Chapter 16 Electromagnetic Waves
(AKA Electromagnetic Radiation)

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16.1 A Tiny Bit of History of the Understanding of Light

Vague history NOT Google AI-ed into detailed exactness

a) All physical waves we know of show interference and diffraction effects.

Note; the words interference and diffraction are synonyms actually. Which term is used depends on context:

Interference is the interaction of wave sets giving rise to constructive interference (they add up) to destructive interference (they tend to cancel).

i) Interference for few sources of waves.

ii) Diffraction for many or a continuum of source of waves.

iii) But these "rules" are NOT strictly obeyed and no one objects.

And light shows these as first noticed only in the 17th century because vaguely speaking the size of these effects scales as

$$\frac{\lambda \text{ (wavelength)}}{\text{Obstacle size}}$$

The size scale of bands (called fringes) of constructive interference (bright fringes) and destructive interference

The obstacle breaks a wave front leading to interference

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and for visible light

$$\frac{\lambda \sim 10^{-6}\ m}{1\ m} = 10^{-6}$$

is very small.

Even for monochromatic light the fringes at the edges of shadows are minute and hard to see.

Since interference is wavelength dependent the diffraction pattern are washed out by polychromatic light and stray reflections from all over in uncontrolled experiments (i.e., casual observations)

Chr. Huyghens in the 17th century thought light was waves based on the very small detected interference and diffraction effects noticed by himself and others earlier on.

b) But geometrical optics treating light just as rays or as streams of particles (as thought Newton) work so well for optical instruments (e.g., telescopes, microscopes) that only after ~1800 was the wave nature of light accepted.

c) The speed of light was first crudely determind in the 17th century and by ~1850 was known close to the modern value $c = 3 \times 10^{8}$ m/s

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16.2 James Clerk Maxwell (1831—1879) And Displacement Current

a) Maxwell is certainly one the great theoretical minds of physics and an experimenter too. He invented the first color photography.

b) In the 1860s, Maxwell collected the already-discovered electromagnetic results and formulated them as a set, but NOT compactly in 4 equations as we know them.
The 4 equation formalism relies on modern vector formalism only gradually introduce after ~1880 (Wiki: vector notation: History)

Maxwell noticed an anomaly with Ampère law.

We're are here and below thinking of the ideal Maxwell, NOT of the actual messy details of exact history where Jamie was always erasing and forgetting where he'd left his notebook, etc.

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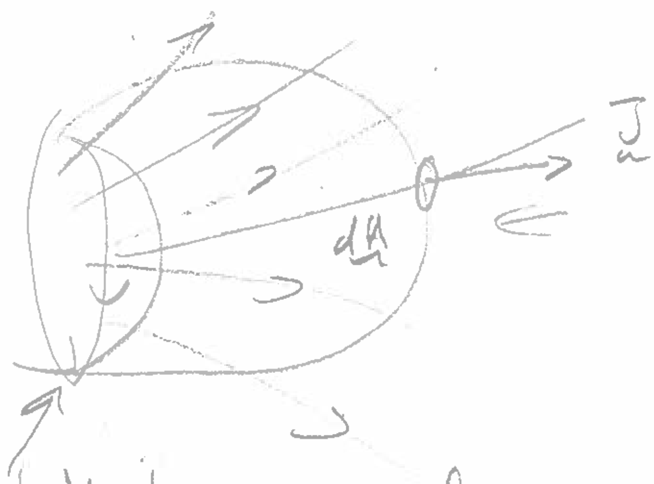
c) Ampère's law in integral form

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I = \mu_0 \int_{\text{linked surface}} \underset{\sim}{J} \cdot d\underset{\sim}{A}$$



Ampèrian loop linking surface areas and any surface area gives the same result for I for steady current due to conservation of charge.

But what if you have capacitor?

and $I_3 = I_1$ for a capacitor charging or discharging

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 \int_{A_1} \underset{\sim}{J} \cdot d\underset{\sim}{A} = \mu_0 \int_{A_3} \underset{\sim}{J} \cdot d\underset{\sim}{A}$$

$$\neq \mu_0 \int_{A_2} \underset{\sim}{J} \cdot d\underset{\sim}{A} = 0$$

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You could say that this a just so, but on the other hand, the B-field is a field somehow created by E-field of moving charge.

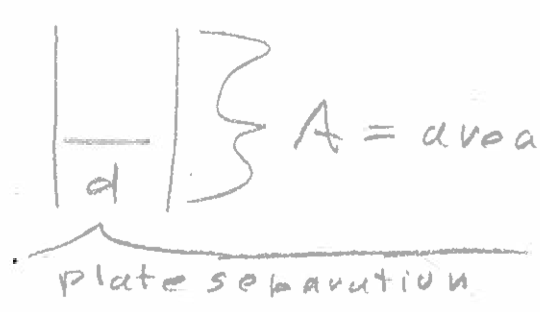
Why should this E-field effect be just NOT calculable across a capacitor gap when it is from both sides.

Well, what is the E-field between the capacitor plates?

$C = \frac{Q}{V}$ in general and

from Gauss' law, the capacitance of (ideal) parallel plate capacitor is

$C = \varepsilon_0 \frac{A}{d}$



$$\varepsilon_0 \frac{A}{d} = \frac{Q}{V} = \frac{Q}{Ed}$$

The E field between the plates which ideally constant

$$\therefore \quad Q = \varepsilon_0 E A$$

This is the actual current flowing into the capacitor plates

$$I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$= \underbrace{\varepsilon_0 \frac{dE}{dt}}_{} A$$

This is ideal capacitor result equals the actual current. The ideal capacitor has E only between the plates and it is constant there

This could be an effective current density

and $I = I_1 = I_3$ from our diagram on p. 02_16006

The then plausibly it is NOT just a coincidence that

$$\oint \underset{\sim}{B} \cdot d\ell = \mu_0 \int \varepsilon_0 \frac{d\underset{\sim}{E}}{dt} \cdot d\underset{\sim}{A}$$

for Area 2 as long as the capacitor approximates the ideal capacitor well enough and contributions from outside the capacitor are negligible

But if the result is only approximately true with errors in evaluating $\frac{d\underset{\sim}{E}}{dt}$ for Area 2, $\longrightarrow$

Then it could be exactly true if applied to the whole surface area A_2 with the exact $\frac{\partial E}{\partial t}$, and if that is true, then $\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ should be a contribution to Ampère's law everywhere.

Maxwell hypothesized this is the exact Ampère and, of course that turned out to be true

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 \int_A \underline{J}\, d\underline{A} + \mu_0 \varepsilon_0 \int_A \frac{d\underline{E}}{dt} \cdot d\underline{A}$$

Now this is the integral form of Ampère law.

The differential form is

$$\nabla \times \underline{B} = \mu_0 \left[\underline{J} + \varepsilon_0 \frac{d\underline{E}}{dt} \right]$$

$\nabla \times$ is the curl differential operator and $\nabla \times \underline{B}$ is the curl of $\underline{B}$

If you know NOT curl, just accept it.

Maxwell called this the displacement current

A modern physicist would probably have called it a virtual current, But for some reason Maxwell decided on displacement.

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16.3 The Electromagnetic Wave Equation

a) Two of Maxwell's equations:

Faraday's law

Ampère's Law with displacement current

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t} \qquad \nabla \times B = \mu_0 \underset{\sim}{J} + \mu_0 \varepsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t}$$

They are very symmetrical and this symmetry probably also made Maxwell think the displacement current had to be a real effect.

The minus sign difference is rooted in conservation of energy and the $\mu_0 \underset{\sim}{J}$ difference is due to the lack of magnetic monopoles.

I said once 2nd order DEs seem necessary for dynamics, but coupled (interacting) 1st order DEs

They are 1st order linear partial differential equations

can amount dynamics too and they often can be combined into 2nd order DEs and this true of Maxwell's equations

Partial because there are more than 1 ~~independent~~ variables: there are 4: time and 3 spatial dimensions.

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b) These two Maxwell equations are coupled to each other and in the absence of $\underset{\sim}{J}$ can be read as

a time varying E-field creates a B-field

and

a time varying B-field creates an E-field.

Maxwell noticed this and wondered what would happen in charge free space if you substituted one into the other thusly

$$\nabla \times [\nabla \times E] = -\frac{\partial(\nabla \times \underset{\sim}{B})}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial^2 \underset{\sim}{E}}{\partial t^2}$$

Now for a vector calculus identity

$$\nabla \times [\nabla \times E] = \nabla(\nabla \cdot \underset{\sim}{E}) - \nabla^2 \underset{\sim}{E}$$

Recall Gauss' law $\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\varepsilon_0}$, but if $\rho = 0$, $\nabla \cdot \underset{\sim}{E} = 0$

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$$\nabla^2 \underline{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \underline{E}}{\partial t}$$

∇^2 = Laplacian operator
∇ = nabla symbol, and ∇f is the gradient of f, where f is a scalar and ∇f is a vector.

$$\nabla^2 \underline{E} = \sum_i \frac{\partial^2 E_i}{\partial x_i^2} = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

This is 3-dimensional linear wave equation that is exactly analogous to the linear wave equation for weak waves on a string and weak sound waves

But there is NO limitation here to weak waves in classical electrodynamics.

But super strong EM-fields due cause failure due to quantum electrodynamic (QED) effects (Google AI),

You can do the same for the B-field and so together

$$\nabla^2 \underline{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \underline{E}}{\partial t^2} \quad \text{and} \quad \nabla^2 \underline{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \underline{B}}{\partial t^2}$$

But $\underline{E}$ and $\underline{B}$ are still coupled via Maxwell's equations cnp. D2_16010,

and so E-field and B-field waves

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So there are NO pure electric waves and NO pure magnetic waves

CANNOT self-propagate through space independently of each other, but they are independent of charge.

c) Electromagnetic waves (EM waves) are created and destroyed by charge (as we discuss later), but they can travel across the universe long after their source system has been destroyed and long before their receiver system has come into existance.

So they are real things, NOT just some mathematical abstraction,

d) And we recognize their phase speed as

$$v_{phase} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \triangleq \frac{1}{\sqrt{4\pi \times 10^{-7} \cdot 9\times10^{-12}}}$$

$$\triangleq \frac{1}{\sqrt{10^{-17}}} = \frac{1}{\sqrt{10}} \times 10^{9} \triangleq 3\times10^{8}\,m/s \triangleq c$$

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and so the vacuum light speed is derived from electromagnetic constants measured from pure electromagnetic experiments.

Maxwell did the phase speed calculation (and though his values were NOT as good as modern ones), still the agreement was striking and Maxwell at least tentatively accepted the idea light was EM waves.

The story will continue in a more qualitative mode.