Ch 8 Conservation of Energy

§8.0

- Actually we've already discussed this
- total energy always conserved for an isolated system, but if changes happen the energy changes form. — always or almost always?

It's hard to make a definitive list because new "types" are often introduced for new contexts & types overlap

Non-exhaustive list of energy types — ~~not~~ types overlap

1) KE
2) PE { really electromagnetic or gravity energy or..
3) Mechanical E = KE + PE as we'll see
4) Thermal energy or internal energy { which like most people, except purists, I call heat or heat energy

Randomized microscopic KE, PE, EMR other forms depending on how one counts.

5) Chemical bond energy
6) Energy in electromagnetic fields including traveling EM fields or EM radiation or light

§ 8.1 Nonisolated systems + Transfer mechanisms

— nonisolated systems don't ~~cons~~ conserve energy since energy can flow in and ~~or~~ out.

— ~~But we believe that if you incorporate the system in surroundings~~ in

Actually a truly isolated system is an idealization except for the universe as whole (it seems).

Is energy conserved for the universe

— This is actually not quite clear even in theory ~~due~~ to mainly maybe to difficulties with gravitational energy in Einstein's general relativity — ~~but~~ we don't have to worry about ~~that~~.

Mechanisms of heat transfer which actually overlap

1) Macroscopic work
   — applied force with a displacement $W = \int \underline{F} \cdot d\underline{r}$

2) Mechanical waves — sound waves, seismic waves, ocean waves
   ~~rope~~ string waves

— but in a ~~an~~other sense it is work — one bit of material does work on another & so on.

3) Heat (Heat transfer) → formally the mechanism of thermal energy transfer

→ essentially work at the micro-level as particles interact and those with more energy share with their poorer neighbors.

I tend to call this heat transfer

4) Matter transfer
— particles carry energy with them when they leave or enter a system.

5) EMR — most long distance of all forms. — Energy can be sent across the universe from near the Big Bang in time to us.

There actually are other ways — gravitational radiation

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In this chapter we concentrate on transfer of energy via macroscopic work.

but we can't be pure, since as we've already ~~all~~ admitted work by friction & drag can transfer energy from KE to heat (i.e., thermal energy).

§ 8.2 Isolated System

— A poor title since it's really about the Work-Energy theorem which shockingly Serway never names!!!

We are only going to deal with macroscopic KE & PE and mechanical energy as we'll see. which all the work-energy thm ~~[illegible]~~

Recall Work-Kinetic Energy Theorem

$$\underbrace{\Delta KE}_{\text{change in KE}} = \underbrace{W}_{\text{work}}$$

work done by ~~all~~ ~~forces.~~ the total net force — NOT just done by any particular force.

Nothing forbids us from breaking W into the parts done by conservative and nonconservative forces.

$$W = W_{non} + W_{con}$$

$$= W_{non} - \Delta PE$$

One can also break into W_{con} by some particular force + W_{other}

Recall $\Delta PE = -W_{con}$

Here we just ~~[illegible]~~ lump all the conservative forces together for simplicity.

$$\therefore \quad \Delta KE = W_{non} - \Delta PE$$

$$\therefore \quad \Delta KE + \Delta PE = W_{non}.$$

Now we define a new form of energy — Mechanical energy.

$$E_{\cancel{mech}} \equiv KE + PE$$

One can use a subscript mech for clarity, but I prefer to just use E and rely on context to identify as mechanical energy as opposed to any other kind of energy.

~~Note~~ Recall what I said about energy categories overlapping

— mechanical energy formally subsumes KE & PE, but it we still find it overwhelmingly useful ~~to~~ recall KE & PE as separate forms of energy.

With mechanical energy, we can now write

$$\left.\begin{aligned} \Delta E &= W_{non} \\ &\text{or} \\ \Delta KE + \Delta PE &= W_{non} \end{aligned}\right\} \text{Work-energy theorem}$$

This is the net work done by all nonconservative forces.

Recall what it means — in some movement there is a change in KE & PE.

If $W_{non} = 0$, $\Delta E = 0$.

i.e., Mechanical Energy is conserved.

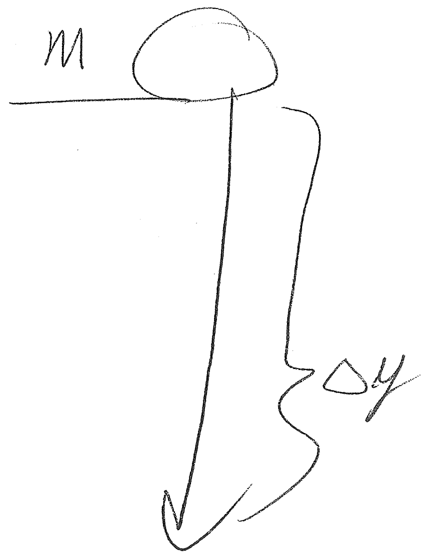
Using this limited form of energy conservation is often very useful in solving different problems as well as.

Special note: Nonconservative forces may be present and doing things, but they do no work in these

Ex 8.1

Ball in free-fall

- in a less tedious manner than Serway



You drop a ball.

The only force that acts is gravity which is conservative.
(we assume zero drag force)

$\therefore W_{non} = 0$

$\Delta E = 0$

~~Say we take the simplest case where $v_i = 0$~~

$\Delta KE + \Delta PE = 0$

$KE_f - KE_i + PE_f - PE_i = 0$

ΔPE since only changes in PE

Let's consider the simplest case where $v_i = 0$.

$\therefore KE_i = 0$

$$\therefore \quad KE_f + \Delta PE = 0$$

or $\quad KE_f = -\Delta PE$

These kinds of problems turn up so often in intro physics classes (if no where else) that often while one just jumps to this step without thinking

$$\therefore \quad \tfrac{1}{2} m v_f^2 = -mg(y_f - y_i)$$

mass independent which often happens in problems where gravity is the only force since mass cancels out.

$$v_f = \sqrt{2g(y_i - y_f)}$$

$$v_f = \sqrt{2g(-\Delta y)}$$ even more simply

~~Say $y_i = 10\,m$~~

Say $\Delta y = -10\,m$

$$v_f \cong \sqrt{2 \cdot 10 \cdot 10} = \sqrt{200} \cong 14\,m/s$$

$\boxed{8\text{-}10}$

As a slight complication say ~~that~~

$$v_i \neq 0$$

In this case

$$\Delta KE + \Delta PE = 0$$

because

$$KE_f - KE_i = -\Delta PE$$

$$KE_f = KE_i - \Delta PE$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - mg\Delta y$$

$$v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$$

Curiously the result only depends on the magnitude of v_i not on its sign

up $v_i \uparrow$ v_i down $\downarrow$ } same v_f which you may recall was a result we obtained for a constant acceleration case.

Recall our timeless kinematic equation

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$v_f = \sqrt{v_i^2 + 2a\Delta y}$$

In this case $a = -g$, $\Delta y = y_f - y_i$

$$v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$$

So we can obtain the same result from applying Newton's laws (and the kinematic equation for constant acceleration)

So our energy approach is an alternative approach.

Actually using Newton's laws ~~tells~~ can tell

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you more.

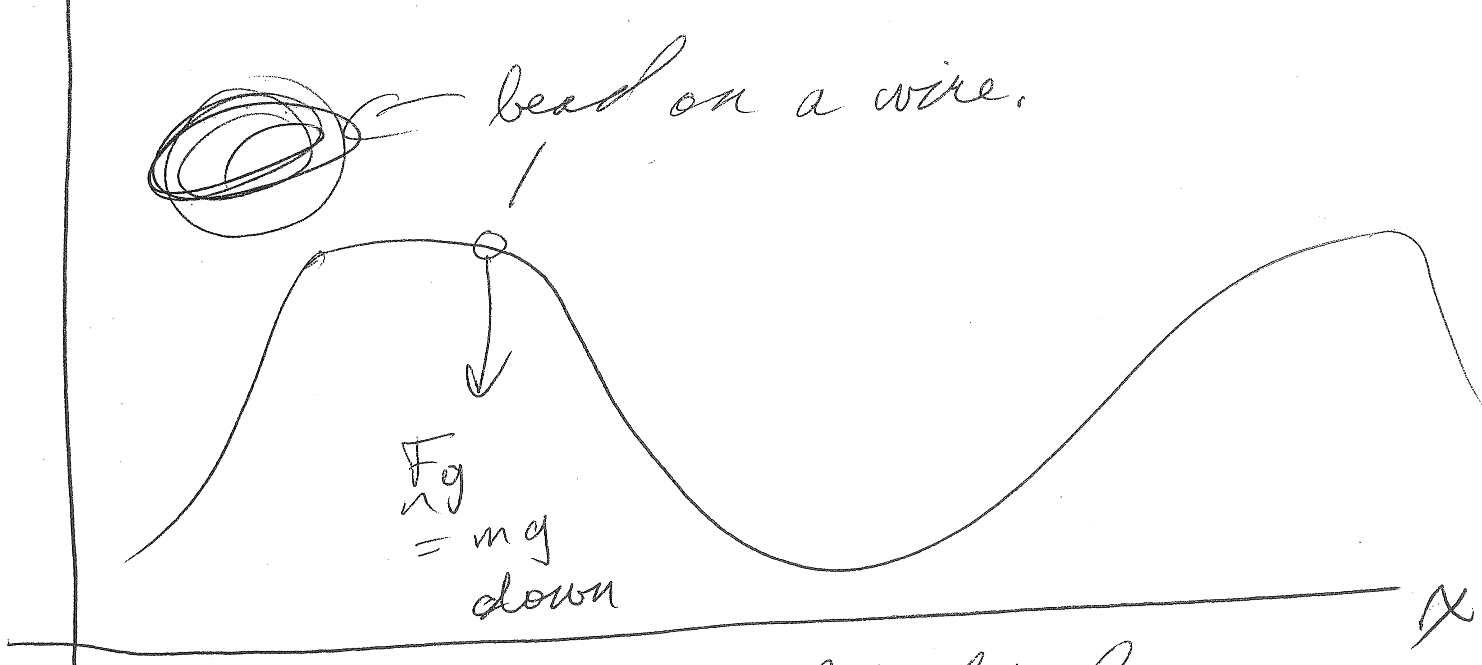
You can determine the whole time evolution for the thrown ball under gravity.

— We can't do this from the energy approach without more tools which are beyond the scope of this course.

So the energy approach offers an alternative, but less than the full Newton's law approach in this case.

But there are other cases where the energy approach offers more advantages.

Example 1 Not from the text but it offers the essence of many of the text examples



- the wire is frictionless and bead slides freely.

- the wire exerts normal forces on the bead. and the only other force is gravity.

- in the jargon I use the wire exerts a force of constraint.

This is NOT a conservative force.

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<u>But</u> it does <u>NO</u> work

$$F_{constraint} \cdot d\underline{r} = 0$$

in all cases.

It does something but not work.

The force of constraint shapes the path of the bead, but does no work

Work energy theorem

$$\Delta E = \underbrace{W_{non\ conservative}}_{\text{0 in this case even though the force of constraint does something.}}$$

$\Delta E = 0$ and mechanical energy is conserved.

~~provided~~

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We are given an initial height y_i and an initial speed

v_i

what is speed v for any height y.

$$\Delta E = 0$$

$$\frac{1}{2}mv^2 + mgy = \frac{1}{2}mv_i^2 + mgy_i$$

$$v = \sqrt{v_i^2 + 2g(y_i - y)}$$

This looks just like a constant acceleration case, but it is not.

— the acceleration is <u>not</u> constant.

The bead can follow a complex motion in general,

but for any y we know its speed v.

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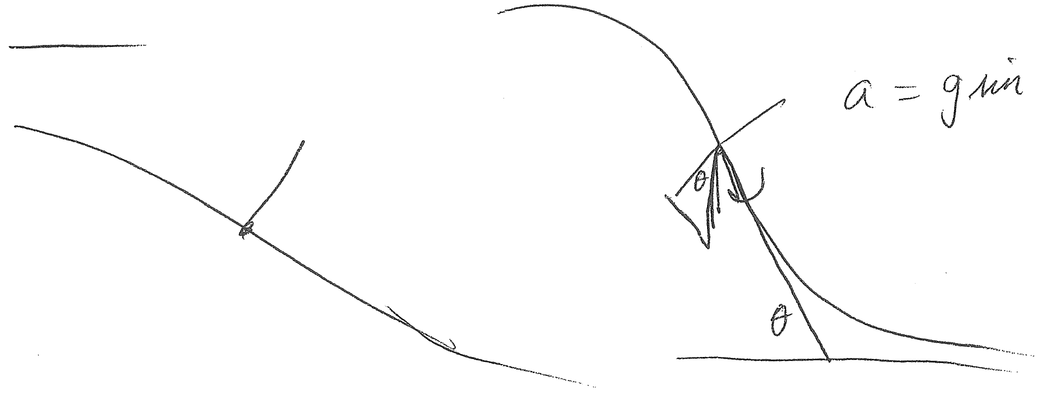
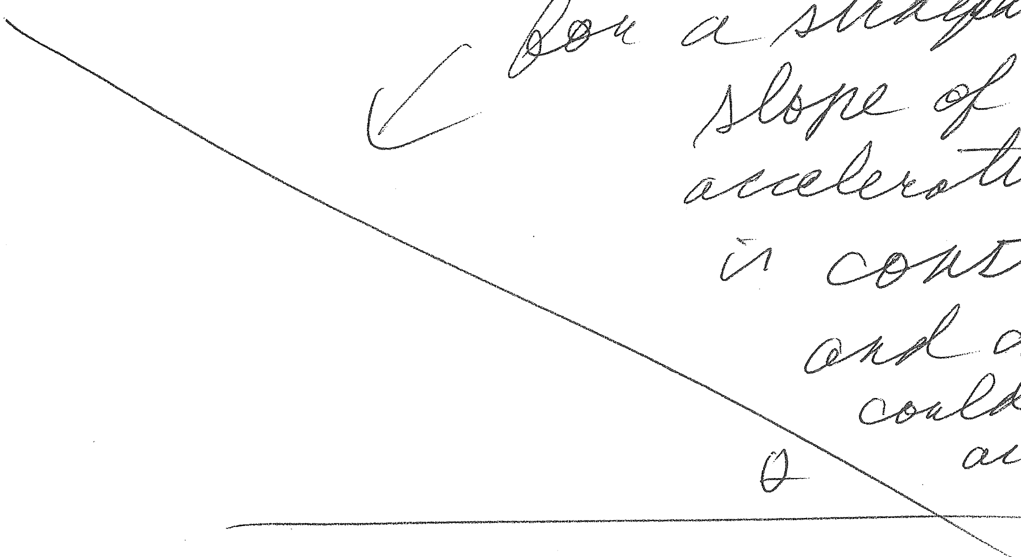
This is only partial information

- What about full information. $x(t)$, $y(t)$, $v(t)$, $a(t)$
- In general, we'd need more information
  - the shape of the wire ~~to start with~~ and initial velocity not just speed.
- In general, because the slope is varying, the acceleration must vary



But to know θ we'd need full information about the wire.

In general, even if we had such information an analytic solution would~~n't~~ NOT be possible and we'd have to do a numerical computation on the computer.



θ

for a straight slope of course acceleration is constant and we could solve as we've done before.

Still the energy approach gives us partial information easily — that's often very useful.

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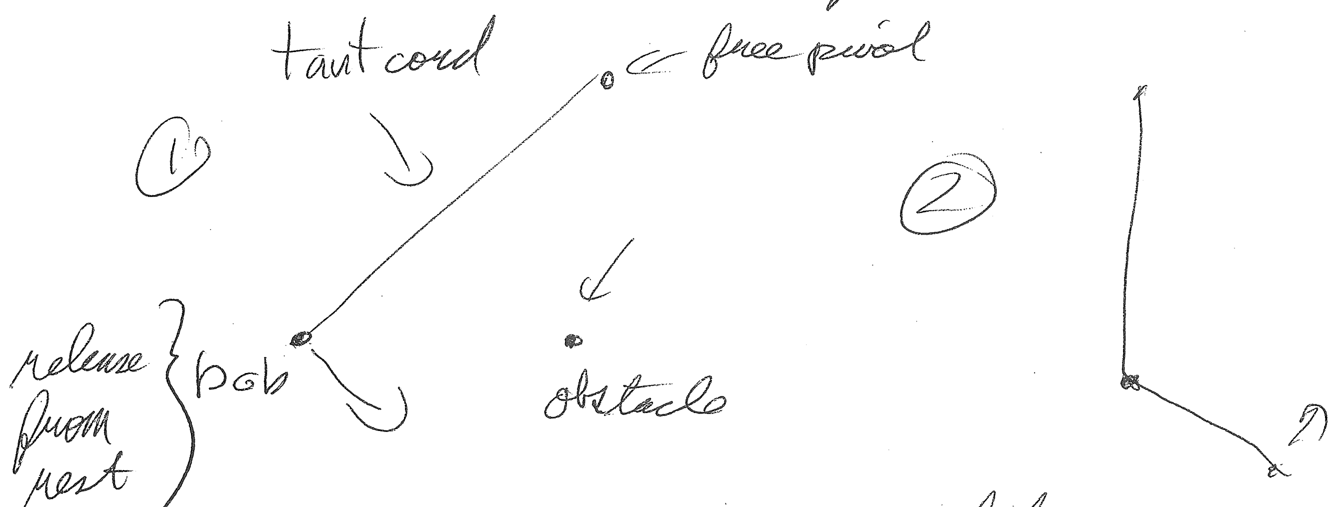
An analogous homey example.

- you have so many gallons of gas. (~~one~~ chemical energy in fact)
- where you'll drive with it depends on many things.
- but about how far you can go is limited by how much energy (fuel) you've got.

Ex 2 A special case of Example 1



how high does the bob go given ideal conditions?

Now we rearrange

$$v = \sqrt{v_i^2 + 2g(y_i - y)}$$

$$y = \frac{v^2 - v_i^2}{2g} + y_i$$

$$v_i = 0$$

what is the final v?

what is final $\overset{v=0}{y}$?

$$y - y_i$$

- assuming no energy loss to work by resistance forces.

in reality there always is such work, but sometimes it can be neglected or accounted for.

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§ 8.3 Cases with Kinetic Friction

— I'm going to be briefer than the text.

$$\Delta E = W_{noncon}$$

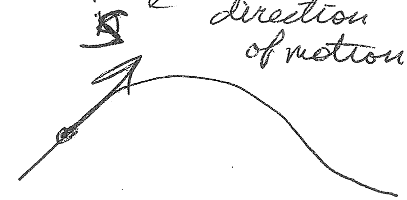
— the work energy theorem.

Work done by kinetic friction

$$W_f = \int \underset{\sim}{F_f} \cdot d\underset{\sim}{r}$$

$$= -\int F_f \, \hat{s} \cdot d\underset{\sim}{r}$$

$\hat{s}$ ← instantaneous direction of motion

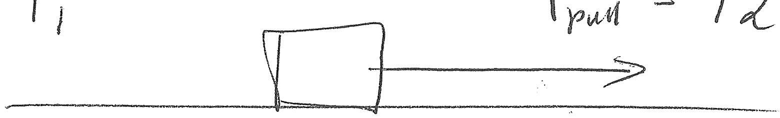


Easiest to illustrate by examples.

Ex 8.4 Pulling a block with friction.

$m = 6\ kg$

a) P_1 $\quad F_{pull} = 12\ N$



$\mu_k = .15$ coeff. of kinetic friction

$F_f = \mu_k F_N = \mu_k mg$ in this case.

~~$\Delta x = 3m$~~

$\boxed{8-21}$

~~we~~ find the speed after the block has moved $\Delta x = 3m$ starting from rest

We could do this problem using Newton's laws, but we can get the speed using the work energy theorem too.

$\Delta PE = 0$ since the ~~system~~ block is on the level

$$KE_f = \Delta KE = \underbrace{F_{pull}\Delta x}_{W_{pull}} \quad \mu_k m g$$

$$\| \qquad\qquad\qquad \text{non-conservative}$$

$$\tfrac{1}{2} m v^2$$

$$v = \sqrt{\frac{2}{m}\left(F_{pull} - \mu_k m g\right)\Delta x}$$

$$= \sqrt{2\left(\frac{F_{pull}}{m} - \mu_k g\right)\Delta x}$$

$$= \sqrt{2\left(\frac{12}{6} - .15 \cdot 10\right)3} = \sqrt{3} = 1.7 \frac{m}{s}$$

ANS 1.8 m/s

8-22

b) My (w) not the text's

Say $F_f = \emptyset$

and $v_i = 10\,m/s$.

If $v_i = 0$, what would happen?

— stable static equilibrium

— the static friction force stabilizes the neutral equilibrium of a frictionless surface.

In this ~~ow~~ case how far Δx does the block go before it stops?

$PE = \emptyset$ always again

$KE_f = 0$.

$\therefore \quad -KE_i = W_f = -\mu_k m g \Delta x$

— minuses cancel out

$$\Delta x = \frac{\frac{1}{2} m v_i^2}{\mu_k m g}$$

$$= \frac{v_i^2}{\mu_k g} \simeq \frac{100}{1.5} = 67\,\mathrm{m}$$

Mass cancels and so a mass independent result.

— this is of course because friction depends on mass.

Again we could have solved the problem for full information using Newton's laws, but this is kind of quick.

§ 8.4 Changes in Mechanical Energy with non-conservative forces

— Same really as § 8.3, but now with $\Delta PE \neq 0$.

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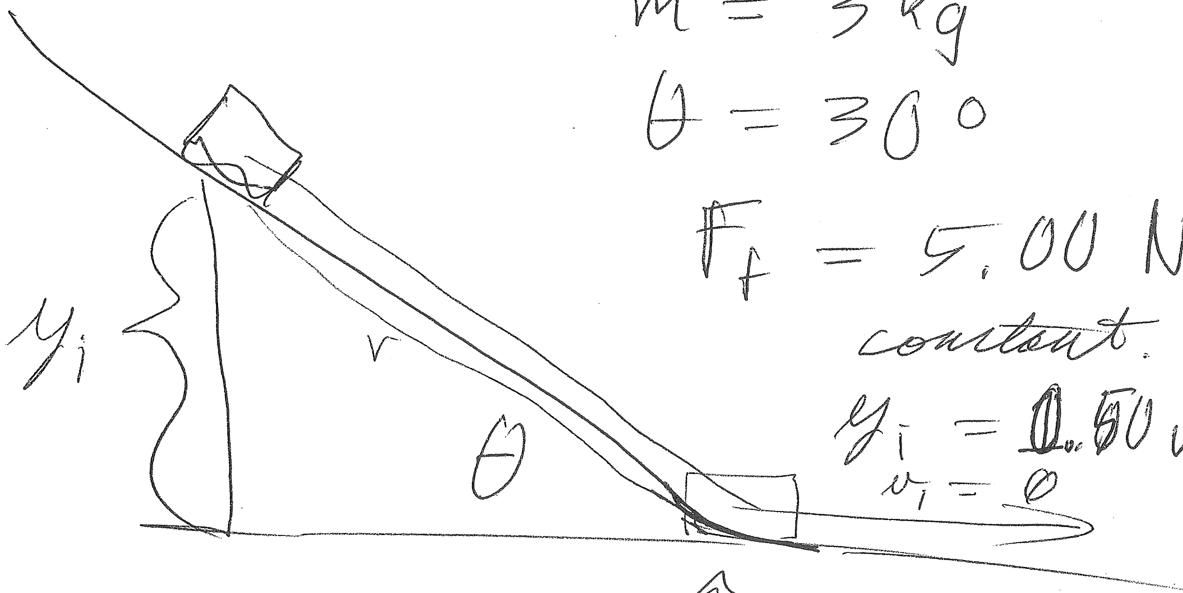
Ex 8.7

— a really exciting sliding down a ramp problem



$m = 3\text{ kg}$

$\theta = 30°$

$F_f = 5.00\text{ N}$ constant.

$y_i = 0.50\text{ m}$

$v_i = 0$

It is really a particle.

{ assume the block negotiates the bend ideally, — gracefully
— no forces except gravity & friction do work.

— the normal forces shapes path but

F_N is always perpendicular to the path and does no work.

a) What is v at bottom?

$y_i = r \sin\theta$

$$\Delta E = W_{non}$$

$$\frac{1}{2} m v_f^2 - 0 + 0 - m g y_i = -F_f \frac{y_i}{\sin\theta}$$

8-25

$$\frac{1}{2} m v_f^2 = m g y_i - F_f \frac{y_i}{\sin\theta}$$

$$v_f = \sqrt{\frac{2}{m}\left(mg - \frac{F_f}{\sin\theta}\right) y_i}$$

$$= \sqrt{2\left(g - \frac{F_f}{m \sin\theta}\right) y_i}$$

$$\cong \sqrt{2\left(10 - \frac{5}{3 \cdot \frac{1}{2}}\right) \cdot 0.5}$$

$$\cong \sqrt{2 \cdot 7 \cdot 0.5}$$

$$\cancel{= \sqrt{14} \cong 3.8 \, m/s}$$

AN

$$= \sqrt{7}$$

$$\cong 2.7 \, m/s \qquad \underline{\text{Ans } 2.54 \, m/s}$$

b) How far does the block slide on the horizontal?

Well we can recycle our analytic solution from earlier with a bit of change

$$\Delta x = \frac{\frac{1}{2} m v_i^2}{F_f}$$

$$\cong \frac{\frac{1}{2} \cdot 3 \cdot 7}{5} \cong \frac{10}{5} \cong 2 \, m$$

$$\underline{\text{Ans } 1.94 \, m}$$

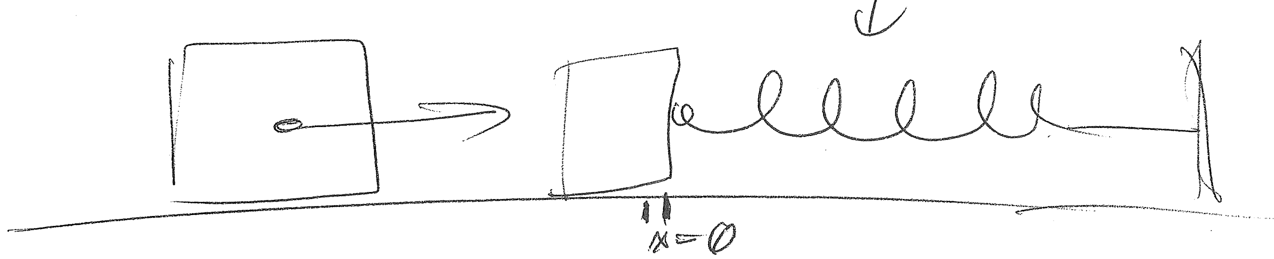
8- 26

Ex 8.8

Block spring collision

b) ~~Part b first~~

Ideal Spring of zero mass



block collides with spring with $v_i = 1.2 \, m/s$

$m = 0.80 \, kg$

$k = 50 \, N/m$ is the spring constant

$\mu_k = .50$ for kinetic friction.

Compute the maximum compression x_{max} displacement.

What is the condition on x_{max}? $v_f = 0$

$$\Delta E = W_{non} \qquad \boxed{8-27}$$

$$= -F_f\, x_{max}$$

$$\underbrace{\Delta KE}_{0-\frac{1}{2}mv_i^2} + \underbrace{\Delta PE}_{\frac{1}{2}kx_{max}^2 - 0} = -\mu_k m g\, x_{max}$$

— no gravitational PE in this problem since everything is on the level.

$$-\frac{1}{2}mv_i^2 + \frac{1}{2}kx_{max}^2 = -\mu_k m g x_{max}$$

$$-v_i^2 + \frac{k}{m}x_{max}^2 = -2\mu_k g x_{max}$$

$$\frac{k}{m}x_{max}^2 + 2\mu_k g x_{max} - v_i^2 = 0$$

A quadratic equation for x

$$x_{max} = \frac{-2\mu_k g \pm \sqrt{(2\mu_k g)^2 + 4\frac{k}{m}v_i^2}}{2\frac{k}{m}}$$

$$= \frac{-10 \pm \sqrt{100^2 + 400}}{120} = \frac{-10 \pm 23}{120}$$

$\underline{ANS.}$ $.093_m \cong .1\,m$ or $-.25\,m$

8-28) The -ve solution is actually unphysical in this case.

→ it is based on the idea that ~~there~~ is a force

$F = -\mu_k mg$

for all

a constant force is conservative and $-\mu_k mg$ moved to the other side is and $\Delta E = 0$ mechanical energy conserved

If this force were real the mass would actually oscillate if it stayed attached the spring between the 2 x values

But actually the frictional force changes sign when the direction of motion changes

This effect is NOT built into our equation.

d) We turn friction off $\mu_k = 0$

$$x_{max} = \pm \frac{\sqrt{400}}{120} \cong \pm \frac{1}{6}$$

$= \pm .167$ m Ans. .15 m

the +ve solution is the real one.

— But if the mass stayed attached to the spring, it would oscillate between the two values

~~Stable equilibrium~~
~~is good for buildings.~~

~~— But sometimes we want~~
~~unstable equilibria~~
~~in mechanical systems~~

~~Can anyone think of~~
~~an example?~~

~~A balance scale~~

~~— the closer to true~~
~~unstable equilibrium,~~
~~the ~~ben~~ more accurately~~
~~one can weigh an object.~~

Actually some slight frictional force that ~~power a~~ slightly stabilize and allows one a "perfect" balance.

§8.5 Power

Hmk examples — ~~actually need a concept not in Ch 7.~~

~~Actually a topic Ch 7 omits~~ — ~~§8.5~~ ~~[illegible]~~ ~~here~~

— The concept of Power,
which is energy
transfer per unit time
in physics jargon

It could be by work or any other transfer mechanism.

8-30

$P = \frac{dE}{dt}$ where E = any energy transferred by whatever mechanism

& the mechanism is work

$$P \equiv \frac{dW}{dt} \quad \Big\{ \text{work}$$

Power

units of J/s in MKS

= watt = W

The watt.

~~Recall~~ Recall $W = \int \underset{\sim}{F} \cdot d\underset{\sim}{r}$

$$\therefore \quad dW = \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$$P = \frac{dW}{dt} = \underset{\sim}{F} \cdot \frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{F} \cdot \underset{\sim}{v}$$

Not the only formula for power since there are other energy transfer processes besides macroscopic work.

There is one Non-MKS unit of power still around as a

Legacy from the bad old days:
the horsepower

$$1\ hp = 746\ W$$
(electrical hp)

actually slightly different definitions exist.

James Watt the improver of the ~~steam~~ engine invented horsepower to rate his engines.
Actually he was flattering horses.
— Only a very powerful horse can do a horsepower of work on things other than its own body and not for so long, I believe.

One could use kilowatts = kW instead. (Car manufacturers believe people still wonder what a horse could do instead.)

You may wonder what is a kilowatt-hour ~~that~~ electric companies bill you for.
It's not power, it's energy

$$1\ kW\text{-}hr = 1\,kW \times 1\,hr \left(\frac{3600\,s}{h}\right)$$
$$= 3600\ kJ = 3.6\ MJ.$$

8-32

So electric companies could bill in MJ and only don't out of sheer obtuseness.

The ~~proliferation~~ ~~maze~~ maze of non-standard energy units is a bother for understanding energy in life

1 kWhr = 3.6 MJ

1 calorie = 4.1868 J (other versions exist of calorie)

1 food calorie = 4.1868 kJ

1 Btu = 1.055.... kJ

~~between the two values.~~

§8.5 Power

— I've anticipated ~~that~~ at the end of Ch 7

So one can just recall

Power P is energy transferred per unit time in general. $P = \frac{dE_{\text{ener}}}{dt}$ Not necessarily mechanical energy

— If the transfer is work

$$P = \frac{dW}{dt}$$

If $W = \int \underset{\sim}{F} \cdot d\underset{\sim}{r}$

$$dW = \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$$P = \underset{\sim}{F} \cdot \frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{F} \cdot \underset{\sim}{v}$$

8-~~34~~

~~$[P] = \frac{J}{s} =$~~ W = watt.

Some Power examples

Ex 8.10

Power delivered by an elevator motor.

a) $m = 1800\,kg$ of elevator and passenger

$F_f = 4000\,N$

a) $v = 3.00\,m/s$ constant.

since v ~~constant~~

$F_{net} = 0$

$F_{net} = F_{el} - mg - F_f$

$F_{el} = mg + F_f$

$\therefore P = F_{el}\,v = (mg + F_f)\,v \cong (18000 + 4000) \cdot 3$

$= 66000\,W$ ANS $6.48 \times 10^4\,W$

b) Say v is not constant
but there is constant upward acceleration a

$$\therefore \quad F_{net} = ma \quad , \quad a = 1.00\,m/s^2$$

$$F_{el} - mg - F_f = ma$$

$$F_{el} = ma + mg + F_f$$

$$v = \int a\,dt$$

we are given $v = 3.00\,m/s$
as the speed to evaluate
the power at.

$$P = F_{el}\,v$$
$$= (ma + mg + F_f)\cdot v$$
$$= (m(a+g) + F_f)\,v$$
$$= (1800 \cdot 11 + 4000)\cdot 3$$
$$\triangleq 24000 \cdot 3$$
$$= 72000\text{ W}$$

ANS 7.02×10^4 W

8-36