension of a variable x: the function function [ ]. One can write

$$[x] = \text{dimension symbol for the quantity of } x \ . \tag{6}$$

For example, say x is a length:

$$[x] = \mathrm{L} \ . \tag{7}$$

The dimension evaluation function distributes over all terms and factors in an equation. Acting on a dimensionless quantity [ ] yields 1. What happens to the units of dimensionless quantities (e.g., radians) in formulae and calculations? Oh, they just appear or disappear as needed by the formulae.

The physical relationships between quantities—which are always multiplicative it seems—dictate how to construct dimension symbols from other dimension symbols: one constructs them by multiplication. For example, an area is equal to the product of two perpendicular lengths. Thus, the dimension symbol for area is L^2. For another example, energy is a quantity that is calculated from formulae that multiply out to give quantities with the dimensions $\mathrm{ML}^2/\mathrm{T}^2$.

As an example of dimensional analysis let's consider the horizontal range formula for projectile motion near the Earth's surface neglecting air resistance. The formula is

$$R = \frac{v^2}{g} \sin(2\theta) \ , \tag{8}$$

where R is the horizontal distance the projectile travels until it returns to its launch height, v is the launch speed, g is the acceleration due to gravity, and θ is the angle of launch from the horizontal (e.g., Serway & Jewett 2008, p. 79).

Applying the dimension evaluation function to both sides one gets

$$[R] = \left[\frac{v^2}{g}\sin(2\theta)\right] = \frac{[v^2]}{[g]}[\sin(2\theta)] = \frac{\mathrm{L}^2/\mathrm{T}^2}{\mathrm{L}/\mathrm{T}^2} \times 1 = \mathrm{L} \ . \tag{9}$$

The equation is dimensionally correct since it yields a quantity that has dimension length—or one can say that is a length.

Dimensional analysis is sort of a form a math with quantities, but without quantity. All dimension symbols have constant values. For example,

$$\mathrm{L} = \mathrm{L} + \mathrm{L} \ . \tag{10}$$

Dimensional analysis does have unit value which is the result of [ ] operating on a dimensionless quantity. But there seems no need to have a zero.

Instead of using the conventional symbols for dimensional analysis, one can just use the units of quantities, usually the MKS units in intro physics. In fact, yours truly often does that. One remembers that the units of dimensionless quantities appear and disappear as needed by the formulae.

6.1. The Other Meaning of Dimensional Analysis: Optional

As well as meaning a procedure for checking formulae, dimensional analysis also means used for a procedure to create formulae.

Say you have a system for which you want to a formula for a particular quantity—but you are pretty clueless as to what the formula is or how to get it.

The dimensional analysis procedure is just to create a formulae that is dimensionally correct and which includes quantities from the system that are relevant. Of course, you could create a pretty lousy formula. But with some physical insight, you might get a formula that gives your order-of-magnitude accurate results or even better. (See § 9 for order-of-magnitude calculations.) The dimensional-analysis formula may help you to find an even more accurate formula.

7. UNIT CONVERSIONS

A unit represents a value and it can be treated liked an algebraic variable—it is an algebraic variable whose value is known.

So conversions are really easy to do although often tedious. They are just algebra with variables whose value you know actually, but never explicitly show.

I like using the concept of factors of unity in doing conversions. Factors of unity are best explained by examples.

And so for example, consider the known equation

$$1\,\mathrm{km} = 1000\,\mathrm{m} \ . \tag{11}$$

The units kilometers and meters are just symbols standing for amounts that are multiplied by numbers. If you divide one side by the other, you have a factor of unity. Say

$$1 = \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} \ . \tag{12}$$

Both the left-hand side and the right-hand side are 1 in value and are dimensionless. The right-hand side is a factor of unity in the jargon I use.

You can always multiply anything by a dimensionless 1 without changing its actual value. So say you want to convert 7 km to its value in meters. Multiply it by the factor of unity: i.e.,

$$7\,\mathrm{km} = 7\,\mathrm{km} \times 1 = 7\,\mathrm{km} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 7000\,\mathrm{m} \ , \tag{13}$$

where the kilometer variable has been canceled out.

Let's do a tougher example. Let's convert 10 m/s into miles per hour.

By the by, what's important in the human context about 10 m/s?

It's about as fast as a human can run.

Of course, one has to be an Olympic class sprinter to run that fast.

OK, the conversion is

$$10\,\mathrm{m/s} = 10\,\mathrm{m/s} \times 1 \times 1 = 10\,\mathrm{m/s} \times \frac{1\,\mathrm{mi}}{1609.344\,\mathrm{m}} \times \frac{3600\,\mathrm{s}}{1\,\mathrm{h}} = 22.36936\ldots\mathrm{mi/h} = 22.37\mathrm{mi/h} \ . \tag{14}$$

where we just inserted the appropriate factors of unity and canceled out the redundant units.

Note the equality 1 mi = 1609.344 m is exact in the modern unit system (Wikipedia: Mile).

About how fast does a human walk in miles per hour?

A human walks about 1 m/s by casual observation, and thus just dividing our last conversion result by 10 gives 2.237 mi/h. I think this is a bit slow—more of as stroll than a walk. References typically say an average human pace is about 4 mi/h. Maybe that's a fastish pace.

And that's all there is to conversions.

Of course, if you stick to pure MKS units you never need to do conversions.

However, frequently in this class and in life you are given quantities not in MKS units and are asked for answers not in MKS units. So you either have to calculate in non-MKS units or do conversions and sometimes, of course, you have to do both.

8. SIGNIFICANT FIGURES: ASSIGNED READING

In the science context, significant means "means something" and insignificant means "means nothing".

Usually, a one speaks of a result as being significant if has some level of accuracy and so has some reliability and as being insignificant if it is completely unreliable.

A measurement or a calculation frequently yields a number with some figures (i.e., digits) that are significant and some that are not. Of course, all the figures could be significant and frequently all are insignificant.

Usually, though not always, the significant figures are the leading ones. Hereafter, we assume that the significant figures are the leading ones.

When reporting the number in detailed report, one usually should only report the significant figures. The insignificant figures are completely unreliable and therefore convey no information and give the misleading impression that they do.

There are rules for determining significant (and insignificant) figures in calculations. But they are only approximate rules.

If you estimate uncertainties and do full uncertainty calculations in your calculations, then a more accurate determination can be made of the number of significant figures in results. Somewhere else you will learn about better uncertainty calculations and more accurate treatments of significant figures. These treatments are usually come up in laboratory courses.

In most intro physics exercises, the better treatment is pointless since the exercises are about the physics with made-up values. So it suffices to use the approximate rules for significant figures.

In fact, in courses of yours truly you don't need to be careful with significant figures, unless the problem specifically asks you to obey the significant figure rules. Your results should have about the number of significant figures of your input values. If you report a few insignificant figures in order to make sure you are not dropping significant figures, that's OK.

What are the significant figure rules.

8.1. Rule 1

When dropping insignificant figures one rounds down or up the to trailing significant figure.

If the insignificant figures amount to number less than half of the 1 in the significant figure place, round down.

If the insignificant figures amount to number more than half of the 1 in the significant figure place, round up.

The insignificant figures are deemed meaningless, but one may wrong. They may have a tiny bit of meaning, and so the rounding rule exploits that possibility.

What if the insignificant figures are exactly half of the trailing significant figure?

The rule is round to the trailing significant figure to an even value. For example if one has 6.45, but the last figure is insignificant, one rounds down to 6.4. For another example if one has 6.55, but the last figure is insignificant, one rounds up to 6.6.

The even rounding rule prevents bias. If one is doing many calculations and assumes that trailing significant figures are even and odd with equal likelihood, the even rounding rule prevents a bias of rounding down more often than rounding up.

People could have chosen an odd rounding rule and had the same bias prevention. But having chosen the even one, we must stick to it.

8.2. Rule 2

In addition (which includes subtraction as a special case of addition), the leading insignificant figure decimal position out of all the terms is the insignificant figure decimal position of the sum.

For example, add 3.15 and 2.1 with only significant figures reported. The leading insignificant figure decimal position is 2nd to the left of the decimal point. Therefore that is the insignificant figure decimal position in the sum. When one adds taking the values as exact one gets 5.25. We now round to significant figures (and we need the even rounding rule) and get 5.2.

The reason of the addition rule is the unreliability of the leading insignificant figure of all the terms contaminates the decimal place of that figure with unreliability in the sum.

8.2.1. Math Argument: Optional

A mathematical argument for the addition rule can be given. Say you have two values $A \pm a$ and $B \pm b$ where a and b are the estimated uncertainties in A and B, respectively. The sum with uncertainty is $A + B \pm (a + b)$. Uncertainties are only estimates and are usually only 1-digit values: i.e., they have only one significant figure and this significant figure is the decimal place of the trailing significant figure of the quantity itself. Thus, the leading figure of $a+b$ (which is the only significant one) should be in the decimal place of the last significant figure of $A + B$. This would be the more exact way of arriving at the last significant figure of $A + B$. This last significant figure of $A + B$ is going to be in the highest in significant figure decimal place of A and B or higher because of the summation of errors $a + b$. But if you havn't been got uncertainties, your minimum estimate for the last significant figure should be the one in the highest significant figure decimal place of A and B.

8.3. Rule 3

When multiplying or dividing values, the significant figures of the result is the minimum of the significant figures of all the input values.

This rule can crudely be justified by saying that the relative uncertainty of the value with the least significant figures is probably the largest relative uncertainty and the result can't be more accurate that the least accurate input value.

A mathematical argument can be given for the multiplication-division rule as giving a reasonable estimate of the significant figures in the result, but it's too intricate for here.

8.4. Rule 4

It's usually best to carry some insignificant figures through intermediate calculations and only round off to significant figures as a last step. This is particularly true if all you are doing are multiplications and divisions where calculating the number of significant figures is easy.

There are four good reasons for doing rounding off only at the end.

First, the rules for significant figures are only approximately accurate. So every time you apply them you may be introducing error. To minimize the introduction of error, best to apply the rules only to the final results.

Second, worrying about the rules at every step is a bother. Just do it at the end.

Third, keeping insignificant figure often allows for checks of your calculations. This is especially true when comparing to other people's calculations.

Fourth, if you are doing uncertainty analysis and your uncertainty estimates were too big, then rounding off for significant figures (as determined by your uncertainty analysis and not by the approximate rules) could cause you to lose real accuracy in your final results that you would not be able to notice at the end of the calculation. If you didn't round off, then your final results might surprise you by their accuracy and alert you to your overestimates of the uncertainties.

But usually best is not always best. If your calculations involve additions and subtractions, you have to keep track of significant figures as you go along. This is particularly in subtractions where a subtraction of nearly equal numbers can reduce the number of significant figures to 0.

9. ORDER-OF-MAGNITUDE CALCULATIONS

An order-of-magnitude approximation of a value is the value rounded off to the nearest power of ten.

Usually, one means the nearest power of ten in a logarithmic sense. This means one uses ordinary rounding off rules on the logarithm of the value. But one doesn't want to take the logarithm since that involves calculational work that one is trying to avoid. One simply writes the number in scientific notation. If the coefficient is less than $10^{1/2} = 3.162\ldots$, one rounds down and if it is greater than that, one rounds up.

If your value is exactly $10^{1/2}$ round to the even power of ten. This even rounding rule prevents bias. If one is doing many calculations and assumes that even powers of ten are as likely to be below as above, then the even rounding rule prevents a bias of rounding down more often than rounding up. Of course, having a value of exactly $10^{1/2}$ is pretty rare.

One makes order-of-magnitude approximation in two cases. One you actually don't know the value very well and can only estimate it to order of magnitude. In this case, there is no rounding, of course. The second case is when you are doing order-of-magnitude values calculations. In this case, all the input numbers are order-of-magnitude values. You may do this because some of your input values are only order-magnitude values and so you can only achieve order-of-magnitude accurate results. Or you may do this because you are just doing an order-of-magnitude calculation for simplicity and you've made the order-of-magnitude approximation for all your values.

As well as order-of-magnitude calculations, one can also do 1-digit calculations which are a lot more accurate and can be done sans calculator. Such calculations mean one just keeps about 1 significant figure in the calculations. There are no hard rules though. One can keep 2 significant figures sometimes and do compensations for dropping significant figures. These help reduce round-off errors and may maintain real 1-digit accuracy. Yours truly is very fond of doing 1-digit calculations for on-the-board examples.

Let's do some examples of order-of-magnitude calculations and 1-digit calculations.

9.1. Example: A Fermi Problem

Fermi problems are ones where order-of-magnitude approximations allow you to find order-of-magnitude results that you couldn't find exactly by any easy means.

Here's a Fermi problem.

How many red cars are there in Idaho?

Well there are I'd say 10^6 people in Idaho to order of magnitude. I know there's more than 10^5, but 10^7 sounds way too big for a smallish-population state in a country of only 300^6.

How many cars does the average person own? Well many own none: three-year-olds, etc. But many own multiple cars. I estimate to order-of-magnitude the average person owns one car.

What fraction of cars are red? Well much less than 1, but probably much higher than 10^{-2}. So I estimate 10^{-1}.

So the number of red cars in Idaho to order of magnitude is

$$10^6 \times 1 \times 10^{-1} = 10^5 . \qquad (15)$$

There are a hundred thousand red cars in Idaho. It wouldn't surprise me if this number were too big by a factor of 10. I would be surprised if it were too small by a factor 10. I just can't believe there are a million red cars in Idaho.

I'll leave it as an exercise to the students to figure out how many woodpeckers there are in Latah County.

9.2. Example: Order-of-Magnitude and 1-Digit Calculations Compared

Say I wanted to evaluate the Planck length which is a unit of length constructed by dimensional analysis out of fundamental constants. Dimensional analysis here has its second meaning (see § 6.1). The constants are $c = 2.99792458 \times 10^8\,\mathrm{m/s}$ (the vacuum light speed), $G = 6.67428(67) \times 10^{-11}\,\mathrm{J\,m\,kg^{-2}}$ (the gravitational constant), and $\hbar = 1.054571628(53) \times 10^{-34}\,\mathrm{J\,s}$ (Dirac's constant). The first thing to note is we can get rid of those joules and kilograms since a length has neither of those units. Remember a joule is $\mathrm{kg\,m^2\,s^{-2}}$. So

$$\mathrm{unit}[\hbar G] = \mathrm{J\,s} \times \mathrm{J\,m\,kg^{-2}} = \mathrm{m^5\,s^3} \ , \tag{16}$$

where unit[ ] is my own unit evaluator function. Now it's clear that

$$\mathrm{unit}\left[\sqrt{\frac{\hbar G}{c^3}}\right] = \mathrm{m} \ . \tag{17}$$

So the Planck length is

$$\sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{1.054571628 \times 10^{-34} \times 6.67428 \times 10^{-11}}{(2.99792458 \times 10^8)^3}} = 1.61625 \times 10^{-35}\,\mathrm{m} \ , \tag{18}$$

where we report the number to correct significant figures according to the rules (which are only approximately right recall).

But say we didn't want to work so hard in the calculation. What is the Planck length to order-of-magnitude? Behold

$$\sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{1.054571628 \times 10^{-34} \times 6.67428 \times 10^{-11}}{(2.99792458 \times 10^8)^3}} \approx \sqrt{\frac{10^{-34} \times 10^{-10}}{10^{24}}} = 10^{-34}\,\mathrm{m} \ . \tag{19}$$

Note the straight order-of-magnitude calculation does **NOT** give the right order-of-magnitude result in this case. This sometimes happens because of the approximations made in dropping the coefficients. But the result is only off by 1 order of magnitude.

A 1-digit calculation does a lot better. Note we will keep 2 digits at times if we think the second digit is significant. Behold

$$\begin{aligned}
\sqrt{\frac{\hbar G}{c^3}} &= \sqrt{\frac{1.054571628\times 10^{-34}\times 6.67428\times 10^{-11}}{(2.99792458\times 10^8)^3}} \\
&\approx \sqrt{\frac{10^{-34}\times 7\times 10^{-11}}{30\times 10^{24}}} \approx \sqrt{\frac{1}{4}\times 10^{-69}} \approx \sqrt{2.5\times 10^{-70}} \\
&\approx 1.6\times 10^{-35}\,\mathrm{m}\ . \qquad (20)
\end{aligned}$$

The 1-digit result is correct to 2 significant figures. This not accidental. The significant figures we dropped only changed values by about 10 % or so, and so we can claim a higher accuracy for the result. Now 1-digit calculations are not always right to 1 significant figure because of round-off errors, but they are usually right to within a factor of 2 or so.

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