Chapt. 7 Energy of a System

0) Intro

The text like all texts has to admit, it is hard to define *energy*.

"Energy is particular a dynamic variable that overall is conserved in ~~any~~ closed system".

All one line definitions are inadequate.

"Energy is everything" aphorism not a definition

I like "~~the~~ energy is the quantified capacity for change."

I think not remote from everyday understanding

Like all aphorisms — not really ~~complete~~ true but it makes a point.

If you have energy, you have the ability to make changes and then changes transform energy among it's different forms

You eat breakfast & have chemical energy and you change things with it and change it into other forms of energy e.g., motion & heat. (Muscles)

But energy is conserved overall.

or both true

→ if you have an isolated system → a set of components of anything cut-off from the outside, ~~then~~ and no

7-2) energy leaves or enters, the amount of energy in the system stays constant even though it can transform.

There are deep theoretical reasons why this should be so — Noether's theorem.

But a basic element of those reason is that energy is conserved in every little event & so must be conserved overall.

There are many kinds of energy and ~~wil wil~~ we'll introduce them as we go along.

— But the categories are overlapping

— Some are by their form ~~their energy~~ nature and some are by the context of the problem.

— energy is actually implicit in Newtonian physics but Newton himself never used the concept.

— Galileo had a glimmer of it and it grew to a basic concept of physics in the 19th century

when people realized they could invent new forms of energy to maintain conservation.

you may wonder is energy "just" an accounting scheme.

— I think not for a number reasons

— ~~with the fact that~~ whenever energy has seemingly disappeared or been created, we've always been able to find a new kind ~~kinds to~~ to maintain

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conservation of energy suggests it's a real thing.

& so does $E = mc^2$ (Einstein's equation) which is a subject of much later talk.

but one interpretation of this is ~~that~~ all energy has resistance to acceleration — i.e., has mass ~~& that~~ (and that is true) and ~~that I think~~ shows it's a real as anything is real.

Why use energy?

① Well in Newtonian physics it offers an alternative and often a superior approach to the pure laws of motion approach.

often energy procedures give partial information very easily. (usually by exploiting conservation of energy)

② the energy concept generalizes to all physics, well beyond Newtonian physics. — it's universally useful

§7.1 System & Environment

— A system is some collection of things that can be treated together in some way.

e.g., a bunch of balls ~~colliding~~ in a box.

— The environment is everything else.

<u>system & environment approach</u> is a ~~s~~implifying model that allows calculation & analysis.

Often the system is itself ~~f~~ simplified. This allows ~~fa~~ easier calculation & analysis

→ your results ~~may~~ may often be very approximate but refinements are then possible.

§7.2 Work done by a Constant Force

& 7.3 conflated on Dot Product.

The text likes to say "work" in physics is distinct from everyday meaning.

It's a matter of opinion, but I'd say it's not ~~so~~ distinct. I think the everyday concept and the physics concept are closely connected.

When you work you exert forces and move things.

In physics we quantify ~~that~~ that idea of work and generalize to all cases of ~~for~~ where there are forces and motion.

We Need Scalar product
or dot product

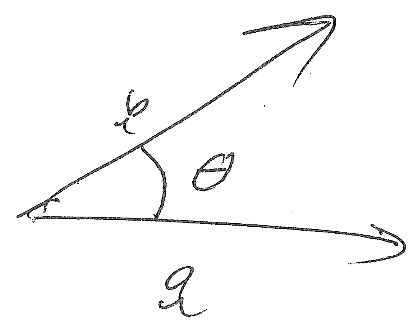
— yes you can multiply vectors,
but you need ~~##~~ special
rules. — which exist because they turn out to be useful + embody useful physics

— there are several ways
and rules: The dot product is one

Say $\underset{\sim}{a}$ & $\underset{\sim}{b}$ are general
vectors.

$$\underset{\sim}{a} \cdot \underset{\sim}{b} \equiv ab\cos\theta$$ {scalar result

dot always
explicit for
dot product.



$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{b} \cdot \underset{\sim}{a}$$ commutative

$$\underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c}$$ ~~associative~~ distributive

An alternative definition is

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = a_x b_x + a_y b_y + a_z b_z$$

The two definitions are equivalent
actually.

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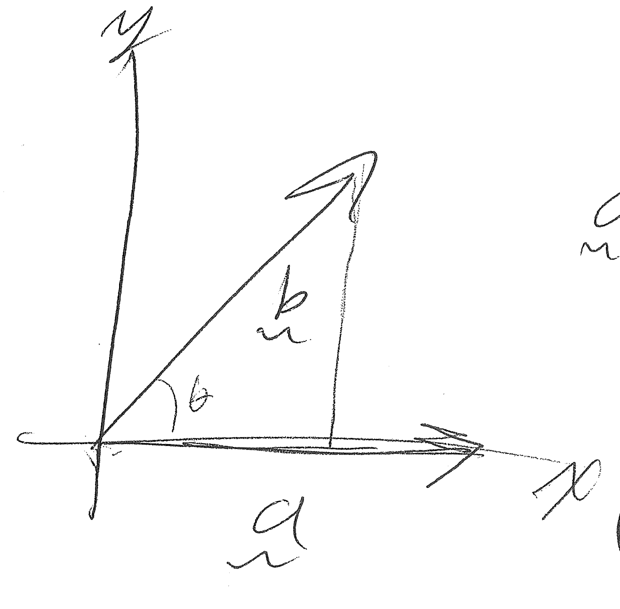
Unit:

~~Say~~ Rember coordinate axes orientation is description — everything should be the same if you change them.

So align x-axis with $\mathbf{a}$ and put the y-axis in the plane defined by $\mathbf{a}$ & $\mathbf{b}$



$$\mathbf{a} = (a_x, 0, 0)$$
$$\mathbf{b} = (b_x, b_y, 0)$$
$$\mathbf{a}\cdot\mathbf{b} = a_x b_x$$
$$= a(\underbrace{b\cos\theta}_{b_x})$$
$$|\mathbf{a}| = a = a_x$$
$$= ab\cos\theta$$

— we need to show $\mathbf{a}\cdot\mathbf{b}$ is a scalar to complete the proof

$$\mathbf{a} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$\mathbf{b} = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$$

$$\hat{x} \cdot \hat{x} = 1 \quad \text{same for } \hat{y} \text{ \& } \hat{z}$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\hat{x} \cdot \hat{z} = 0$$

$$\therefore \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z .$$

Now that we have the ~~scalar~~ dot product, the physics definition of work for a constant force is

$$W = \mathbf{F} \cdot \Delta \mathbf{r}$$

$\mathbf{F}$ constant

$\Delta \mathbf{r}$ is displacement vector

$$= \underbrace{F \Delta r \cos\theta}_{\text{a scalar quantity}}$$

The work done by force $\mathbf{F}$ acting

7-10 then displacement $\Delta \underset{\sim}{s}$

(is often how people express the physics/math operation)

my symbol for unit of → units

$$[W] = [F \Delta s] = N \cdot m$$

$$= kg \frac{m^2}{s^2}$$

The text gets coy at this point, but this has a special name

$$\text{joule} = J = kg \frac{m^2}{s^2}$$

Joule for James Joule, 19th innovation in the concept of energy & its conservation

(who actually pronounced his name Jowl rhymes with bowel.
but we like joule rhymes with drool)

— It is the MKS unit of work
energy and work

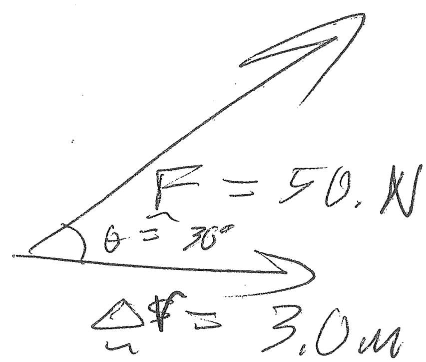
is a process of energy transformation

~~comes~~

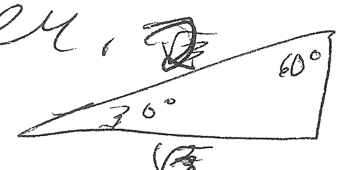
– the energy comes from one form is transformed to another form thru work.

→ we'll worry about forms later.

Ex 7.1



man pulling a vacuum cleaner.



$$W_{\text{done}} \text{ done by } \underset{\sim}{F} = \underset{\sim}{F} \cdot \Delta \underset{\sim}{r} = 50 \cdot 30 \cdot \cos 30°$$

$$= 50 \cdot 3.0 \cdot \frac{\sqrt{3}}{2}$$

$$= 150.0 \cdot \frac{\sqrt{3}}{2} \quad \cancel{2500\ J}$$

$$= 150 \cdot .9 \cong 130\ J$$

ANS 130 J.

~~§7.4 Work Done by a Varying Force~~

What if $\theta = 90°$, $\underset{\sim}{F} \cdot \Delta \underset{\sim}{r} = F \Delta s \cos\theta$

for $\theta = 90°$ $= 0$

$\cos 90° = 0$

and no work is done by $\underset{\sim}{F}$. No energy transferred.

Similarly

if $\Delta \underset{\sim}{r} = 0$

$$W = \underset{\sim}{F} \cdot \Delta \underset{\sim}{r} = 0$$

– If I push on this wall macroscopically no work is done

– ~~No macroscopic energy is transferred~~

In this case ~~to~~ no
Macroscopic energy of
motion (kinetic energy)
is transferred from
me to the wall.

— even if I strain & strain.
— Now at the micro level there
are transfers
— the wall really flexes
and so some KE is
transferred to flexing
motion that turns into
heat energy actually.
— but the wall gains ~~to~~
no macroscopic energy
and that is what we are
counting.

It doesn't start macroscopically moving.

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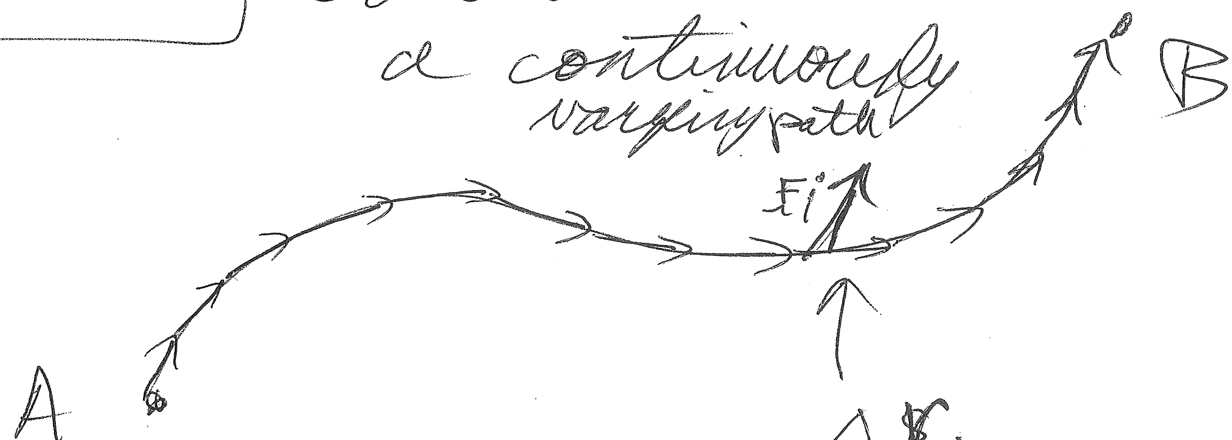
§ 7.4 Work Done By a Varying Force

Consider the work done by a ~~varying~~ constant force over a small displacement

$$\Delta W = \underset{\sim}{F} \cdot \Delta \underset{\sim}{s}$$

Now say we calculate a series of small steps in which $\underset{\sim}{F}$ varies as we move along and so does the direction of the displacements.

7-16 Consider a continuously varying path



Δr_i approximates the path

F_i is the force at the midpoint of Δs_i

$$W_{AB} \cong \sum_i F_i \cdot \Delta r_i$$

If we take the limit as the steps Δr_i become vanishingly small, but infinitely numerous we have an vector integral

May not have seen a vector integral like this, but it's comprehensible

3 summed scalar integrals.

$$W_{AB} = \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_{\mathbf{r}_A}^{\mathbf{r}_B} (F_x\,dx + F_y\,dy + F_z\,dz)$$

force a function of position $\mathbf{r}$

That the limit is the integral is something validated in calculus

— the Fundamental theorem of Calculus tell us ~~that~~ the antiderivative of a function is its indefinite integral (integral without specifying the endpoints).

— In 3-d ~~these~~ ~~two~~ operations are mostly beyond our scope although ~~they~~ might turn up. In 1-d ~~things~~ are simpler

Vector calculus needed

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and within our calculus know-how.

Say the path and force are confined to 1 d — the x-axis

$$W_{AB} = \int_{x_a}^{x_B} F\, dx$$

just x-component, which is +ve for the positive direction & -ve for the negative direction.

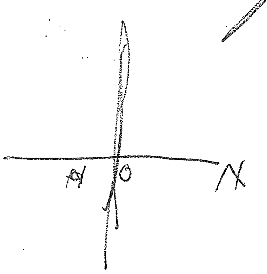
Say $\mathcal{F}$ is the antiderivative of F: $F = \frac{d\mathcal{F}}{dx}$.

Then

$$W_{AB} = \mathcal{F}(x)\Big|_{x_a}^{x_b} = \mathcal{F}(x_b) - \mathcal{F}(x_a)$$

To actually do this integral ~~there~~ analytically F must have an analytic form — in general, it won't, but in our next example force it will.

Consider the 1-d force given by



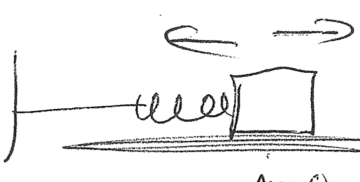
$$F = -kx$$

$$[k] = \left[\frac{F}{x}\right] = \frac{N}{m}$$

where k is a constant.

This force has several names

a) Hooke's law force since this is Hooke's law



b) Spring force – since it applies to ideal springs + real springs to a degree

c) linear force since it is linear in x

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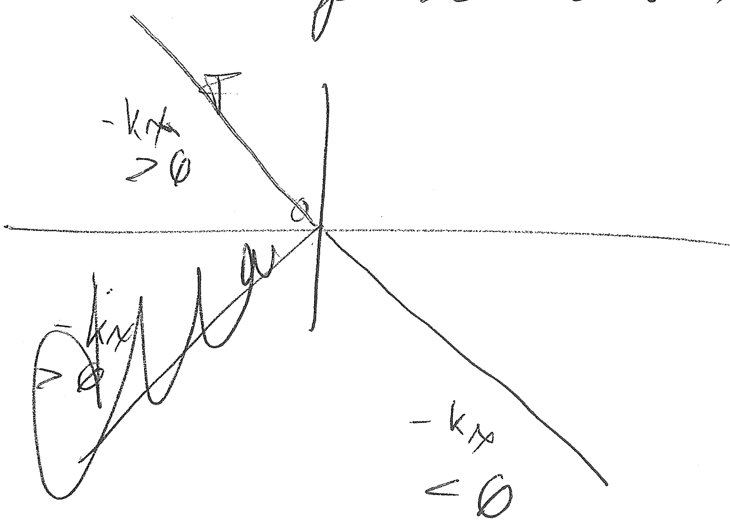
d) linear restoring force since it's linear in x and it tries to return ~~the body it acts on back to~~ ~~the eq~~ the force's equilibrium position $x = \emptyset$.



e) Simple harmonic ~~oscillator~~ ~~or~~ motion force law since a particle acted on by this force law **alone** exhibits simple harmonic motion as we'll see at a later date.

The ~~linear~~ restoring force at first glance seems like a very special kind of force with limited application.

— Actually it has an immense range of application.

— as we'll see if a body is in stable equilibrium

$F = 0$ in equilibrium. If small displacements cause a force that pulls/pushes back to equilibrium then it is stable

e.g.

stable

unstable

(and we'll discuss this ~~state~~ a bit later — but a lot of things are in it like this building), then in most cases ~~the~~ for small displacements from static equilibrium, there is a linear restoring force

87-20c)

It's all over the place
in physics & engineering
— It's safe to say that
most of you will never
be permitted to forget ~~it~~ it in
your whole career.

What is W done by an ideal spring on an object going from $x = 0$ to x_B

$$W_{0B} = \int_0^{x_B} F\,dx = \int_0^{x_B} (-kx)\,dx$$

$$= -\frac{1}{2}kx\Big|_0^{x_B} = -\frac{1}{2}kx_B^2$$

The work in going from the equilibrium point of zero force is always negative since the force ~~opposes the~~ is opposite the direction of ~~motion~~ displacement.

More generally

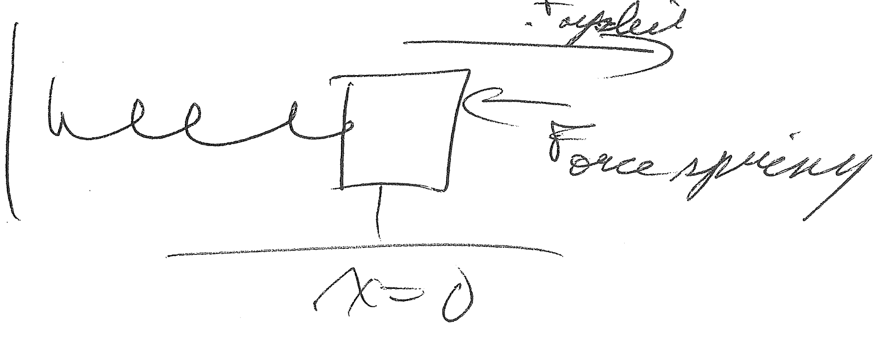
$$W_{AB} = \int_{x_A}^{x_B} (-kx)\,dx = -\frac{1}{2}kx^2\Big|_{x_A}^{x_B}$$

$$= -\frac{1}{2}k(x_B^2 - x_A^2) \lessgtr 0 \begin{cases} x_B > x_a \\ x_B < x_a \end{cases}$$

~~if $x_B > x_a$~~

Another interesting related result is the work done by an applied force to

§7-22 displace an object acted on by a ~~[illegible]~~ linear force without acceleration.

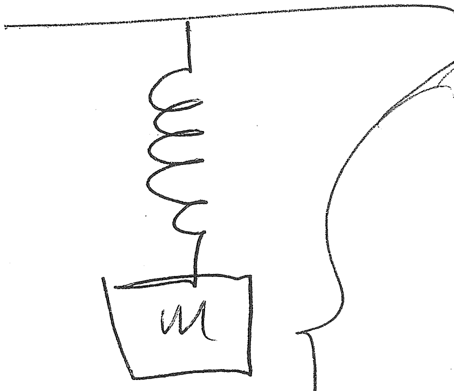


$$F_{app} = -F_{spring}$$

$$\therefore W_{app\ A B} = -W_{spring\ A\ to\ B}$$

$$= \frac{1}{2}k\left(x_B^2 - x_A^2\right)$$

Ex 7.5 Measuring k for a spring.



Hang a ~~static~~ static mass.

— this is after any oscillations have died out — a process for Ch. 15

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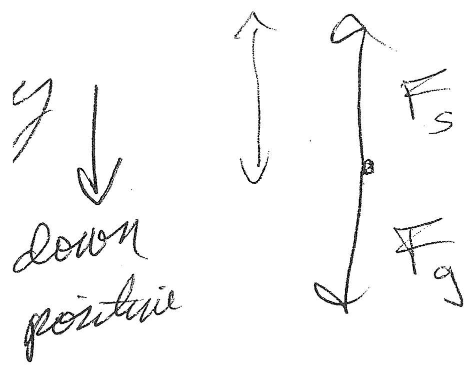

FBD

F = ma is always true
and it is always true
component by component

$$\therefore \quad m\cancel{a} = 0 = F_s + mg$$

$$= -ky + mg$$

displacement
from equilibrium
— a positive value

$$\therefore \quad k = \frac{mg}{y} = \frac{.55_{kg} \cdot 9.8\, m/s^2}{.02\, m}$$

$$\cong \frac{.5 \cdot 10}{1/50} \frac{N}{m}$$

$$= 250 \frac{N}{m}$$

Ans $270 \frac{N}{m}$

$\frac{kg}{s^2}$ but $\frac{N}{m}$ are preferred k units just a convention

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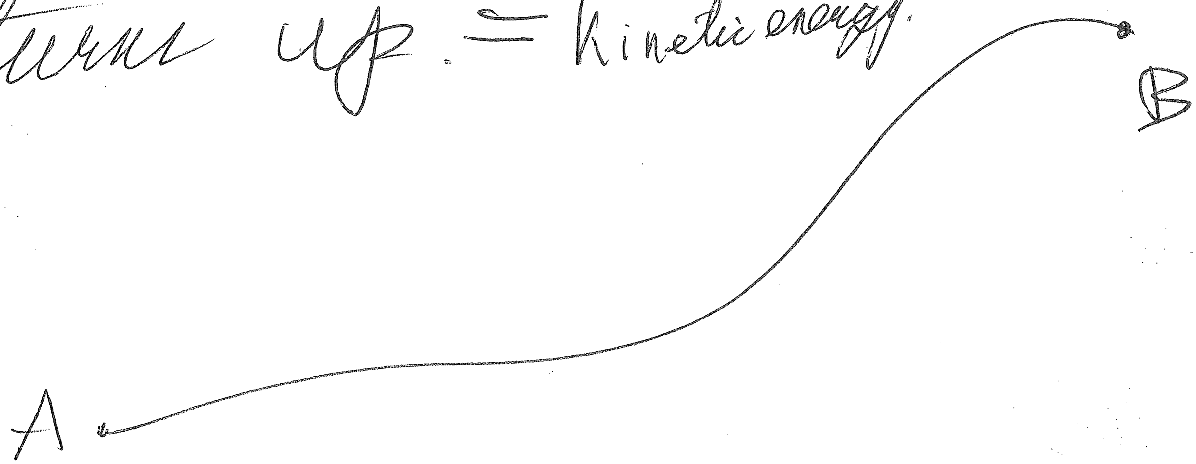
§ 7.5 Work-Kinetic-Energy Theorem

This is a really beautiful result that shows where one form of ~~kinetic~~ energy turns up = kinetic energy.



Say ~~an~~ object moves from A to B.

What is the ~~net force~~ total or net work done on it by all forces (or the net force) 27-25

$$W_{\substack{AB \\ \text{total}}} = \int_{\underset{\sim}{r}_A}^{\underset{\sim}{r}_B} \underset{\sim}{F}_{\text{net}}(\underset{\sim}{r}) \cdot d\underset{\sim}{r}$$

Here we can use $\underset{\sim}{F}_{\text{net}} = m\underset{\sim}{a}$ — Newtonian ~~dynamics~~ physics enters our ~~use~~ work-~~cont.~~ concept

$$W_{\substack{AB \\ \text{total}}} = \int_{\underset{\sim}{r}_A}^{\underset{\sim}{r}_B} m\underset{\sim}{a} \cdot d\underset{\sim}{r}$$

only valid for net force & net work. — Not for the work done by any particular force.

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt}$$

$\#7\text{-}26)$ and in a calculus result

Expect ~~[illegible]~~ equation in the infinitesimally small dt limit.

$$\left.\begin{aligned} d\underset{\sim}{v} &= \frac{d\underset{\sim}{v}}{dt}\,dt \\ &= \underset{\sim}{v}\,dt \\ \therefore \; \underset{tot}{W_{AB}} &= \int_{\cancel{t}_A}^{\cancel{t}_B} m\,\frac{d\underset{\sim}{v}}{dt}\cdot\underset{\sim}{v}\,dt \end{aligned}\right\}$$

a change in integration variable.

But

$$\frac{d\left(\frac{1}{2}v^2\right)}{dt} = \frac{d\left(\frac{1}{2}\underset{\sim}{v}\cdot\underset{\sim}{v}\right)}{dt}$$

$$= \frac{1}{2}\left[\frac{d\underset{\sim}{v}}{dt}\cdot\underset{\sim}{v} + \underset{\sim}{v}\cdot\frac{d\underset{\sim}{v}}{dt}\right]$$

product rule.
- applies to the dot product to since it applies to each term
$\underset{\sim}{A}\cdot\underset{\sim}{B} = A_xB_x + A_yB_y + A_zB_z$

$$= \underset{\sim}{v}\cdot\frac{d\underset{\sim}{v}}{dt}$$

$$= \int_{\cancel{t}\,v_A}^{\cancel{t}\,v_B} m\,\frac{d\left(\frac{1}{2}v^2\right)}{dt}\,dt$$

← can be formally integrated

$$= \frac{1}{2} m v^2 \Big|_{v_A}^{v_B}$$

$$= \underbrace{\frac{1}{2} m v_B^2} - \frac{1}{2} m v_A^2$$

We define <u>kinetic energy</u> to be

$$\underbrace{KE \equiv \frac{1}{2} m v^2}$$

(book uses K, but KE is more mnemonic)

units $kg \frac{m^2}{s^2}$ = joule just like work of course.

Leibniz in the made use of ~~the~~ quantity mv^2 and called it vis viva (living force)

Thomas Young in 1807 called it energy for the first time.

B7-28

In the 19th century, it was figured out that a $\frac{1}{2}$ was needed and since other forms of energy were introduced $\frac{1}{2}mv^2$ became

<u>kinetic energy</u> or

energy of motion

– kinetic is derived from the Greek and is related to the word cinema, but <u>cinematic energy</u> is something else altogether).

Something Steven Spielberg has.

As we've seen kinetic energy comes up using Newton's 2nd law and so is implicit in Newtonian physics as formulated by Newton, but he didn't know this.

The Work-kinetic energy theorem itself has just been derived

$$W = \Delta KE$$ or $\Delta KE = W$

in concise form

understood to be net work done on a particle by the net force.

change in kinetic energy $KE = \frac{1}{2} m v^2$

The energy to do the work comes from somewhere and we'll look into some of those somewhere's

— or goes somewhere

$W < 0$ and $\Delta KE < 0$

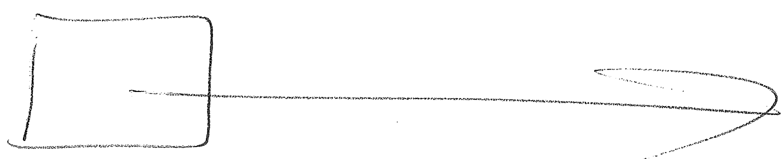
7-30

but $KE \geq 0$

always actually

since $KE = \frac{1}{2} m v^2$ always positive

Ex 7.6 1-d example.



we pull a block

with $F = 12\,N$

for $\Delta x = 3\,m$

and $m = 6\,kg$.

- This is the only force in the horizontal direction.
- Our surface is frictionless.

$$W = F\Delta x = 12 \cdot 3 = 36\,J$$

change in KE by Work kinetic energy

theorem $\Delta KE = W$

$= 36\ J$

Say $v_i = 0$

$\therefore \Delta KE = KE_f - \underbrace{KE_i}_{=0}$

$= KE_f = \frac{1}{2} m v_f^2$

What is v_f?

$$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2 \cdot 36}{6}}$$

$$= \sqrt{12} \cong 3.5\ m/s$$

I've just conflated these 3 sections in my own order.

Ans 3.5 m/s too.

§7.6 78 Potential Energy

– potential energy

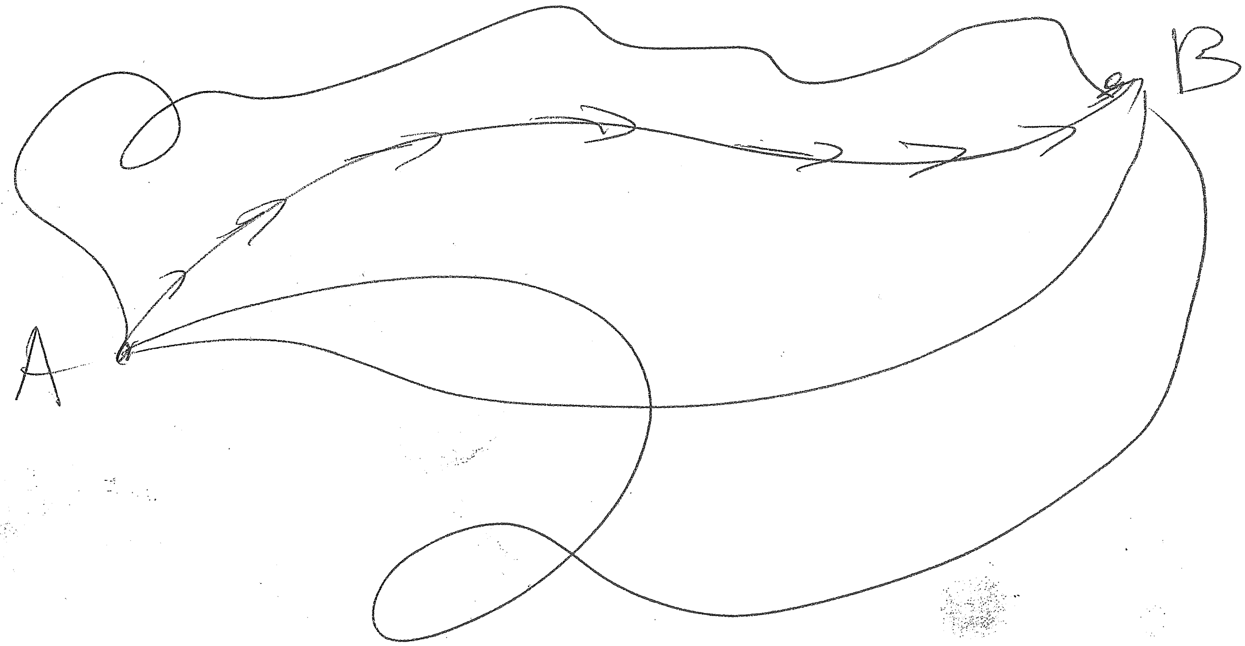
is the energy of position

in a ~~field of domain~~ region of a field force.

$\boxed{7-32}$

"field force" ...
"region" has to interpreted a bit ~~liberally~~ broadmindedly sometimes

Say that you move an object from A to B and ~~is a host~~ at both end points in a field of force.



$$W_{\cancel{applied}} = \int_A^B F_{\cancel{app}\ field} \cdot d\vec{r}$$

If this work is constant no matter what path you take

~~to overcome counter~~

exactly the field forces — not work for any other force ~~reason, e.g., accelerate the body overcome~~ friction

$F_{app} = -F_{field}$

Then you can define a <u>potential energy</u> associated with that <u>field force</u> — which is an energy of position in the field force region

~~Note sometimes the F applied can be opposite the motion if the field force is trying to accelerate the body.~~

We'll see this on p. 7-?

Note ~~the~~ that ~~fact~~ $W_{app} \left\{ \begin{matrix} > \\ < \\ = \end{matrix} \right\} 0$

are all possible.

In fact, also if

W_{AB} by any path 1

$= W_{AB}$ by any path 2

$W_{BA} = -W_{AB}$

reversing path 2

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and the work countering the field force for any closed path must be zero.

So W_{AB} independent of path implies $W_{\text{any closed path}} = 0$

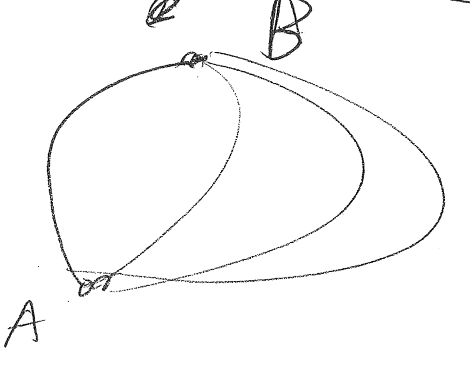
And the reverse is true too

$W_{\text{any closed path}} = 0$

implies W_{AB} is independent of the path between A + B.

W_{AB} W_{BA} on a closed path.

If you go A to B by a path, any path you come back on any path and set $W_{closed} = 0$



then clearly $W_{\cancel{BA}}$ is path independent and since B & A are general. W_{AB} is the same for any closed path too.

Now where does the potential energy ~~come~~ fit in.

Say $W_{AB} = \int_A^B \mathbf{F}_{\cancel{field}} \cdot d\mathbf{r} > 0$ for some object.

Applied force

~~just counteracts~~ the field

F_{field}

~~Then you we done~~ ~~the field done~~ object has is work by field on the ~~field~~ object and transferred energy out of ~~to~~ the field (~~energy from somewhere~~ & form) to maybe KE or chemical energy or heat energy via friction

Say $W_{AB} = \int_A^B \mathbf{F}_{\cancel{field}} \cdot d\mathbf{r} < 0$

then the field has ~~has~~ work done ~~work~~ on it ~~the object~~ and energy is transferred energy to it from somewhere e.g., KE on a pushing force.

(energy to some form — ~~maybe KE~~ or heat energy of ~~object~~ & surroundings via friction.

7-38a

the energy stored is ~~[illegible]~~ the potential energy.

We need to

~~So~~ we give the Potential energy a symbol.

— text uses U
— I use PE

both are common but PE is more mnemonic I think.

~~$\Delta PE = W = \int_A^B \vec{F}_{app} \cdot d\vec{r}$~~

~~done exactly counter field force from AB~~

~~but $\vec{F}_{app} = -\vec{F}_{field}$ itself.~~

The work creates a change in the potential energy

$$\Delta PE = -W_{field} = -\int_A^B \vec{F}_{field} \cdot d\vec{r}$$

field from AB

−ve sign: $W<0$ is energy stored in field
− and $W>0$ is energy removed from field

or ~~$PE = W$~~ ~~$\Delta PE = \Delta W$~~

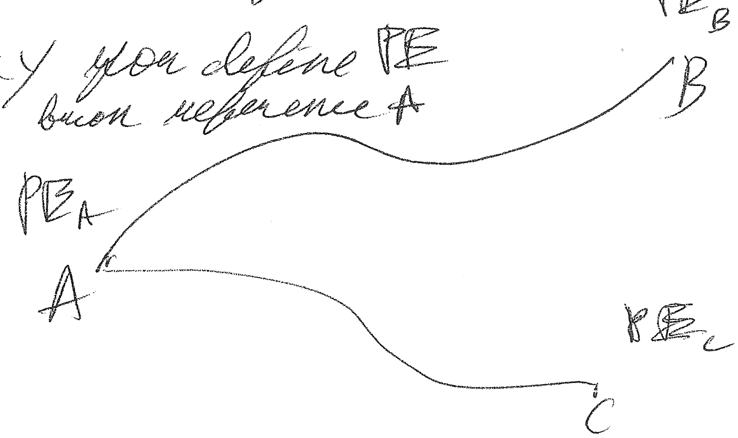
change in field PE: } $\Delta PE = -W$ in abbreviated general form

Since $\Delta PE = -W$

independent of endpoint.

– One can say each point in space has a ~~PE~~ definite PE value.

Say you define PE based on reference A



ΔPE_{AB} same by any path

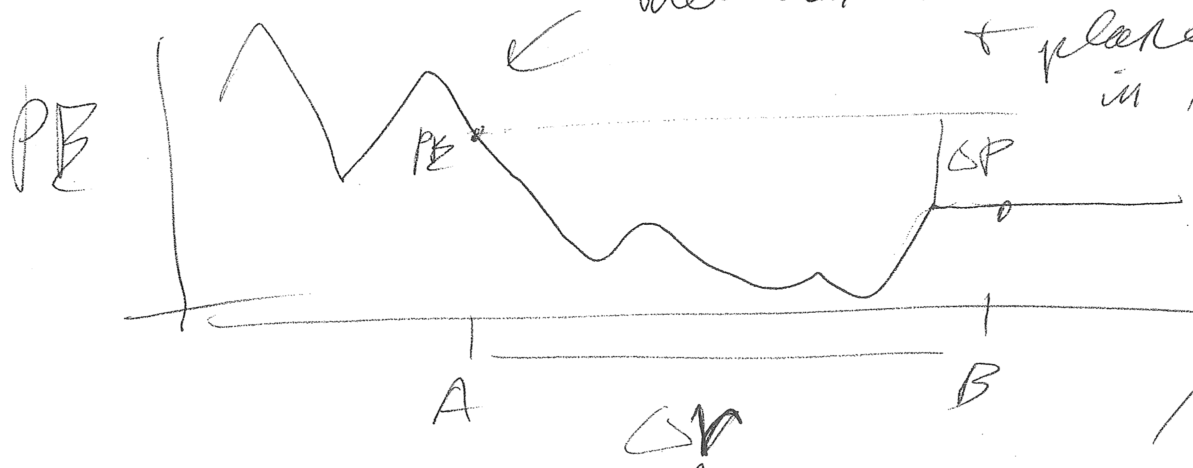
ΔPE_{AC} same by any path

$\therefore \underline{\Delta PE_{BC}} = \underline{\Delta PE_{BA} + \Delta PE_{AC}}$

~~also~~ — must be true by any path route from B to C

– Very much like height.

mountains + valleys + planes in ~~ΔP~~ PE in space



PE in 3-d space is a height in an abstract ~~4-th~~ PE-dimension

Doesn't matter how you go from A to B, the height change is the same

7-36c

Thus the heights defined from any point are the same as defined from any ~~height~~ to within a constant.

— So definite value of PE with location in space is ~~constant~~ consistent.

— But you can always add an arbitrary constant to the PE values.

— only differences in PE come into results (other than PE values themselves)

$$PE_A' = PE_A + C$$

$$PE_B' = PE_B + C$$

$$\Delta PE_{AB}' = \Delta PE_{AB}$$

Constant and so zero level of PE is arbitrary. Sometimes there is a conventional choice.

One can go backward

6

$$dPE = -\underset{\sim}{F}_{field} \cdot d\underset{\sim}{r}$$

for a differential small displacement $d\underset{\sim}{r}$

Omit

... — text does this on p. 184

$$dPE = \underbrace{\left(\frac{\partial PE}{\partial x} + \frac{\partial PE}{\partial y} + \frac{\partial PE}{\partial z}\right)} \cdot d\underset{\sim}{r}$$

$$\underset{\sim}{F}_{field} = -\nabla PE \qquad \nabla PE \text{ gradient of PE}$$

(a bit of vector calculus)

say $d\underset{\sim}{r} = \hat{s}\,ds$ where $\hat{s}$ is aligned with ~~PE~~ $\underset{\sim}{F}_{field}$

The

$$\underset{\sim}{F}_{field} = -\frac{dPE}{ds}\hat{s}$$

since

$$\underset{\sim}{F} \cdot \hat{s} = -\frac{dPE}{ds}$$

or $dPE = -\underset{\sim}{F} \cdot ds\,\hat{s}$

In 1-d

$$F_x = -\frac{dPE}{dx}$$

for instance

we will use this section & forget it.

7-36b

Just consider 1-d
since we don't have
vector calculus

$$dPE = -F_x\,dx$$

for example,

$$F_x = -\frac{dPE}{dx}$$

α [illegible] $\Delta PE = -W$

7-37

This form of the formula is abbreviated ~~and~~ general.

Note Just as work [illegible] field force is path independent, so is the work done by the field force.

It is implied that ~~ΔW is~~ the W is the work done by the field force you are thinking of in some change in position. → it's not total work or work done by other forces that may have been acting too.

ΔPE is the change in potential energy associated with the ~~ge~~ field force you are thinking of.

Jump back to 7-26b

Forces with ~~cons~~ associated potential energy are called conservative forces

(~~and have their work done independent of path~~)

7-28

— because energy put into the ~~field~~ can be gotten back directly from the field force by action of the force.

~~So~~ the field ~~does~~ conserves energy from going somewhere else

— total energy is conserved overall as we asserted, but conservative forces do so in a rather direct way.

Whether a force is conservative depends not only on its nature but also on the physical system in which the force acts. (context on local structure of the force)

Forces which ~~have~~ are conservative and have PE

— gravity in all contexts I know of

— linear force

— electric force (in many but not all contexts)

— magnetic force in ~~some contexts~~ though ~~often~~ not in some important ones and so some say it is not a conservative force.
— unless you point out the cases where it is.

— nuclear forces

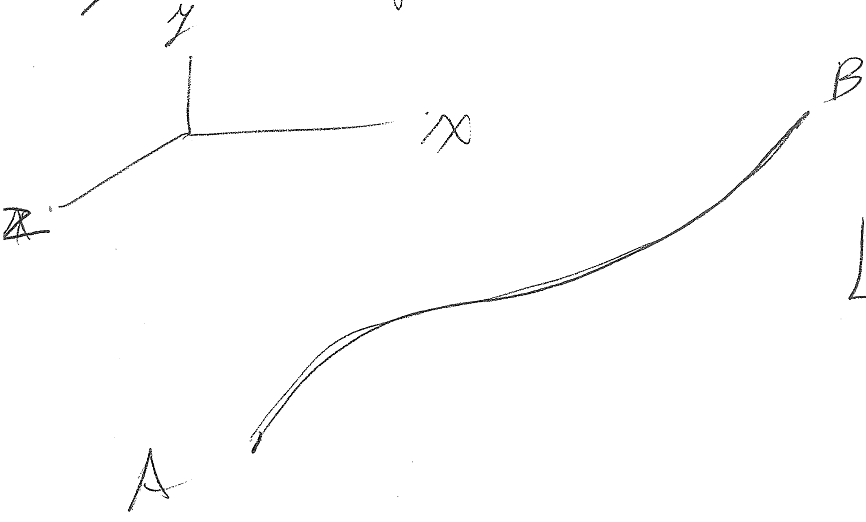
Ex Gravity

— But not in general

— only near the Earth's surface where g can be taken as constant and

We take up as positive.

$\underset{\sim}{F}_g = mg(-\hat{y})$ for an object



work done by

$$W_g = \int_{\underset{\sim}{r}_A}^{\underset{\sim}{r}_B} \underset{\sim}{F}_g \cdot d\underset{\sim}{r}$$

$$= \int_{\underset{\sim}{r}_A}^{\underset{\sim}{r}_B} (F_{xg}\, dx + F_{yg}\, dy + F_z\, dz)$$

7-10

$$= \int_{y_A}^{y_B} F_{g\,y}\,dy = -\int_{y_A}^{y_B} mg\,dy$$

$$W_{AB} = -mg(y_B - y_A)$$

Units

$$[mg\,y]$$

$$= kg\,\frac{m}{s^2}\,m$$

$$= J$$

of course. We didn't need to ~~check~~ except for sanity.

The work done is path independent.

It only depends on the end positions.

∴ gravity near the Earth's surface is conservative.

and we can say

$$\Delta PE_{AB} = -W_{AB} = mg(y_B - y_A)$$

In abbreviated general form

$$\Delta PE = mg\Delta y$$

for gravity with y upward taken as positive.

Another key point about <u>Potential energy</u>. → which we discussed before on p. 7-36c.

— Only changes in PE come into any physical prediction.

— So the ~~ze~~ zero of PE can be chosen arbitrarily for convenience.

— Sometimes there is a conventional choice — but there's none really ~~for~~ gravity near the Earth's

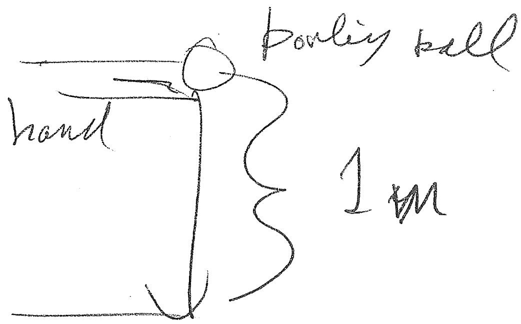
7-42

surface.

Say $y = 0$ for PE is usually chosen just for mental convenience of the problem ~~at~~ in hand.

Ex 7.8

$\therefore$ $PE = mgy$ where $y = 0$ gives $PE = 0$



Say we take ground as $PE = 0$ i.e., as $y = 0$

$m = 7\,kg$

$\Delta PE = mg\Delta y$ in general

$$\Delta PE = mg(0 - 1\,m)$$
$$\cong 7 \cdot 10(-1)$$
$$= -70\ J$$

~~Note $[mgy]$~~

~~$= kg\,\frac{m}{s^2}\,m$~~

~~$= kg\,\frac{m^2}{s^2} = J$~~

~~So the units of PE are indeed Joules which follows nicely since PE has the same units as work.~~

But if we take the level as $y = 0$, then

$\Delta PE = mg(-1 - 0)$
$= -10\,J$ the same then

Case 2 for PE

elastic
or linear force PE

The linear $F = -kx$
is 1-d in our scope

$$W_{AB} = \int_{x_a}^{x_B} (-kx)\,dx$$

$$= -k\frac{x^2}{2}\Big|_{x_a}^{x_b} = -\left(\frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2\right)$$

$$= \cancel{-k x^2 \big|_{x_a}}$$

7-~~#1~~)

$$\Delta PE = -W = \tfrac{1}{2}kx_b^2 - \tfrac{1}{2}kx_a^2$$

$$= \tfrac{1}{2}kx^2 - \tfrac{1}{2}kx^2$$

or to give an abbreviated general form

$$\Delta PE = \Delta\left(\tfrac{1}{2}kx^2\right)$$

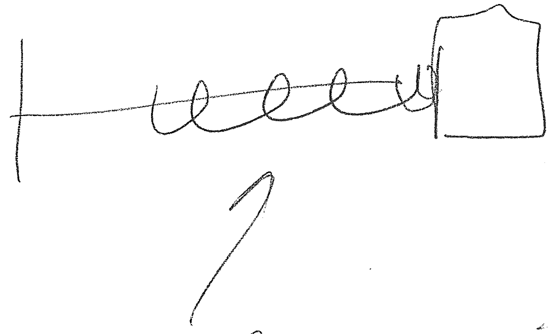
The zero point is arbitrary, but it is conventional & convenient to choose $PE = 0$ for $x = 0$

$$\therefore\ PE = \tfrac{1}{2}kx^2$$

is the conventional absolute PE formula.

Th-4PE

Conceptional example



ideal spring.

- start it oscillating.
- at $x = 0$, $PE = 0$, $KE \neq 0$
  KE_{max} since v_{max}
- at turning points $x = \pm X_{max}$ $PE = \frac{1}{2} k x_{max}^2$ is maxim, $KE = 0$ since $v = 0$.

In Chapter 8, we see that sum of KE & PE energies is conserved and $PE + KE = \text{Constant}$ in this case.

7-~~4~~6

Where is and what is potential energy actually?

Well it is somewhere and it is something.

Best example is the electric field case which ~~is~~ is beyond our scope but is in the 2nd semester.

Here we'll just say

A charge creates an electric field which causes an electric force on another charge.

→ and the electric field has an energy that one calculates from the field.

↳ a vector field → a little arrow at every point in space

a real physical thing (light is a time varying combined electric & magnetic field)

q_1

Hold in place

q_2

7-~~4~~7

Bring up another charge
on the electric
field causes a force
on q_2

But q_2 has it's own electric
field and the combined charges
create a joint field
and ~~electric fi~~ that
energy of ~~that~~ joint field
is different ~~than when they are apart.~~ than the sum of the separated fields when the charges are far apart.

and you can calculate
~~that~~ energy from the fields by an integral
(~~defined over~~ over space)
and the energy change
in bringing the charges
together & that energy
change is exactly
the ΔPE of bringing
the charges together.

7-~~7~~8

So the PE is in the ~~field~~ field structure and that is extended in space and so that is where it is.

Same is true of the other field cases (magnetism, nuclear?) — though I myself have never ~~wort~~ done a gravity calculation even as an homework problem. (There may be some qualification for gravity because it is a tricky force)

The PE concept allows you to skip all those field calculations of energy at the expense of being a bit abstract.

What of the linear force & PE?

$$F = -kx$$

$$PE = \frac{1}{2}kx^2$$

— Well just considered as an abstract force law, there is no ~~field in space~~ other (their energy)

to describe the PE
as and no
real place to say it is

But say you consider
a real spring ~~that~~ has
approximately the linear force.

— when you compress
or expand the spring
you change the chemical
bond structure in the matter.

but chemical bonds are
just electric ~~fields~~ forces at
the atomic level.

— So the ΔPE is in the changed
microscopic electric field
structure.

7-~~4~~10 ~~8~~ 7, 9

Energy Diagram and Equilibrium

A particle (or more generally a system) is in (Mechanical) Equilibrium if

$$\underset{\sim}{F}_{net} = 0$$

{ For a system this must be true for all parts & points

which means $\underset{\sim}{a} = 0$ from $\underset{\sim}{F}_{net} = m\underset{\sim}{a}$.

Equilibrium is inertial-frame independent since acceleration is inertial-frame independent as we discussed long ago.

Static equilibrium is if $\underset{\sim}{a} = 0$ and $\underset{\sim}{v} = 0$

But static equilibrium is inertial-frame dependent.

Obviously something $\boxed{7-51}$
at rest in one inertial frame can be in uniform straight line motion in another.

But for simplicity and concreteness in discussion we often just consider that we are in the frame where an equilibrium is static mostly.

Let's just consider 1-d ~~case point~~ particle case in detail.

~~Now recall~~

Now let's assume ~~all the forces that cons~~ we have some conservative force as a function of x: $F(x)$

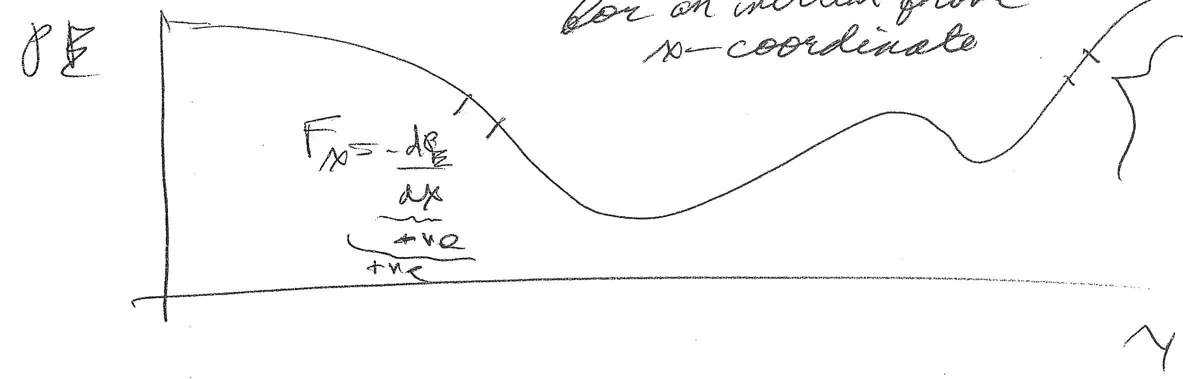
~~Therefore~~ Since it is ~~conservative~~ conservative it has an associated $PE(x)$

$$F(x) = -\frac{dPE}{dx}$$

$$\left\{ \begin{array}{l} \Delta PE = -\int_{x_0}^{x} F\,dx' \\ \frac{dPE}{dx} = -F \end{array} \right.$$

7-52

One can plot a PE landscape.
for an inertial frame x-coordinate

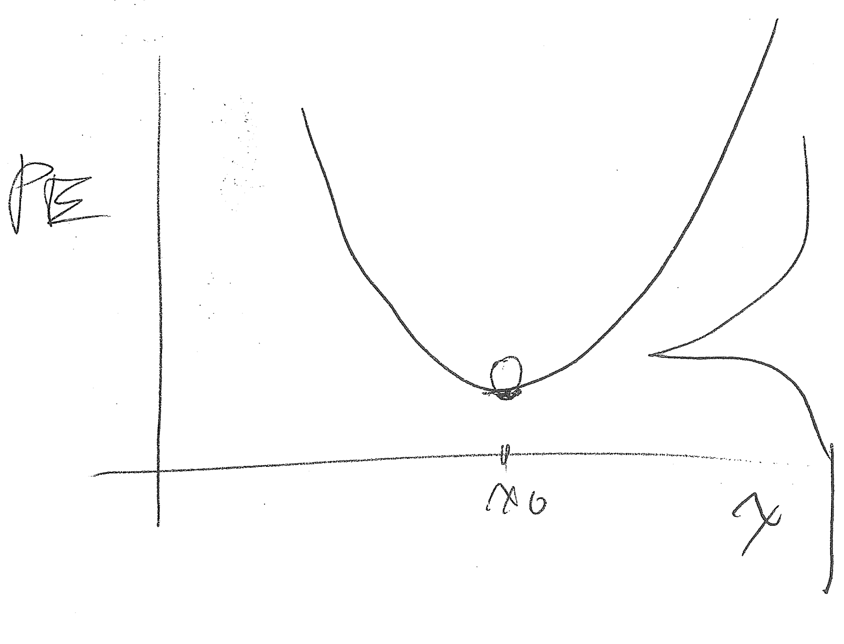


slope is $F_x = -\underbrace{\frac{dPE}{dx}}_{\text{positive}}$

-ve

The force F_x always tries to push you down the slope.

1) What of the case at the bottom of a potential energy well

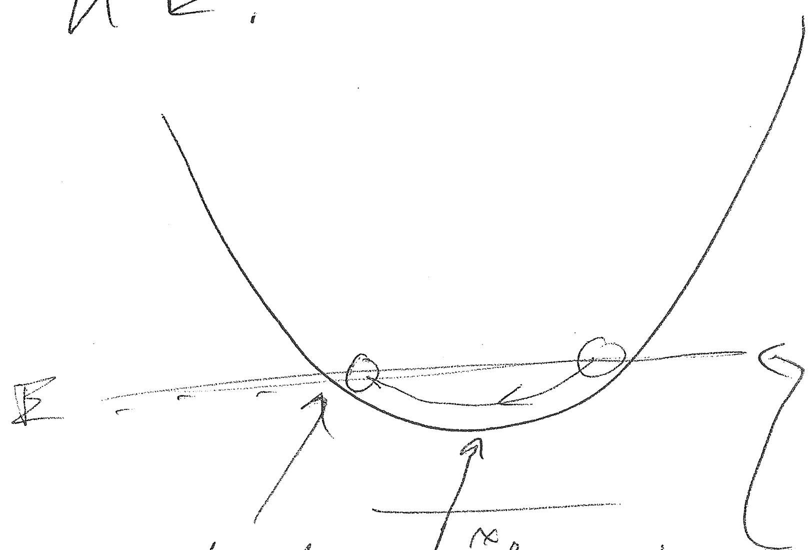


$\frac{dPE}{dx} = 0$ since it is a minimum

$\therefore F_x = 0$

And the bottom of a well is an equilibrium position.

Say the particle is at rest at the bottom of a well and then is either displaced a bit or given some KE.



Then it will oscillate back and forth

and as we'll discuss in Ch 8

mechanical energy $E = KE + PE$ will ~~stay~~ constant if no non-conservative forces act

at rest
all PE
$E = PE$
(turning point)

KE_{max}
and PE_{min}
— ~~also~~ no forces act ~~but no~~
— so instantaneously in ~~state of~~ equilibrium, but not static in the frame of the coordinate system.

This is a perpetual oscillation.

We call ~~x_0 a stable~~ the minima of ~~potent~~ PE wells

7-~~5~~7

Stable equilibria

— Perturbation from ~~the~~ static equilibrium position do <u>not</u> cause the particle to escape far away

— It stays trapped in the well.

In absence of <u>Non-conservative forces</u>

— one gets perpetual oscillation.

In most real cases there is <u>friction which</u> is Non-conservative

and it transforms the mechanical energy to heat energy and the oscillations damp out and the particle comes to rest eventually.

Which is a pretty commonly observed phenomena actually.

~~b) How case of Potential~~

Omit?

What is PE close to the minimum? x_0

We can Taylor's series about x_0.

— Show of hands on those who have seen Taylor's series.

brief description

So all or half?

since x_0 is a minimum, 0

$$PE(x) \cong PE(x_0) + (x-x_0)\left.\frac{\partial PE}{\partial x}\right|_{x_0} + \frac{1}{2}(x-x_0)^2\left.\frac{\partial^2 PE}{\partial x^2}\right|_{x_0} + \ldots \text{ higher order terms in } (x-x_0)$$

truncated to.

If $x - x_0$ is very ~~very~~ small, the higher order terms ~~vanish~~ can be considered negligible.

$$PE(x) \cong PE(x_0) + \frac{1}{2}(x-x_0)^2\left.\frac{\partial^2 PE}{\partial x^2}\right|_{x_0}$$

$$F(x) = -\frac{d\,PE}{dx} = -(x-x_0)\underbrace{\left.\left(\frac{\partial^2 P}{\partial x^2}\right)\right|_{x_0}}_{\equiv k}$$

$$= -k(x-x_0)$$

7-~~5~~6

This is just the linear restoring force.

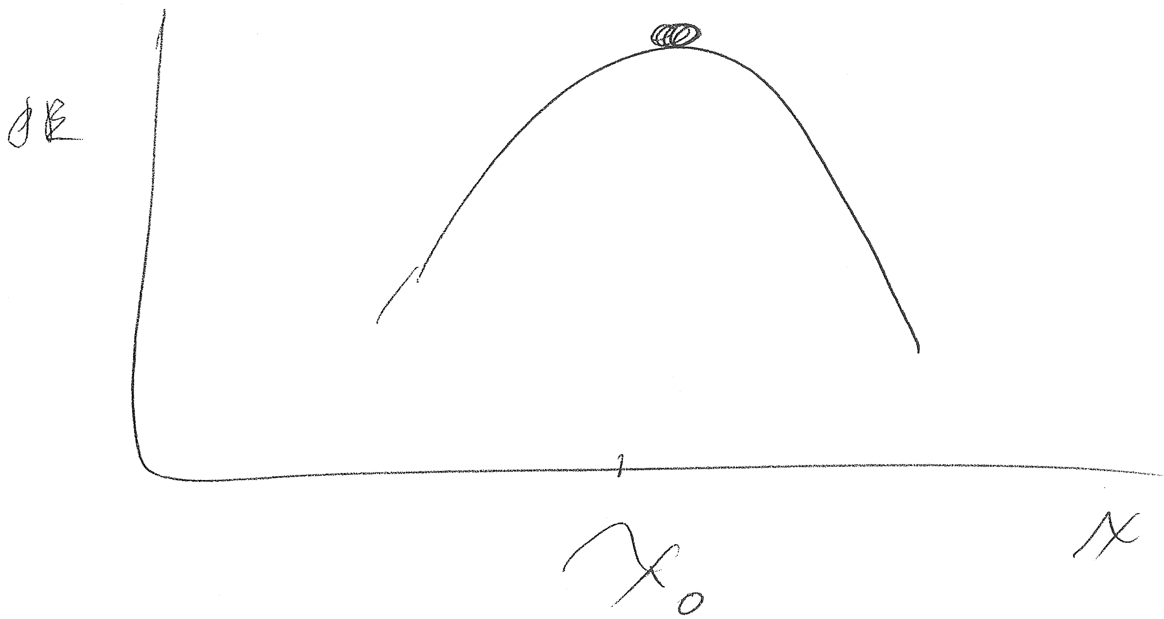
So for small perturbations about stable equilibrium the force on the particle is the linear (restoring) force

This is one reason (the main one) that the linear force is so important.

— It's actually everywhere where one has stable equilibrium.

— Not just particles but structures in 3-d like this building

b) Now consider a Potential Energy hill



Well at top a PE maximum

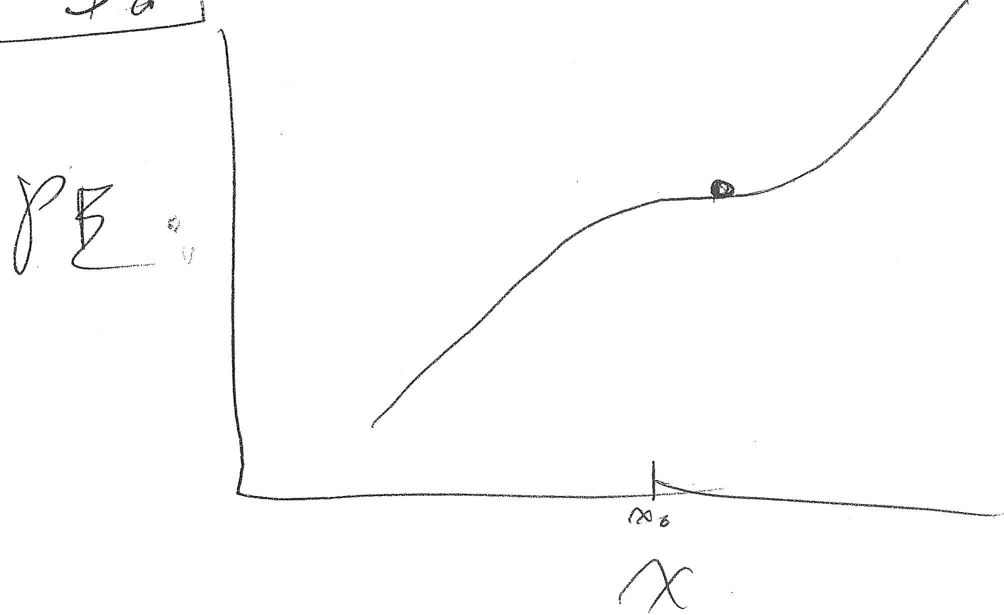
$$F(x_0) = -\frac{dPE}{dx} = 0$$

and there is an equilibrium.

But any tiny perturbation in position or an injection of KE and the conservative force will cause the particle to escape far away and not come back.

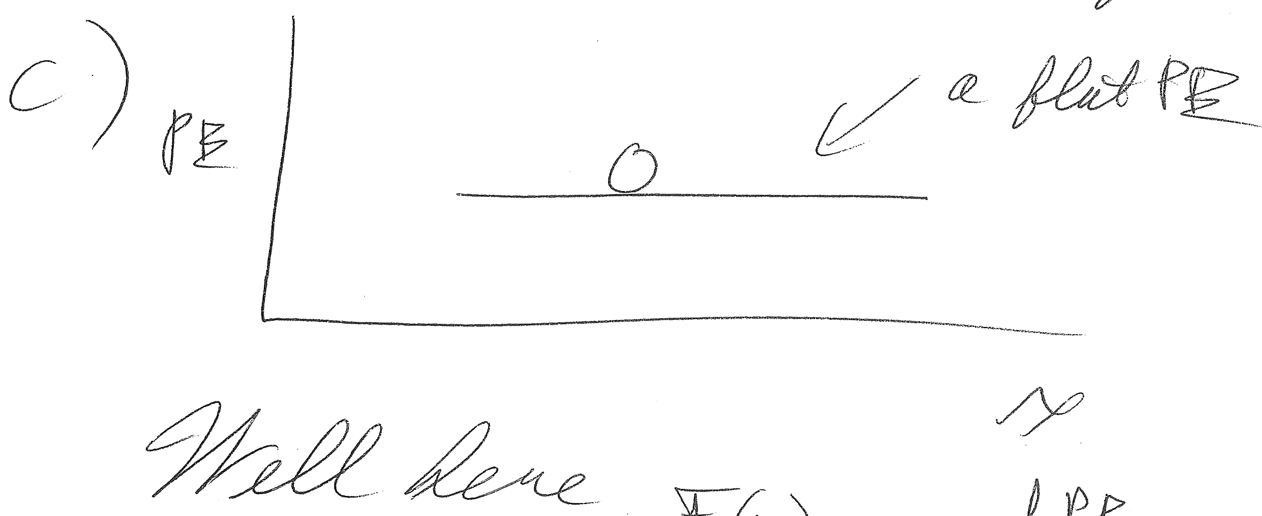
This case is called unstable equilibrium.

7-58



A stationary point like this $F(x_0) = -\frac{dPE}{dx}$ which is neither a maximum or a minimum is also an unstable equilibrium

c)



Well here $F(x) = -\frac{dPE}{dx} = 0$ over a finite region

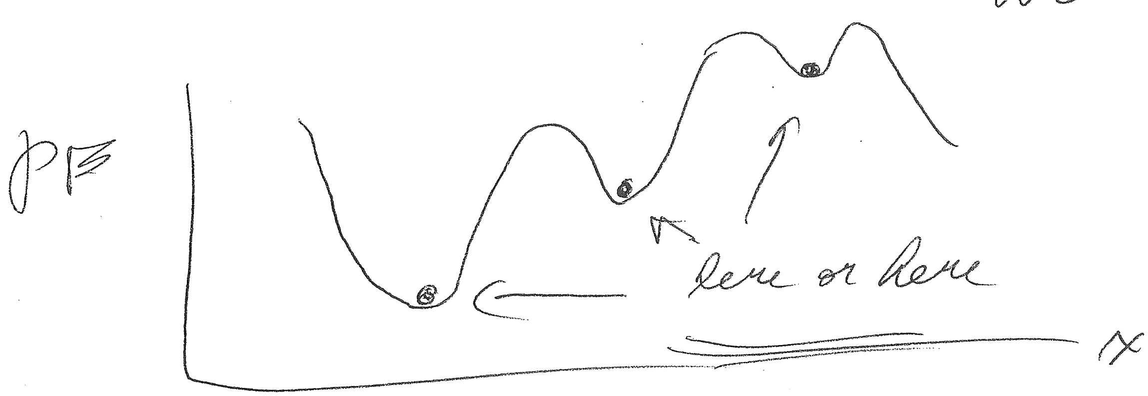
— a particle that
is put in this region
with zero KE will stay
where it is put in static equilibrium.

— If you move it a bit it
will stay in the new place
in static equilibrium

— if moving at constant velocity it is in non-static equilibrium

This is called <u>neutral equilibrium</u>.

d) A fourth case not mentioned
in the text is a particle in
a localized Potential energy
well.



The particle is in stable equilibrium
for small perturbations,
but not large ones.

7-~~6~~0

This is called
metastable equilibrium.

— Actually all stable equilibrium are metastable really.

— but whether you say stable or metastable depends on the case

stable if ~~very stable~~ the PE well is deep according to some ~~your~~ criterion

metastable if the PE well is shallow by some criterion.

The concepts of stable, unstable, neutral, metastable equilibrium generalize to 3-d and structures, but it's beyond our scope to consider those cases in detail.

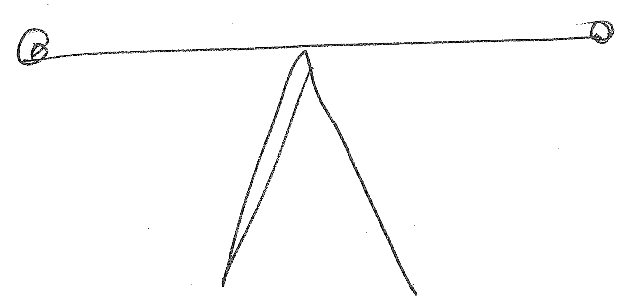
engineers will consider them one day.

Stable equilibrium is good for buildings.

But sometimes we want unstable equilibria in a mechanical system.

Can anyone think of an example?

~~A~~ A balance scale.



– the more unstable, the more accurately (potentially anyway) one can weigh an object.

Actually some slight frictional force at some level will stabilize the equilibrium and allow a perfect balance.

7-62