ROTATIONAL KINEMATICS

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ABSTRACT

Lecture notes on what the title says.

Subject headings: keywords — keywords

1. INTRODUCTION

In this lecture we are concerned with **ROTATIONAL KINEMATICS**.

Kinematics recall is the description of motion without reference to causes.

The causes in Newtonian physics are **FORCES** and when causes are included one is dealing with dynamics.

But in rotational cases we can often use **TORQUES** as the causes.

TORQUE is quantity that combines displacement and force and provides a compact, efficient way to deal with forces in rotational cases.

But **TORQUE** is part of **ROTATIONAL DYNAMICS** and we leave it to the next lecture.

In fact, **ROTATIONAL KINEMATICS** in general is complex.

So we will largely concern ourselves with the special case of **RIGID-BODY ROTATION AROUND A FIXED AXIS**.

This is a very special case—but technologically it is very important—it includes wheels and many of the devices of human technology.

And it turns up sometimes in the natural world.

The Earth rotating on its axis for example. Well approximately since the Earth isn't completely rigid.

Other planets to the same level of approximation.

Stars? They are not rigid enough, except as a crude approximation.

We actually specialize further to consider the constant angular acceleration case in detail.

CONSTANT ANGULAR ACCELERATION is analogous to **CONSTANT ACCELERATION** and the mathematics is virtually identical to that of **CONSTANT ACCELERATION**.

2. RADIANS AND ALL THAT

This should be a review—even a review of a review.

There is nothing sacred about dividing the circle into 360°.

The ancient Babylonians chose this division circa 500 BCE (e.g., Neugebauer 1969, p. ?)—and they didn't tell us why.

The guess is that they liked whole number factors for easy manipulation and 360 has a lot of whole number factors—there are 24 for them—there is/was a problem to find them all.

It also maybe that 360° was convenient for astronomy—the royal science—the Queen of the Sciences—at which they were quite adept. The Sun goes around the Earth relative to the fixed stars in 365.25636042 days (this is a modern accuracy number in modern standard days) which is the sidereal year (Wikipedia: sidereal year): the year for the Sun to return to the same position relative to the fixed stars—which are not really fixed. By choosing to divide the circle into 360°, the Sun moves approximately 1° per day. This was a handy approximation—and the Babylonians knew it was an approximation.

The division into 360° is in fact arbitrary—but convenient if you are an ancient Babylonian.

You can divide the circle anyway you like.

There is a mathematically natural division which among other things simplifies the calculation of arc lengths and simplifies calculus formulae.

This division is a division into radians.

2.1. Radians

The arc length s of a fraction f of a circle of radius r is

$$s = f(2\pi r) \ , \tag{1}$$

where $2\pi r$ is the circumference of the circle.

It's just part of the nature of 2-dimensional Euclidean space that the circumference of a circle of radius r is $2\pi r$.

We just define the angle in radians subtending s to be

$$\theta = \frac{s}{r} = 2\pi f \ , \tag{2}$$

where s and r are measured in the same length units.

One can say that angle in radians is a dimensionless quantity since it is the ratio of two lengths. But from an another perspective, one can say that the two lengths are different and then a radian is a unit of one kind of length per a unit of another kind of length. It's in the eye of the beholder.

For the record, we note the following relations:

$$\theta = \frac{s}{r} \ , \qquad s = \theta r \ , \qquad r = \frac{s}{\theta} \ . \tag{3}$$

In degree measure, the fraction of a circle subtend by an angle is

$$f = \frac{\theta_{\rm deg}}{360^\circ} \ , \tag{4}$$

where $\theta_{\rm deg}$ is the angle measured in degrees.

Substituting equation (4) into equation (2) gives

$$\theta = \frac{s}{r} = 2\pi f = \frac{2\pi}{360^\circ}\theta_{\rm deg} = \frac{\pi}{180^\circ}\theta_{\rm deg} \ . \tag{5}$$

Thus the conversion relations between angles in radians and angles in degrees are

$$\theta = \frac{\pi}{180^\circ}\theta_{\rm deg} = 0.017453\ldots \text{ radians/degree} \times \theta_{\rm deg} \approx \frac{1}{60} \text{ radians/degrees} \times \theta_{\rm deg} \tag{6}$$

and

$$\theta_{\rm deg} = \frac{180^\circ}{\pi}\theta = 57.295\ldots \text{ degrees/radian} \times \theta = 60 \text{ degrees/radian} \times \theta \ . \tag{7}$$

We note that

$$1 \text{ radian} = 57.295\ldots^\circ \approx 60^\circ \qquad \text{and} \qquad f(\theta = 1) = \frac{1}{2\pi} = 0.159154\ldots \approx \frac{1}{6} \ . \tag{8}$$

2.2. Angular velocity and Angular Acceleration

ANGULAR VELOCITY, usually given in radians per unit time, is

$$\omega = \frac{d\theta}{dt} \ . \tag{9}$$

The small Greek omega ω—it's not double-u w—is the almost universal conventional symbol for **ANGULAR VELOCITY**.

An **ANGULAR VELOCITY** is also an **ANGULAR FREQUENCY** for which the symbol ω is also used. In repeating systems of any kind—they may be oscillating or periodic systems, but not necessarily—you have a quantity cycles per unit time which is frequency f.

It turns out that $2\pi f$ is often useful—and this is called angular frequency ω. The system is not going around in geometric sense—unless it is actually a rotational motion—but it is in an abtract sense, and so **ANGULAR FREQUENCY** turns out to be useful.

One has to kind the somewhat distinct meanings of **ANGULAR VELOCITY** and **ANGULAR FREQUENCY** clear.

ANGULAR ACCELERATION, usually given in radians per unit time per unit time, is

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \ . \tag{10}$$

The small Greek omega α is a conventional symbol for **ANGULAR ACCELERATION**. It don't think it is as widely used as ω for angular velocity, but it's common in intro texbooks anyway.

Note that arc length, tangential velocity, and tangential acceleration can be obtained from angle, angular velocity, and angular acceleration:

$$s = r\theta \ , \qquad v = r\omega \ , \qquad a = r\alpha \ . \tag{11}$$

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