Chapter 2 Motion in One Dimension

SO

Kinematics is ~~the~~ study of motion without reference to causes (i.e., forces).

it's descriptive of motion

position, velocity, acceleration.

— ~~in~~ this chapter we deal only with translation along a line → 1-d motion — real exciting stuff.

usually the x-axis

x

conventional for horizontal

but ~~sometimes~~ along ~~the~~ y axis.

y

convention for vertical.

And we only consider "particles"

(we) 'll only consider "particles" for some time,

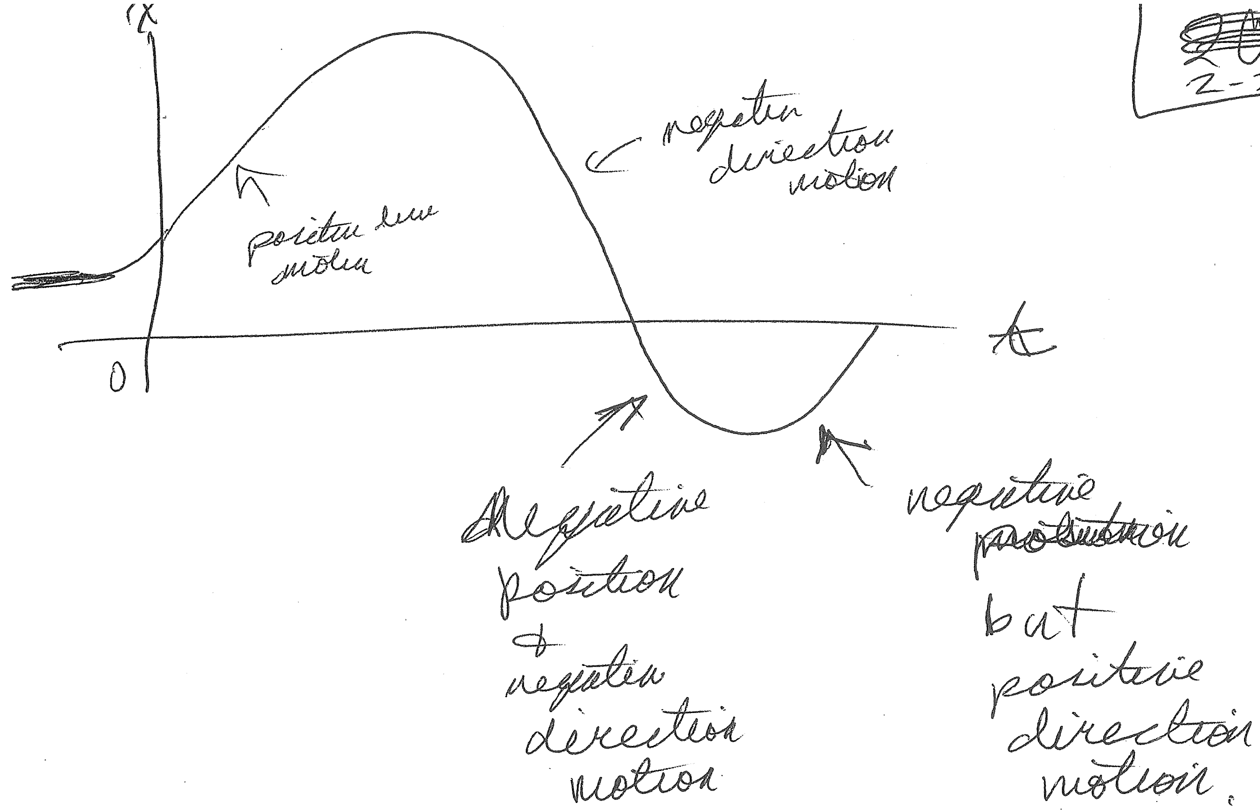
in the sense we mean here → a body ~~whose~~ without internal ~~structure~~ or whose internal ~~structure~~ can be ignored.

but eventually we get to ~~extended~~ objects with parts

— Doesn't have to be small.
— a car can be a particle in ~~this~~ sense.

§ 2.1 Position, ~~velocity~~ speed displacement, distance

We are dealing with 1-d motion, but we can plot it on a ~~graph~~ 2-d graph
x vs t



displacement of a "particle" is change in position

$$x_{initial} = x_i$$

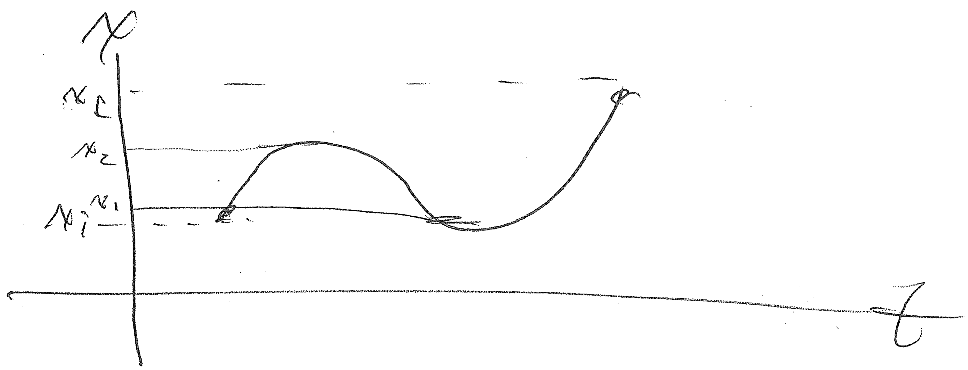
$$x_{final} = x_f$$

∴ displacement from initial to final is $\Delta x = x_f - x_i$

Δ is Greek capital delta — used to mean "change in"

~~20~~ 2-4

distance is the length of the path a particle follows.



$\Delta x = x_f - x_i$ = displacement

but distance $= (x_2 - x_i)$

$+ (x_2 - x_1)$

$+ (x_f - x_1) = \cancel{\Delta} s$

I use s for path length here. —SJ

do too sometimes

<u>Displacement</u> is a <u>vector</u>

— it has both a magnitude and a direction in 1-d (but not usually in higher Dimensions)

the direction can be ~~neg~~ expressed by sign

Only 2 direction — and so sign suffices for all cases.

$x_i \longrightarrow x_f$

$\Delta x = x_f - x_i > 0$ a positive direction displacement

$x_f \longleftarrow x_i$

$\Delta x = x_f - x_i < 0$ a negative direction displacement

distance is always positive in a formal sense.

Velocity → displacement per unit time.

It's a ratio of two quantities we usually consider more basic

— so also a vector.

— sign again ~~tells~~ direction in 1-d

Average velocity ~~for~~ for time Δt

is $v_{avg} = \frac{\Delta x}{\Delta t}$ { displacement in Δt

maybe depend on view point

A definition not traceable back to some more basic idea of velocity I think

~~2~~ 2c
~~2-6~~

Speed is distance per unit time

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

v is used for both velocity & speed (by me) — so context must decide.

Of course, one has to have some definition of time to measure time. — We won't go into the whole saga But we have oscillatory system that we believe measure equal time period = clocks

Actually I and many others say velocity when we mean speed. Usually context ~~tells~~ what we/I/you they mean.

§ 2.3 ~~2.2~~ Constant Velocity Case

~~Avg speed~~ & Instan

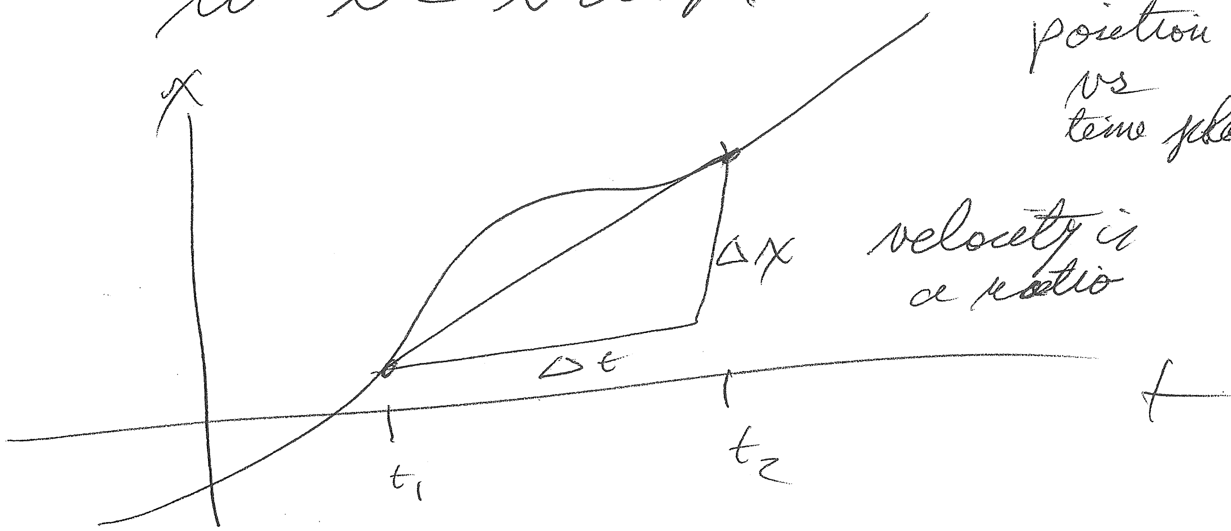
— a pretty easy special case

v constant

$x = x_0 + vt$

§ 2.2 Instantaneous velocity & speed

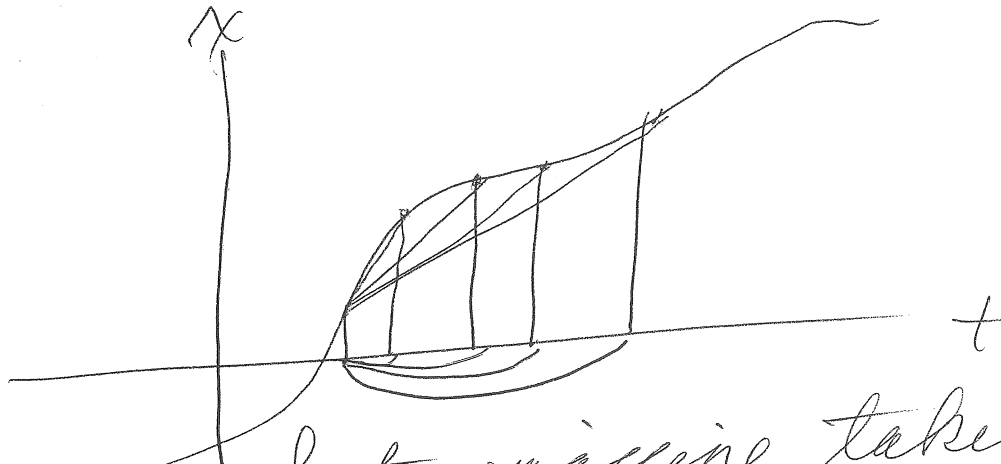
— which we usually just call velocity and speed to be brief.



position vs time plot.

velocity is a ratio

$$v_{avg} = \frac{\Delta x}{\Delta t} \text{ for } t_1 \text{ to } t_2$$



but imagine taking shorter and shorter time intervals.

2-8

Δt would get smaller, but so would Δx

In the limit as $\Delta t \to 0$,

~~so would~~ Δ

$\Delta x \to 0$.

But their ratio which is velocity would not go to zero or be undefined in general in the limit $\Delta t \to 0$

That ratio is the instantaneous velocity

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$

the derivative of x with respect time is what this is.

— for ~~those~~ well advanced in calculus, this is an old story — for the others you'll see it calculus soon.

So $v = \frac{dx}{dt}$

The dx and dt are standard calculus symbols.

— the "d" prefix means differential

So dx is the differential of x, dt the differential of t.

differentials are kind of funny animals

Δx means change in x

dx means infinitesimally small change in x.

but infinitesimal only has meaning in certain limit operations — differentiation + integration.

How does one take a ~~differ~~ derivative of x?

$\boxed{2-10}$

If no analytic function $x(t)$ is given, One can only make an approximate evaluation

$$v = \frac{dx}{dt} \simeq \underbrace{\frac{\Delta x}{\Delta t}}$$

for small measured Δt and Δx.

If $x(t)$ is given as an analytic function, then differentiation formulae exist.

Let us do one that is of immediate use in this class.

Say $x = At^n$, $\frac{dx}{dt} = Ant^{n-1}$

A is a constant

n is an exponent or power.

units

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{x(t+\Delta x) - x(t)}{\Delta t}$$

definition of a derivative

$$x(t+\Delta t) = A(t+\Delta t)^n$$

$$= A\Big(\underbrace{t^n + n t^{n-1}\Delta t}_{\text{first two terms of the binomial theorem}} + \underbrace{O(\Delta t^{l \geq 2})}_{\text{all the others have } \Delta t^l \text{ where } l \geq 2}\Big)$$

$$= \lim_{\Delta t \to 0} A\left[\frac{t^n + n t^{n-1}\Delta t + O(\Delta t^{l\geq 2}) - t^n}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0} A\Big[n t^{n-1} + \underbrace{O(\Delta t^{l \geq 1})}_{\text{goes to } 0 \text{ as } \Delta t \to 0}\Big]$$

$$\frac{dx}{dt} = A n t^{n-1}$$

'2-12

So in fact a ~~for~~ general formula for power law differentiation for p = integer

$$x = A t^p$$

then $$\frac{dx}{dt} = A p t^{p-1}$$

But we've only proven the integer case.

Actually p can be any real number, but $p = 0$ is a special case result 0 is still right

<u>Special case</u>

$n = 0$, $\quad x = A$ constant

$$v = \frac{dx}{dt} = 0$$

if position is constant, the velocity is zero

$n = 1$, $\quad x = At$ a linear function

$$v = \frac{dx}{dt} = A$$ velocity is a constant

$n = 2$, $\quad x = At^2$

$$v = \frac{dx}{dt} = 2At$$ velocity is a linear function

which is a very important result for this chapter and physics.

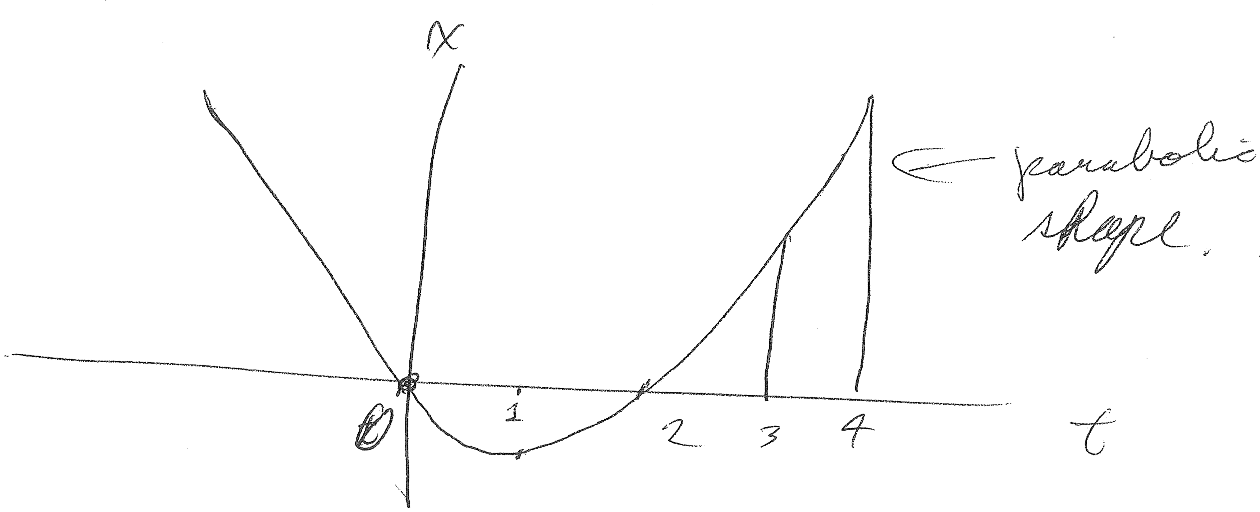
Derivative ~~of~~ for speed can be found the same way

$$v_{speed} = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

Ex. 2.3

A particle position varies as $x = 2t^2 - 4t$

x in meters
t in seconds



Let's find the average velocities from $t = 0$ to some t_{final}

2-14

$t_f = 4$, $v_{avg} = \dfrac{2\cdot 4^2 - 4\cdot 4 - 0}{4}$

$= \dfrac{16}{4} = 4\ \frac{m}{s}$

$t_f = 3$, $v_{avg} = \dfrac{2\cdot 9 - 12 - 0}{3}$

$= \dfrac{6}{3}$

$= 2\ m/s$

$t = 2$, $v_{avg} = \dfrac{2\cdot 2^2 - 4\cdot 2 - 0}{2}$

$= \dfrac{0}{2} = 0$

You have just moved back to where you started and the displacement is zero.

$t = 1$, $v_{ave} = \dfrac{2\cdot 1^2 - 4\cdot 1 - 0}{1}$

$= \dfrac{-2}{1} = -2\ m/s$

$t = \frac{1}{2}$, $v_{av} = \dfrac{2\left(\frac{1}{2}\right)^2 - 4\cdot\frac{1}{2} - 0}{1}$

$= \dfrac{1/2 - 2}{1} = -\frac{3}{2}\ m/s$

What is v at $t = 0$?
— The instantaneous velocity?

$$v = \frac{dx}{dt} = 2 \cdot 2t' - 4$$
$$= 4t - 4$$

which is general and applies for all time

$$v(t=0) = -4$$

And ~~at t=2.5~~ $v(t = 2.5s) = 4 \cdot (\frac{5}{2}) - 4$
$= 10 - 4$
$= 6$ m/s as in text.

§2.3 Constant Velocity

Say v is a constant

Rather clearly

$$\left\{ \frac{dx}{dt} = v \right.$$

$x = x_0 + vt$ is the position at any time where

I and others often use subscript 0 to mean time zero value.

x_0 is the position at time zero.

Also $\Delta x = v \Delta t$

2-16 Now

$$v_{avg} = \frac{x_f - x_i}{\Delta t}$$

In this case

$$x_f = x_0 + v t_f$$
$$x_i = x_0 + v t_i$$

$$v_{avg} = \frac{v(t_f - t_i)}{\Delta t}$$

$$= v$$

So ~~the ave~~ if the velocity is constant the average velocity for any interval is the average velocity.

(A not very surprising result.)

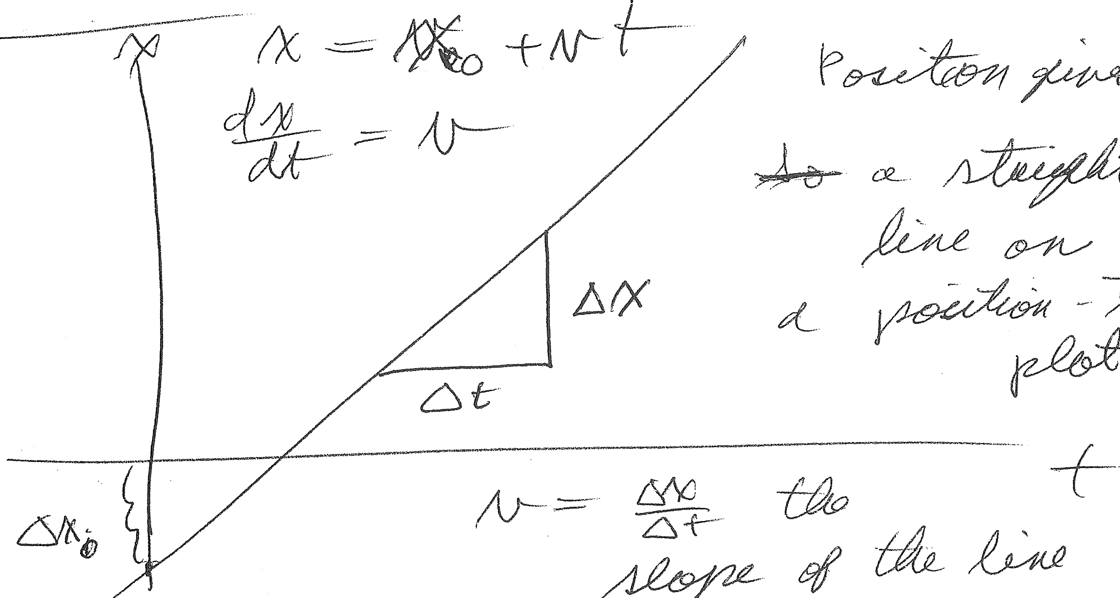
~~where x_0 is the initial position and t is the time since time zero~~

$$\Delta x = v \Delta t$$

0 out
no point

$$x = x_0 + vt$$

$$\frac{dx}{dt} = v$$



Position gives ~~so~~ a straight line on a position-time plot.

$v = \frac{\Delta x}{\Delta t}$ the slope of the line

Ex. 2.4 a runner runs $\Delta x = 20\,m$ in $\Delta t = 4\,s$

$$\therefore v = \frac{\Delta x}{\Delta t} = \frac{20}{4} = 5\,m/s$$

which is not real fast.

Olympic sprinters can do 100 m in 10 s or ~10 m/s.

The text calls the constant velocity an ~~analysis~~ model

– a model that helps understand common situation

Analysis model
is not a real common term.
I just call the constant velocity case a special of interest because it is important. It does occur in nature and in many human systems.

2.4 Acceleration

velocity is the rate of change of displacement

- acceleration is the rate of change of velocity

It is a vector also — which means in 1-d it can be negative.

slowing down when moving in the +ve direction

— or speeding up moving in the negative direction

	vector	scalar analog
displacement	$x(t)$	$s(t)$ path location
velocity	$v(t) = \frac{dx}{dt}$	$v = \frac{ds}{dt}$ speed
		really no special symbol
acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$a = \frac{d^2s}{dt^2}$

no special symbol and often called acceleration too.

2-20) ~~Graphical illustration~~

2.6 Constant acceleration

an important special case
(an analysis model
in Serway jargon)

— why important?

1) well it's easy to understand
and calulate with and
so allows one an ~~easy~~ ~~what~~ way
to understand acceleration
— the prototype acceleration case.

2) Constant acceleration are
not so common in nature
but humans can arrange them
for special purposes.

(except for the trivial zero acceleration case.)

3) There is one important special

case in nature
— freefall near the Earth's surface when air resistance can be neglected.

Fundamental theorem of calculus: the differentiation is reversed by integration and vice versa. (Wiki)

Say a is constant
the antiderivative or integral is

$\cancel{v = v_0 + a}$

$$v = at + v_0$$

$$\left(\frac{dv}{dt} = a \right)$$

v_0 is a initial constant velocity at time zero

The antiderivative of v is

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$

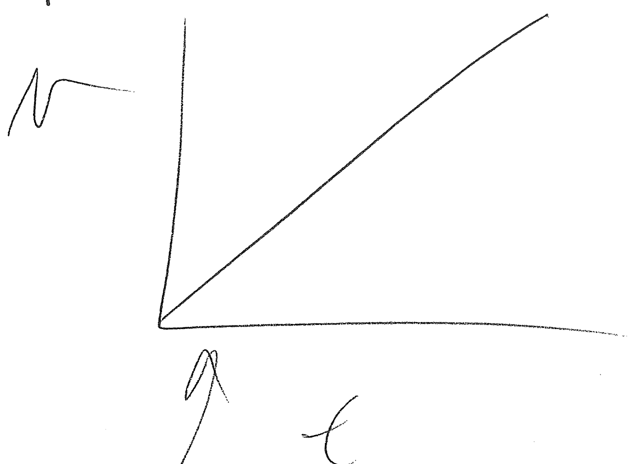
x_0: initial position at time zero.

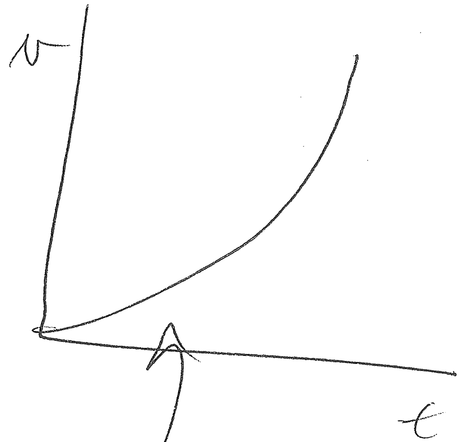
$$\frac{dx}{dt} = at + v_0 = v$$

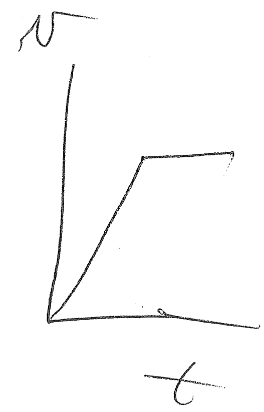
as it should.

-22 | Omit ?

Graphical illustration







constant acceleration
- slope a constant

increasing acceleration since slope increases
Not a constant acceleration case.

constant acceleration phase followed by a zero acceleration phase.

This is a <u>2-phase</u> situation.

- some ~~Most~~ of the homework #2 problems are <u>multiphase</u> problems. Each phase has constant acceleration

– the ~~final~~ conditions of one phase are the initial conditions of the next.

4 or 5 Kinematic equations for constant acceleration a

1 $v = at + v_0$

2 $x = \frac{1}{2}at^2 + v_0 t + x_0$

~~$\Delta x = \frac{1}{2}a$~~

in most problems this can be set to zero ~~or~~ arranged to be zero.

or ~~Δx~~ $x_{new} \equiv x - x_0$.

There are really all you need to solve a constant acceleration problem.

~~Unknown They are algebra~~

~~Unknowns~~

Variables v, a, t, v_0 and Δx ~~and Δx~~ (which could be ~~$x_{new} = x - x_0$~~)
$= x - x_0$

So 5 variables.

Need some extra piece of information to solve for x & x_0 separately

~~x_0 can be solved for without some~~ point in space given as a reference ?

2-24

The two equations are related by calculus, but are algebraically independent — you can't make one out of the other by algebraic means.

— So how many unknowns can you solve for?

2 in general out of 4 variables

(in ~~special cases of sets of equations you~~

General rule.

If you have N independent equations, you can solve for N unknowns

— in special cases, you may not ~~to~~ able to solve for N unknowns, but that's another story

- We can actually construct by algebraic means 2 (or 3) more kinematic equations.

they will not be independent of the first 2 and so they can't help you solve for more unknowns.

you can still only solve for 2 out of 5 variables.

But these 2 (or 3) extra equations can speed up the solution and are often convenient to use.

2-2b

4) eliminate a from (2)

$$v = at + v_0$$

so $$a = \frac{v - v_0}{t}$$

~~eliminate a from (2)~~

$$\therefore \Delta x = \tfrac{1}{2}at^2 + v_0 t$$

$$= \tfrac{1}{2}(v - v_0)t + v_0 t$$

$$= \underbrace{\tfrac{1}{2}(v + v_0)}\,t$$

recognized as v_{avg} for over time t

$$v_{avg} = \tfrac{1}{2}(v + v_0)$$

One can so it is consistent with

$$\frac{x_2 - x_0}{t} = \frac{\tfrac{1}{2}at^2 + v_0 t}{t}$$

$$= \tfrac{1}{2}at + v_0$$

$$= \tfrac{1}{2}(v - v_0) + v_0$$

$$= \tfrac{1}{2}(v + v_0)$$

~~the~~ our standard defn

3) eliminate t from (2)

~~$v - v_0 = at$~~

~~$v^2 + v_0^2 - 2vv_0 = a^2t^2$~~

$$t = \frac{v - v_0}{a}$$

$$\therefore \Delta x = \tfrac{1}{2}a\left(\frac{v - v_0}{a}\right)^2 + v_0\left(\frac{v - v_0}{a}\right)$$

$$= \cancel{\tfrac{1}{a}\left[\tfrac{v^2 + v_0^2}{2} - 2vv_0 + \ldots\right.}$$

$$= \frac{1}{a}\left[\tfrac{1}{2}(v^2 + v_0^2 - 2v_0 v) + v_0 v - v_0^2\right]$$

$$= \frac{1}{a}\left[\tfrac{1}{2}v^2 - \tfrac{1}{2}v_0^2\right]$$

$$v^2 = v_0^2 + 2a\Delta x$$

the timeless equation.

5) eliminate v_0 from (2)

this one is not in text but logically it should be,

$$v_0 = v - at$$

$$\Delta x = \tfrac{1}{2}at^2 + (v - at)t$$

assuming $x_0 = 0$ by arrangement

$$= -\tfrac{1}{2}at^2 + vt$$

Summarize 5 kinematic eqns for constant acceleration

		Missing variable
1)	$v = at + v_0$	Δx
2)	$\Delta x = \tfrac{1}{2}at^2 + v_0 t$	v
Timeless equation 3)	$v^2 = v_0^2 + 2ax$	t
4)	$\Delta x = \tfrac{1}{2}(v + v_0)t$	a
5)	$\Delta x = -\tfrac{1}{2}at^2 + vt$	v_0

2-28

~~In most~~

In problems you must have 3 known variables ~~to~~ solve for ~~the~~ 2 unknowns.

— but ~~usually~~ often the problem asks for only 1 of the 2 unknowns

One is most wanted
the other is least wanted } one might say.

The easy way to solve ~~it to~~ for the most wanted is to choose the kinematic equation where the least wanted is ~~at~~ missing.

Then it is a one unknown in one equation problem and easy to solve.

One can always solve for the least wanted unknown from the equation missing the most wanted.

— You ~~don't~~ really only need the first 2 kinematic equation. All solution can be obtained from them, but at the expense of extra algebra — and the extra mental effort in setting up the algebra.

In ~~practice~~ equation 1, 2, 3 get most of the work.

— no physical reason for this.

— it's just the way problems are usually set.

2-30

Ex 2.7

a) a jet lands ~~on~~ a carrier from $v_0 = 63\,m/s$ at ~~touchdown~~ cable

- it stops in $t = 2$ from an arresting cable.
- assume constant acceleration.

What is the acceleration?

knowns

$v_0 = 63\,m/s$

$t = 2\,s$

$v = 0$ implicit when the jet has stopped ~~this is time~~

unknowns

$x = ?$

$a = ?$

x_0 can be set to zero - point of ~~touch~~ cable catch

x is least wanted
for the moment
a is most wanted
So we choose the equation missing x.
(eq. 1)

$$\therefore \quad a = \frac{v - v_0}{t} = \frac{0 - 63}{2}$$

$$= -31.5 \ m/s^2$$

all the units of input are MKS, so the result is MKS.

b) Now what is x?
Now we have 4 knowns
— so any equation with x in it can be used to solve for x.

> let's use 4

$$x = \tfrac{1}{2}(v + v_0)t$$

$$= \tfrac{1}{2}(0 + 63)\cancel{2}$$

$= 63 \, m$ by coincidence same numeral as for time.

2-32

2.7 Free-Fall

— in the absence air resistance (or when it can be neglected such as in most our problems which say neglect it) and near the Earth's surface.

the acceleration due to gravity (acting alone) is a constant.

[actually it's not quite constant but varies a bit with latitude and altitude and local geological structures

K-33

these variations
are quite measurable and
are used in geology.
But they are below
human perception

The magnitude of this
acceleration is ~~a~~
given the symbol g

$$g = 9.8\ m/s^2$$
$$\cong 10\ m/s^2$$

is the
fiducial
or reference
value
for this course

$g_{eq} = 9.780\ m/s^2$
$g_{pole} = 9.832\ m/s^2$
at sea level

There is a decrease
with altitude.

standard gravity
$g_0 = 9.80665\ m/s^2$
is a ~~define~~
conventionally
defined
mean
value
for ~~E~~ Earth
(Wik)

2-34

We'll go into the dynamical reasons for g later

force of gravity

If you take ~~gravity as~~ positive ~~positive~~ downward

$a_y = g$ (- if problem is all down.)

if you take positive upward

$a_y = -g$ (if problem is up & down.)

y is the conventional symbol for the vertical coordinate just as x is for the horizontal coordinate.

It's a well known story ~~that~~ Galileo (1564-1642) drop balls from ~~the~~ Leaning Tower of Pisa to demonstrate ~~the constance~~ that gravitational acceleration was constant (neglecting air resistance effects)

He probably really did this.
- his first biographer Viviani (who knew Galileo) says he did.
- why doubt it.
- but we only have the bare story, no details.

It was probably more of a physics demonstration than an experiment.

Demo with pen & paper
- air resistance
  - in vacuum they'd fall the same
- denser objects less affected
- pen & Serway.

$x = \frac{1}{2}gt^2$

same x for same t and same start position, no matter what object.

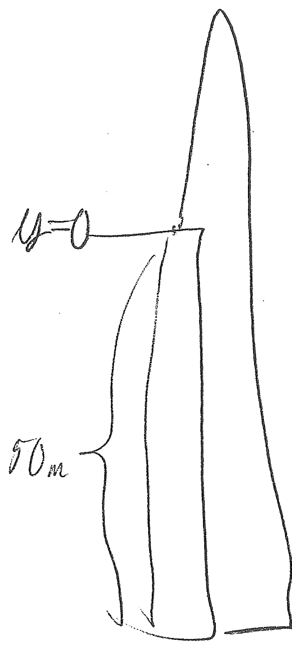
Ex 2.10

neglect air resistance

Ball thrown up from top of building

$v_0 = 20 \, m/s$

y initial is set to 0.



take upward as +ve direction
- My rule is that if all motion is downward, take down as positive.
- Otherwise take up as positive.

~~this up to~~ the choice is conventional

2-36

and doesn't affect
what happens
just the description.

a) find the maximum height
of ball.

$a = -g$ implicit given

$v_0 = 20 \, m/s^2$

$\cancel{y} = ?$

$v = ?$

$t = ?$

find

3 unknown.

Can we solve?

Yes $v = 0$ is an ~~implicit~~ implicit known.

- at y maximum the ball is at rest for an instant.

We don't know time and don't want to know it.

So the timeless equation is the one to use

$$v^2 = v_0^2 + 2a\underbrace{\Delta y}$$

Δy in this case

$$\Delta y = \frac{v^2 - v_0^2}{2a}$$

$$\doteq \frac{0 - 400}{2(-10)}$$

$$= 20\,m$$

b) Determine time to maximum height.

— well any kinematic equation would work now but eq (3) might be quickest

$\cancel{y = \frac{1}{2}(}$

$$v = at + v_0$$

$$t = \frac{v - v_0}{a} = \frac{0-20}{-10}$$

$\cancel{10s}$ $\doteq 2s$

units must work to MKS if all inputs were MKS.

2-38

c) Determine when ball returns to height from which it was launched

equn (2) looks best now

$$y = \frac{1}{2} a t^2 + v_0 t$$

$y = 0$, $a = -g = -9.8$, $v_0 = 20$

frequently multiple solution — usually only one matches the conditions of problem.

$t = 0$ solution is the initial time solution

$$0 = \frac{1}{2} a t + v_0$$

$$t = -\frac{v_0}{\frac{1}{2} a} = -\frac{2 v_0}{a}$$

$$\hat{=} -\frac{2 \cdot 20}{-10} = 4 s$$

d) Determine velocity when it has returned to the same height

— the timeless equation looks best now.

$$v^2 = v_0^2 + \underbrace{2a\Delta y}_{0}$$

$$v = \pm v_0 = \pm 20\ \mathrm{m/s}$$

two solutions

the conditions of the problem tells us which applies.

The ball was at the initial height twice — on launch and on return.

$v = 20\ \mathrm{m/s}$ is its velocity on launch

$v = -20\ \mathrm{m/s}$ is the height on return.

2.8 Kinematic Eq. Derived from Calculus

— well we already did this on p. 2-21.

— it reviews some calculus.

$\boxed{2-40}$

Finish with a book problem or two if needed.