

2026 aug 01

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ev $\rightarrow$: Electric Charge
and Electric Fields
in the Electrostatic limit
(the sources of the electric
field are at rest or
can be approximated
as being at rest)

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Introduction, Overview, Preview of Some Aspects, And Context in Which Aspects

5.1 Classical Electromagnetism (E&M) ^{usual} ^{and} ^{Enrich} (or Classical Electrodynamics) (short-hand and Touching on Many Other Topics of the Course

a) The basics of (Classical) E&M were presented in the 19th and early 20th centuries. Of course, there was a long history of developments and discoveries going back to classical Antiquity and developments in techniques and applications continue.

We will NOT do this since NOT educational good and NOT everyone has had

b) A logical/mathematical presentation of E&M would start with

Vector calculus but this is an over and as we go through the course we intro bit

Maxwell's equations (a fundamental axiom of classical E&M)

Vacuum form in SI (MKS) units (G-519)

$$\begin{aligned}\nabla \cdot \underline{E} &= \frac{\rho}{\epsilon_0} \quad \left\{ \begin{array}{l} \text{Gauss' law} \\ \text{differential form} \end{array} \right. \\ \nabla \cdot \underline{B} &= 0 \quad \left\{ \begin{array}{l} \text{Gauss' law for magnetism} \\ \text{differential form} \end{array} \right. \\ \nabla \times \underline{E} &= - \frac{\partial \underline{B}}{\partial t} \quad \left\{ \begin{array}{l} \text{Faraday's law of induction} \\ \text{differential form} \end{array} \right. \\ \nabla \times \underline{B} &= \mu_0 \left(\underline{J} + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \right) \quad \left\{ \begin{array}{l} \text{Ampère-Maxwell law} \end{array} \right.\end{aligned}$$

4 partial vector differential equations

Note, squiggle under

= Vector in my handwritten notes.

It is a common, but not universal formalism

The interaction between $\underline{E}$ and $\underline{B}$ makes the dynamic equations even though they are 1st order

ρ is electric charge density

$\underline{J}$ is electric current density (a vector)

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$\underline{E}$ is electric field
the cause of the
electrostatic
or Coulomb force

$\underline{B}$ is magnetic field
the cause of the
magnetic force

(Why B not M ? Some historical reason)

ϵ_0 vacuum permittivity } constants
 μ_0 vacuum permeability } we will
deal with
later

Recall differential equations (DEs)

tell how things change
with respect to other things.
Hence, they contain derivatives.

or
which
you always
need

boundary
conditions
which
are
included
initial
conditions

Most physical laws are formulated
in terms of DEs.

For physical law DEs tell you
what is true at every point
in spacetime (within their
realm of applicability)

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which may be large or small,
— and for Maxwell's equations
it's very large).

However, you have to
connect the spacetime dots
to solve for a system
and get its behavior.

This requires imposing
boundary conditions
(including initial conditions).

d) Maxwell's equations tell you
E-field and B-field
evolve in response
to electric charge and current
and to themselves.

However, you also need
to know

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how matter (which carries charge and current)

responds to $\mathbf{E}$ & $\mathbf{B}$ -fields.

This response is specified by the Lorentz force
(G-198)

$$\mathbf{F}_L = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

point charge

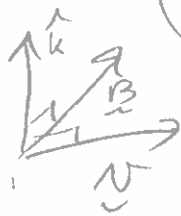
particle velocity

Vector (cross product)

A second fundamental axiom of classical EM in the nonrelativistic limit when relative velocities $v \ll c$

Recall your vector (cross) product behavior

$\mathbf{A} \times \mathbf{B} = AB \sin \theta \hat{k}$ and the right-hand rule.



The magnetic part of the Lorentz force is velocity dependent

Recall in the classical limit (i.e., the Newtonian physics limit)

$$\mathbf{F}_{\text{net}} = m \mathbf{a}$$

for a point mass relative to an inertial frame.

If only the Lorentz force acts $\mathbf{F}_{\text{net}} = \mathbf{F}_L$,

$$\text{and } m \mathbf{a} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

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Actually, there is an important complication, we do NOT treat in this course beyond mentioning.

An accelerating charge radiates electromagnetic radiation (EMR) and this causing the charge to lose energy and decelerate relative to an inertial frame (G-432-4). There is a sort of "friction" called the radiation reaction force.

In general an E+M system can be complex:

- changing E & B-fields
- accelerating charge

(G-434)
which we usually treat as negligible in this course except when we discuss EMR

All things interacting requiring an overall self-consistent solution — but

But in this course, we consider relatively simple systems.

this true in all complex physical systems

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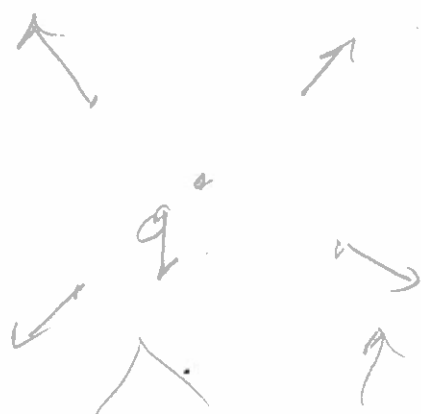
e) Electric Field, Magnetic Field, the Electromagnetic Field

Now it seems there are
two distinct fields: E-fields
and B-fields.

However, they are really two
manifestations of the
same thing the electromagnetic field.

To illustrate the point.

A: charge a rest causes
an E-field



charge

representative
E-field vectors

— they have
direction in real space
but their size is their
strength in an abstract space

But a moving
charge causes
an E-field
and a B-field.

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But if you move with a moving charge, you see only an E -field.

So E -field & B -field mix their identities depending on motion of source and observer.

But it's still true E -field and B -fields seem very distinct in most manifestations and we will NOT in this course go into a general treatment of the special relativistic transformations.

f) Electromagnetic Radiation (EMR)

One of Maxwell's discoveries in the 19th century was that you can derive from Maxwell's equations separate wave equations for E -fields and B -fields, but they are implicitly coupled by Maxwell's equations.

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As a preview in vector form

$$\nabla^2 \underline{E} = \frac{1}{(\frac{1}{\sqrt{\epsilon_0 \mu_0}})^2} \frac{\rho^2 E}{\rho t^2} \quad , \quad \nabla^2 \underline{B} = \frac{1}{(\frac{1}{\sqrt{\epsilon_0 \mu_0}})^2} \frac{\rho^2 B}{\rho t^2}$$

and for 1-d for plane waves

ie E-
d B-fields
are NOT
dependent
id both are
needed for

$$\frac{\rho^2 E}{\rho x^2} = \frac{1}{(\frac{1}{\sqrt{\epsilon_0 \mu_0}})^2} \frac{\rho^2 E}{\rho t^2} \quad , \quad \frac{\rho^2 B}{\rho t^2} = \frac{1}{(\frac{1}{\sqrt{\epsilon_0 \mu_0}})^2} \frac{\rho^2 B}{\rho t^2}$$

There are, in fact,
the standard
linear wave equation



(Textbook Vol. 1 p. 764)

and $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c = 2.99792458 \times 10^8 \frac{m}{s}$

The phase velocity
the velocity of
wave propagation
not the speed
of oscillation

vacuum
speed of
light speed
relative to
all inertial
frames.

It is frame
invariant
which is an axiom
of special relativity

Note ϵ_0 and μ_0 are
constants set by electromagnetic phenomena
NOT light observations.

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What Maxwell conjectured was light was a self-propagated electromagnetic.

It needs both E- & B-fields to self propagate.

EMR can be created and destroyed by electric charge, but is independent of charge as it propagates.

Light can travel ~~across~~ the observable universe long after its source has been destroyed, long before it sink has come into existence.

Maxwell's conjecture was verified by the discovery of radio waves by Heinrich Hertz (1857-1894) in 1887.

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So Maxwell's equations united electricity, magnetism, and optics as one physical field.

Note geometrical optics is a simple limit of $\vec{E}$ & $\vec{M}$ optics (called physical optics)

which we cover later in text-book Vol 3 Ch. 2

Geometrical optics ignores the wave nature of light and treats light as rays.

$\vec{E}$ & $\vec{M}$'s existence also verified that $\vec{E}$ & $\vec{B}$ -fields are real physical things in space, NOT just imaginary aids to calculating forces.

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9) Classical Electromagnetism + Special Relativity

In the 19th century, when it was recognized that light was a wave phenomenon obeying Maxwell's equations the idea was there had to be a medium for light and the name given to this unnoticeable medium was the luminiferous ether and Maxwell's equations applied relative to this medium.

But by the end of 19th century this medium had not been found anywhere.

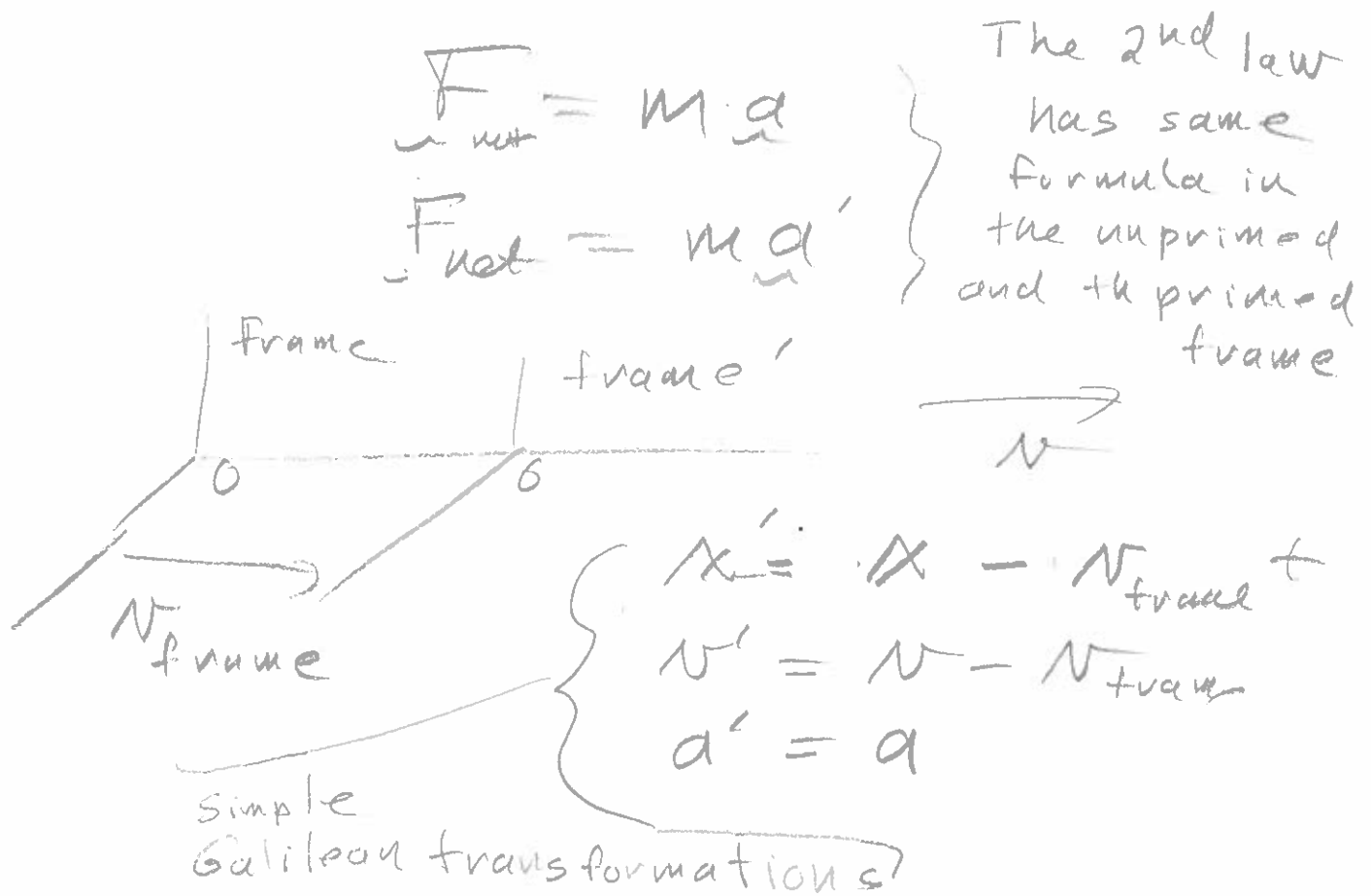
Einstein (the ideal Einstein of explication and not complicated historical person) strongly came to believe

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that Maxwell's equations
applied relative to all inertial frames
and in vacuum.

The waves were just ~~of~~
the E - & B - fields themselves
and NOT in any medium.

BUT Maxwell's equations
were NOT inertial frame invariant
under the Galilean transformations



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But Maxwell's equations were already known to be invariant under the Lorentz transformations.

→ The formulae stayed the same and $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ was the invariant light measured by all observers

Einstein argued that

Maxwell's equations were correct and the Lorentz transformations were correct physical laws

and Newtonian physics

was only asymptotically correct in the low relative velocity limit;

i.e., when $v_{\text{relative}} \ll c$

and this is true

and special relativity was discovered in essentials

in 1905.

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We will NOT write
the Maxwell equations
in explicit special relativity
form, but we will return
to special relativity
in Textbook Vol 3, Ch05

h) Fields, Vector Fields, & Field Theory

In physics, a field is any
quantity with a value at any point
in space time ^{within their realm} _{of application.}

Scalar fields just have a magnitude:
e.g., temperature, pressure.

Vector fields have magnitude
and a direction: e.g.,
velocity field in a fluid,
E-field, B-field.

Tensor fields Let's NOT go into
them here
explicitly.

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Only certain fields have their own intrinsic dynamics and the electromagnetic field is the prime example and dynamics are described by Maxwell's equations.

What of the gravitational field.

For Newtonian gravity, I believe most people would say that field theory is NOT applicable. For one thing pure Newtonian gravity does NOT predict gravitational waves.

Now General relativity (which is our best theory gravity) is a field theory → a tensor field

theory and it does predict gravitational waves which have been directly observed since 2015 (LIGO) and predicted by Einstein & Rosen 1936.

There is no DE for the Newtonian gravitational field.

It just is wherever matter tells it to be. Shift a mass

and its gravitational field just moves with the mass.

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General relativity is well beyond the scope of this course.

1) Quantum Electrodynamics (QED)

(of which I know almost nothing)

QED is the relativistic quantum mechanical version of electromagnetism, Quantum mechanics (QM) is the theory of microscopic particles.

Classical electromagnetism (EM) is the macroscopic asymptotic limit of QED. } We'll touch on QM many times

My perspective is that classical EM is an exact theory in the macroscopic asymptotic limit and NOT just an approximate theory and believe most people agree with this perspective.

Similarly, Newtonian mechanics is an exact theory in the classical limit (i.e., the macroscopic, low-relative velocity, weak gravity field limit).

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To be a bit more general, classical field theory is the macroscopic asymptotic limit of quantum field theory (QFT) (of which I know almost nothing)

Quantum field theory deals with all fundamental forces



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There is no lack of ideas for quantum gravity or TOE, but which if any of those is right, we do NOT know.

At present, it is (or nearly is) impossible to experimentally study gravity at the quantum level.

The gravitational force between atoms is far too weak to measure.

9.2 Electric Charge

a) Electric charge is a fundamental property of matter

Fundamental means
can't be explained
an further
in the context
of some grand
theory.

Stuff with rest-mass-energy
unlike photons
which have mass-energy
but no rest-mass-energy
to anticipate special relativity
and also zero charge

In the case of charge, it is absolutely fundamental
to our current knowledge.

A just so.

b) Charge comes in two flavors

~~positive~~ positive and negative

as named by none other than Benjamin Franklin
(1706-1790).

Positive and negative makes sense
because in many contexts they
can cancel each other.

$$\text{Net charge} = Q_+ + Q_-$$

is conserved

overall in the universe it seems
and locally (i.e. in any absolutely
enclosing closed surface),

Q_+ & Q_-
are
common
symbols
for quant.
of charge

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As far as we can tell, the universe is neutral overall; i.e.

$$Q_{+universe} + Q_{-universe} = 0$$

In fact, most systems on a large enough scale are neutral or asymptotically neutral. Physical context decides on what is a large enough scale.

This tendency to local neutrality is due to the fact unlike charges attract
like charge repel.

(Further elucidation p. _____)

Note, net charge is conserved, but positive charge and negative charge are NOT conserved individually.

Equal amounts of positive to negative charge can be created (pair creation)

or annihilated (pair annihilation)

microscopically (Further elucidation p. _____)

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c) Charge is Quantized

quanta

i.e., it comes in definite indivisible smallest amounts noticeable at the microscopic scale.

Quantum mechanics is called quantum mechanics because many physical entities are quantized — they come in discrete amounts. They do NOT form a continuum like real numbers.

At the macroscopic scale charge can often be treated as a continuum like water at the macroscopic scale and not as bunch of H_2O molecules.

There are actually two quantizations:

i) First,

a proton (p) and antiproton ($\bar{p}$) and positron (e^+) has $q = e$

an antiproton ($\bar{p}$) and electron (e^-) has $q = -e$

where e = elementary charge unit

and $e \equiv 1.602\,176\,634 \times 10^{-19}$ coulomb (C)
 $\approx 1.602 \times 10^{-19} C$

which is exact by definition
The coulomb (C) is the Mks unit of charge

and $1 C/s = 1$ ampere (amp)

is the familiar unit of electric current

The elementary charge unit

is noticeable in many contexts.

Note, there are neutral particles, neutrons, neutrinos, photons

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1) Second, Quark charge

proton
uud
 $\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$

neutron
udd
 $\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$

tauons
strange
charm
up quarks
valence

u u
d

neutron
d d
u

particle antiparticle

$$\text{up (u)} = \frac{2}{3}e, \quad \bar{u} = -\frac{2}{3}e$$

$$\text{down (d)} = -\frac{1}{3}e, \quad \bar{d} = +\frac{1}{3}e$$

other flavors and anti flavors
have the same set of values.

I do NOT think these charge quantities
can be detected directly

— but maybe in some sense
in a quark-gluon plasma that
the LHC can create
for some tiny fraction of a second.

We will not often refer to
quark charges.

1) Note

quantum
mechanical
particles

fundamental particles are
(leptons [electron, positron, etc.]
and quarks
and rest-mass-less particles
(photons, gluons)

and composite particles are
(protons, neutrons, nuclei, atoms, molecules, etc.
asymptotically in the unperturbed limit)
are absolutely identical →

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In way no set of macroscopic objects can be.

This is in principle NOT just in practice.

For example

say you had an electron created in the Big Bang and one created by pair creation a second ago.

There is no way to tell them apart except by tracing their past histories

And if they get close enough to have a joint wave function

(a quantum mechanical specification of state), you

in principle lose track

of which is which

and then you don't know which was "old"

and which was "young"

Common wave function symbol is capital Greek letter

$$\Psi = \Psi$$

Wave function

are continuous functions, but in many important contexts they form discrete (i.e., quantized) sets of wave functions.

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In fact, quantum mechanical particles
have no identity or age.

They are just manifestations of
their types.

Why do we believe this?

Quantum mechanics posits
this as axiom

and quantum mechanics is the
best verified of all physics theories.

It's never wrong.

In applying it you often
have to make large approximations
because it's a very difficult
theory to apply.

Also often you make mistakes
in calculation and experiment
but no anomaly has ever
resisted explanation

All modern electronics, including your phone,
verifies quantum mechanics.

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The upshot is that if quantum mechanics (QM) says something is true, we just believe it.

But there are mysteries in QM which we'll mention at appropriate times.

Quantum field theory (QFT) is the special relativistic version of QM.

5.3 The Electric Field

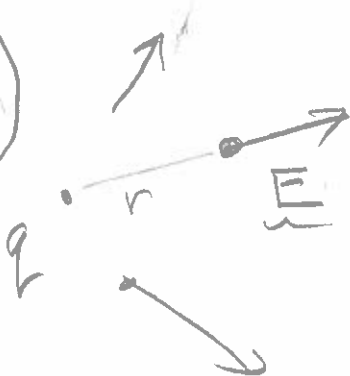
In electrostatic where charges are at rest one can be so approximate.

a) For a (classical) point particle (at an

Coulomb's law for the E-field (G-82) which can be derived from Maxwell's equations (p. 02-05003), but it's beyond our scope to show how.

$$\underline{E}(\underline{r}) = \frac{kq}{r^2} \hat{r} \begin{cases} \infty & \text{for } r=0 \\ 0 & \text{for } r=\infty \end{cases}$$

origin of spherical coordinates



$\Rightarrow q$ is the electric charge of the point particle.

$\Rightarrow r$ is the distance to where the $\underline{E}$ is evaluated

$\Rightarrow \hat{r}$ is the radial unit vector

02-05028 $\Rightarrow$ and $k = \frac{1}{4\pi\epsilon_0} = 8.987551785972(14) \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
 $\approx 10^{10}$ MKS units

is Coulomb's constant

From p. 02-05004, the vacuum permittivity

$$\epsilon_0 = 8.8548188(14) \times 10^{-12} \frac{\text{C}^2 \text{s}^2}{\text{kg m}^3}$$

$$\approx 10^{-11} \text{ MKS units}$$

Newton-meter
Coulomb

[Note vacuum permittivity in older jargon is the permittivity of free space]

The coulomb (C) is the macroscopic unit of charge (see p. 02-05023)

and $1 \text{ C/s} = 1 \text{ ampere} = 1 \text{ amp}$
the familiar unit of electric current,

b) Fiducial E-field value

prop
ector
quiggle
or
magnitude

$$E = \frac{kq}{r^2} \approx \frac{10^{10} \cdot (1 \text{ C})}{(1 \text{ m})^2} = 10^{10} \text{ N/C}$$

which is an enormous force per coulomb

The electric force on any point charge q is

$$F = qE \quad \left\{ \begin{array}{l} \text{Follows from the Lorentz force} \\ \text{with } B=0 \text{ (see p. 02-05006).} \end{array} \right.$$

Fiducial force between point charges of 1 Coulomb each is

$$F = 10^{10} \text{ N}$$

02-05030)

Ans. In almost all human and other contexts too matter is nearly neutral. All the electrons in hydrogen are balanced by protons.

$\therefore Q_{\text{net}} \approx 0$ for most systems.

But in circuits, we often have currents of order $1 \text{ amp} = 1 \text{ C/s}$, and so why so - why do wires NOT have deadly strong electric charges?

Ans. Wires are mostly neutral — all the current of free electrons (which are the charge carriers) are flowing through positively charged ions at rest.

Of course 1 amp is a pretty deadly current.

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Recall $1 \text{ N} \cong 0.22481 \text{ lb} \cong \frac{1}{5} \text{ lb}$

So actually 1 C of (net) charge
is an enormous and dangerous
amount of charge in human environments.

Ex. What is the amount
of negative charge
in 1 kg of hydrogen $\swarrow 1 \text{ kg}$

$$q = (-e) \cdot N_H = -e \frac{M}{A u}$$

elementary charge unit

$$e = 1.602\,176\,634 \times 10^{-19} \text{ C}$$

exact by modern definition

(see p 02-05023)

atomic
weight
of H
 $= 1.0080(20)$
standard

atomic
mass unit
(AMU)
 $= \text{dalton}$

$$= 1.660\,539\,068\,92(92) \times 10^{-27} \text{ kg}$$

$$1 \cong (-1.6 \times 10^{-19}) \left(\frac{1}{1.7 \times 10^{-27}} \right)$$

$$q = -10^8 \text{ C}$$

and so why isn't a 1 kg
of hydrogen deadly?

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[02-05031]

Flowing thru the human body

1 amp even for 10ms is pretty bad

but 0.5 milliamps is imperceptible
even for long times (10s)

(Wik: Electrical injury)

c) Superposition Principle (G-61)

$$\vec{E} = \frac{k q \hat{r}}{r^2}$$

is the electric field



for a point charge everywhere in space
when it is at rest or can be
approximated as being
at rest.

This is the electrostatic limit
or the realm of electrostatics.

What if you have multiple charges or a
continuum of charge?



system of
point charges

dV is
differential
volume

dq is
differential
charge



continuum of charge
with $dq = \rho dV$

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The superposition principle of electromagnetism is that the net electric field $\underline{E}(\underline{r})$ at some point $\underline{r}$ is just the sum of individual E-fields of point charges q_i or differential charges dq .

Note, Maxwell's equations (see p. 02-05003) allow a differential equation solution for E-fields for continuous charge distributions.

Thus

$$\underline{E}(\underline{r}) = \sum_i \frac{k q_i}{|\underline{r} - \underline{r}'_i|^2} \hat{r}'_i$$

Note a differential quantity is formally an entity in limiting process; e.g., from calculus

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

where dx and dt are differentials.

In physics, we tend to think of differentials as being small enough for any smallness requirement we put on them is true or more formally asymptotically true.

→ As here, the differentials have an asymptotically true ratio.

$$\underline{E}(\underline{r}) = \int \frac{k \rho(\underline{r}')}{|\underline{r} - \underline{r}'|^2} \hat{r}' d\tau'$$

Note, the superposition principle seems perfectly natural and it is implicit in Maxwell's equations (p. 02-05003), but it did NOT have to hold.

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For example say $E(\underline{r}) = \sqrt{\sum_i \left[\frac{kq_i}{|\underline{r} - \underline{r}_i|^2} \right]^2} \approx \frac{\sum_i \hat{r}_i}{N}$

for N point charges,

The superposition principle would NOT hold.

→ If this were true, reality would be very strange.

d.) Point Particles

Classical point particles do NOT exist,

but they are very useful
in introducing concepts
and thinking about concepts.

You can think about a
classical point particle
with charge q_i (a point charge q_i)
as representing a differential
amount of charge $dq_i = \rho(\underline{r}_i) dV$
where $\rho(\underline{r}_i)$ is charge density
and dV is differential volume.

02-05034

Note: the integral and DE solution for E-field will in general be hard and often requires numerical and/or approximate analytic solutions.

But what quantum mechanical point particles?

fundamentally
elementary
particles
in
current
physics,
composite
particles
are made
up of them:
e.g.,
neutrons,
protons,
nuclei,
atoms,
molecules.

They may or may not exist in some sense.

leptons (e.g., electrons and positrons)

and quarks (see p. 02-05029)

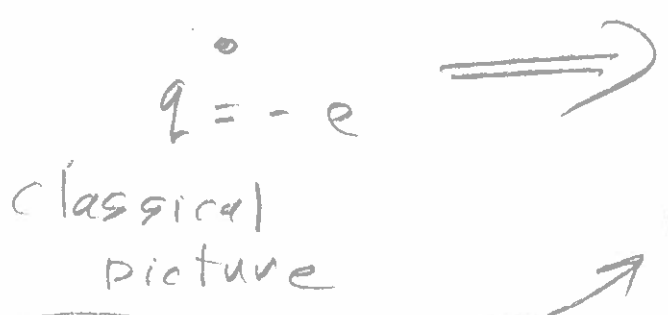
may or may not be points

in some sense — but
only quantum field theorists know
(and they aren't telling), but

QM particles exist in a continuum
superposition of positions.

So a point particle electron is
spread out in a "cloud"

For example an isolated electron



There is no sharp edge.
The charge density goes to zero
as distance increases from cloud center

$$\int \rho_e dV = -e$$

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02-0503!

Here $\rho_q = -e |\psi|^2$ is the

charge density

and ψ is the wave function which is intrinsically

a complex number function.

The absolute value of ψ is $|\psi|^2$

and is called the probability density;

but yours truly thinks
existence density

would be a better name

since that is how it is treated

in calculations

Since composite particles are made of elementary particles, they too exist in a continuum superposition of positions

The idea of a point particle

existing in a continuum superposition of positions

is "just something you get used to in quantum mechanics."

The conclusion here is that charge effectively
is spread into a continuum, but you can treat

quantum particles as classical point particles or a continuum averaged
over a macroscopic scale if convenient to do so,

QM seems to be intrinsically a complex number theory. Most people believe if you try to dispense with them, you just use them in another form.

(Sci Am 2023 April)

Mass density of a particle of mass m is $\rho_m = m |\psi|^2$

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5.4 The Electrostatic Force or the Coulomb's Law Force

a) First recall Newton's 2nd Law (AKA $f=ma$)

$$\mathbf{F}_{\text{net}} = m \mathbf{a} \quad \text{for a classical point particle}$$



system
of point particles
making up an object at all.

$$\mathbf{F}_{\text{net}} = \underbrace{\sum_i \mathbf{F}_{i,\text{ext}}}_{\text{external forces}} + \underbrace{\sum_{\substack{i,j \\ i \neq j}} \mathbf{F}_{ij}}_{\text{internal forces}}$$

Note $m_i = \rho(\mathbf{r}_i) dV$
where $\rho(\mathbf{r}_i)$ is
a continuum of density
which is actually case
for quantum mechanical
particles as mentioned
on p. 02-05035.

$$\begin{aligned} \text{But } \sum_{\substack{i,j \\ i \neq j}} \mathbf{F}_{ij} &= \sum_{\substack{i,j \\ i \neq j}} (-\mathbf{F}_{ji}) \quad \text{by the 3rd Law} \\ &= - \sum_{\substack{i,j \\ i \neq j}} \mathbf{F}_{ij} \quad \text{by relabeling} \\ \therefore \sum_{\substack{i,j \\ i \neq j}} \mathbf{F}_{ij} &= 0 \end{aligned}$$

But at the macroscopic scale,
we usually/always average
the quantum mechanical
mass density over
macroscopic scales.

Note, the 3rd law is actually NOT always valid
in the classical limit, but there is
always some generalization that is (Goldstein-8),
and so we do NOT usually worry about this fine-point.

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For a system of particles?

$$\vec{F}_{i\text{ext}} = m_i \vec{a}_i$$

$$\therefore \vec{F}_{\text{net}} = \sum_i \vec{F}_{i\text{ext}} = \sum_i m_i \vec{a}_i = m \left(\frac{\sum_i m_i \vec{a}_i}{m} \right)$$

where $m = \sum_i m_i$ is total mass

and $\vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m}$

is the center-of-mass acceleration

$$\therefore \vec{F}_{\text{net}} = m \vec{a}_{\text{cm}}$$

So this form applies to the center-of-mass motion.

To learn about the motions of the parts, you need to consider individual $\vec{F}_{i\text{ext}}$ and $\vec{F}_{ij}$

external force

internal forces

b) Coulomb's Law for point charges

Recall (from p. 02-05006) Lorentz force with $\vec{B} = 0$ is $\vec{F} = q\vec{E}$

Thus, from p. 02-05027, Coulomb's law for $\vec{E}$ -field, we get the Coulomb's Law for force

This is what is meant by Coulomb's law (on the electrostatic force law) without qualification.

$$\vec{F}_{ij} = \frac{k q_i q_j}{|\vec{r}_i - \vec{r}_j|^2} \hat{V}_{ij}$$

Force of charge i due $\vec{E}$ -field of charge j ,



Recall from p. 02-05028, $k = \frac{1}{4\pi\epsilon_0} = 8.987551785 \times 10^9 \approx 10^{10} \text{ N m}^2/\text{C}^2$

02-05038

Note if $q_1 q_2 > 0$, the force is repulsive

if $q_1 q_2 < 0$, the force is attractive

Mnemonic "like charges repel"
"unlike charges attract,"

Note Coulomb's law is very similar to Newton's law of universal gravitation

$$\vec{F}_{ij} = - \frac{G m_i m_j}{|\vec{r}_i - \vec{r}_j|^2} \hat{r}_{ij}$$

where m_i and m_j are point masses

Points about Coulomb force and Gravity and $G = 6.67430(15) \times 10^{-11} \text{ Mks}$

- i) Both depend on particle "charges"
charges q_i for Coulomb's law } masses m_i for grav. law.

But two flavors for charge
and likes repel, unlikes attract

One flavor for mass and likes attract,

- ii) Both arise from a field

$\vec{E}$ -field for E & M

Gravitational field $\vec{g}$ for grav. force

This means Coulomb's force can be "canceled" but gravity cannot. There is NO antigravity in a straight-forward sense.

(2026 aug 01)

(02-05039)

but E-field obeys E&M dynamics
as expressed by Maxwell's equations (p 02.0500
(which is relativistically correct))

and Newtonian gravity (classical gravity)

does NOT have a complete dynamics
- (and is NOT relativistically correct)

→ General relativity (GR)
which is a relativistic
theory of gravity does
have a complete dynamics
expressed by
Einstein's field equations
and the geodesic equations
(Which is well beyond
the scope of this course)

ii) They are both long range
inverse-square law forces.

iv) The great similarities of E&M and gravity
led Einstein to pursue what he
called a unified field theory
for the last 30 years or so of his life.

The unified field theory would unite
E&M and gravity

12-05040)

He did find various versions, but we now believe they are all wrong because his approach was wrong — he avoided all quantum mechanics.

The unification of EM, nuclear weak force, and nuclear strong force in the standard model of particle physics is a quantum field theory and the expectation is a quantum gravity (of which GR is the macroscopic limit) will eventually be united with the other forces.
(Maybe the expectation is wrong).

d) Coulomb Force + Gravity Force Strength Comparison

Two Fiducial Cases

$$\begin{aligned} \text{i) } \frac{F_g}{F_E} &= \frac{G m_1 m_2}{k q_1 q_2} = \frac{G}{k} \frac{(1\text{kg})^2}{(1\text{C})^2} \\ &\approx \frac{10^{-10}}{10^{10}} = 10^{-20} \end{aligned}$$

Recall 1C is an enormous net charge in the human context (see p. 02-05029)

$$\begin{aligned} \text{ii) } \frac{F_g}{F_E} &= \frac{G}{k} \frac{m_{\text{electron}} \cdot m_{\text{proton}}}{q_{\text{electron}} \cdot q_{\text{proton}}} \\ &\approx \frac{10^{-30} \cdot 10^{-27}}{10^{-19} \cdot 10^{-19}} = 10^{-20} \cdot 10^{-19} \approx 10^{-39} \end{aligned}$$

(2026aug01)

(02-05041)

You can see in a sense gravity is a very weak force.

Atoms, molecules, human scale rigid solids are held together by the electrostatic force

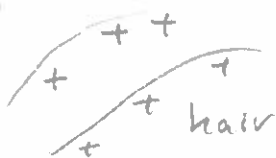
NOT gravity which at the microscopic scale must be quantum gravity but at that scale gravity is unmeasurable.

Gravity between kilogram masses can be measured in the lab

(and has been done since the Cavendish experiment of 1797-1798 (wik)), but between atoms it's still immeasurably small, which one reason we don't have an establish quantum gravity theory now.

e) Demonstration of Weakness of Gravity

Need bits of paper and Comb



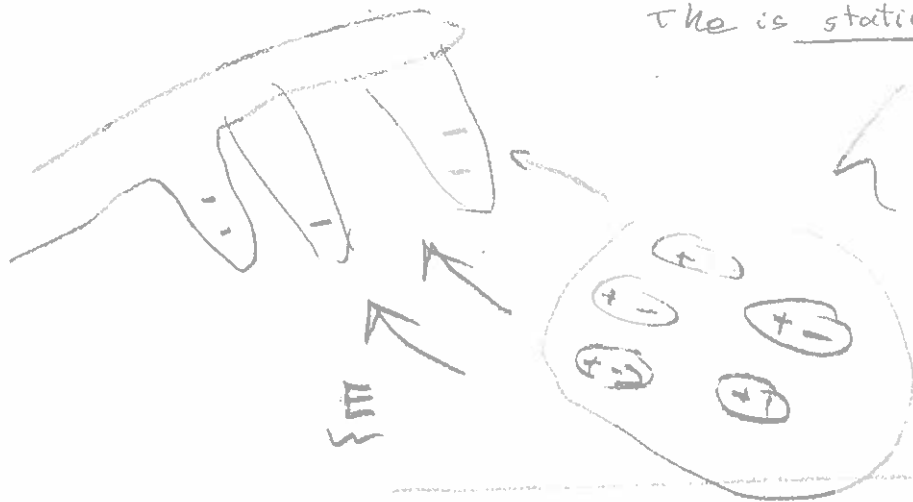
Some kind of rubbing action between materials causes kinetic energy to separate charge and create non-neutrality in combs and hair

02-05042

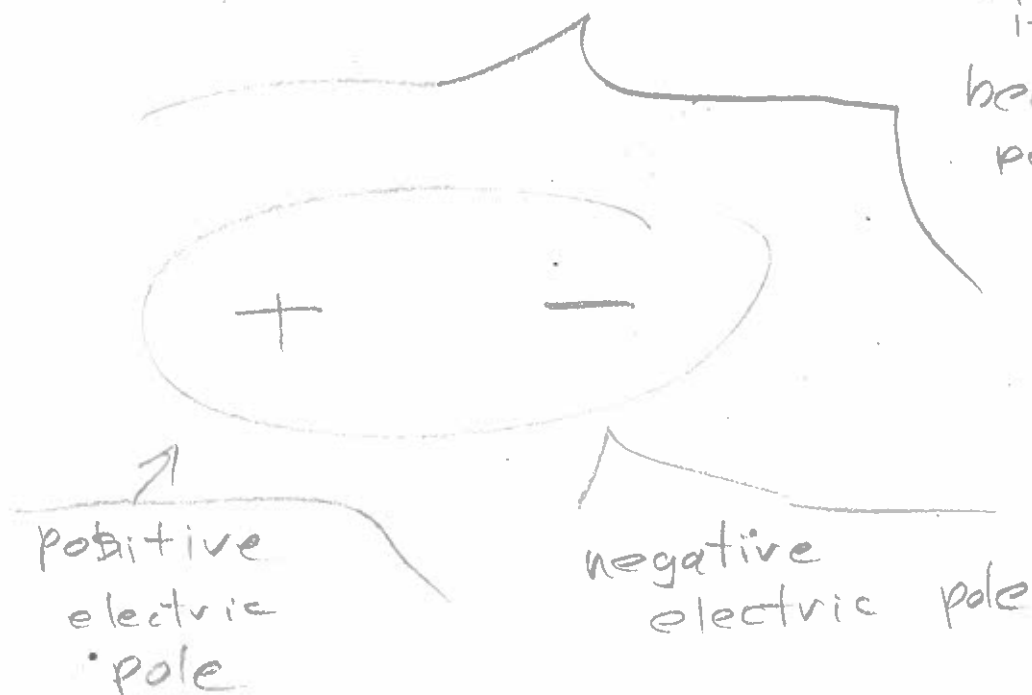
— electrons flow onto comb surface
and it becomes negatively charged

(Google AI)

and they can't quickly discharge
since the comb is NOT a conductor.
The is static. Static electricity.



Paper also Not
a conductor
and stays
neutral,
but
it molecules
become
polarized
by the
E-field



Since the E-field falls off with
distance (the inverse-square law
in action), the +ve pole
more attracted than the -ve pole
repelled.

[2026 aug 01]

[02-0504]

The upshot is a net attraction
of the paper bits to the comb

And this net attraction

largely cancels the
gravity of the whole Earth.

Other than
lightning and electric fish, the oldest
known electrical
phenomena, but
still poorly known.

In fact
electron
is the
Greek
word for
amber
which
was
noted
for
static
electric
properties
(wiki
electricity)

The creation of static electricity
is called the triboelectric effect
or triboelectric charging and
it is an immensely complicated effect
And to this day is still needing more elucidation.
(see Ahart, J. 2026 mar 18, Nature).

By complex surface interactions, the kinetic energy of rubbing overcomes

f) Electrostatic Force and

Small Scale Structure

small scale attraction and
move charge from one
surface to another

Atoms, molecules, and ions
cannot exactly neutralize.

Quantum Mechanics forbids
electrons and protons from
exactly overlapping and in
most contexts from combining to
make neutrons.

22-05044)

So on small scales the electrostatic force is very strong and holds atoms, molecules, ions, and rigid solids together in a way determined by quantum mechanics.

Also the electrostatic force plus QM makes atoms, molecules, and ions strongly incompressible.

Thus, the strong pressure force resisting the compression of solids and liquids

But because of the strong tendency to neutralization on the macroscopic scale in many contexts the electrostatic force at the macroscopic scale just appears as a contact force with special cases being

- Normal force
- friction
- viscosity in fluids
- tension
- pressure force
- etc.

2026aug01

02_V5045

So the long-range inverse-square law nature of the Coulomb force is only apparent in relatively few contexts

But the electrostatic force dominates small scale structure:

objects up to small asteroids

g) Medium Scale?

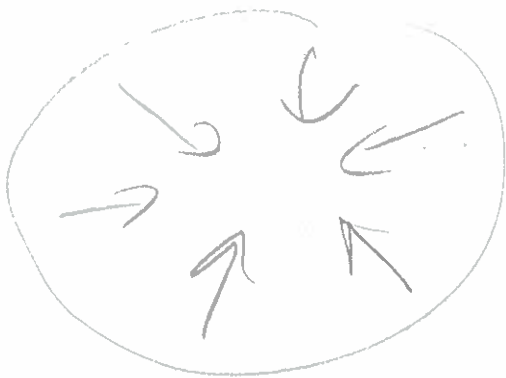
large asteroids,
planets,
stars,

pressure-supported astro-bodies

A combination
of gravity

and pressure force (a special case
of electrostatic force)

Pressure-Supported Astro-Body



- gravity pulls in
- pressure pushes out
and there is hydrostatic equilibrium

What of Mountains?



The pressure force is strong enough to support huge columns
But it's isotropic, and so mountains slump.

22-05096

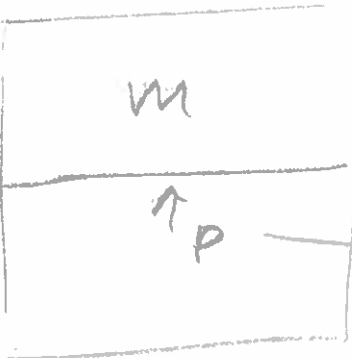
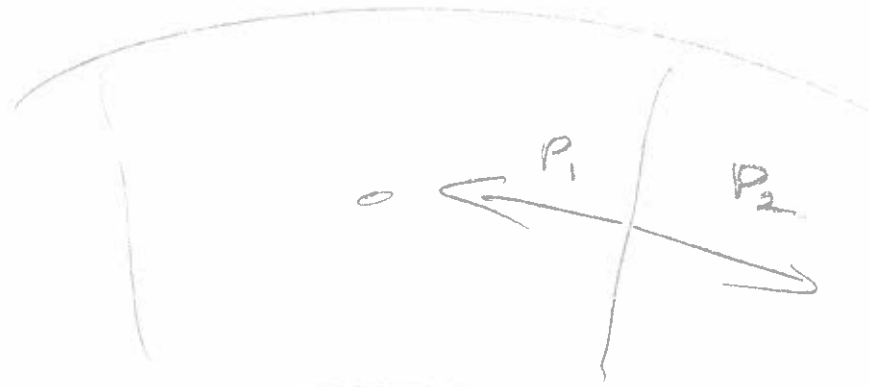
But

column of
air

or column
of Earth's
interior

don't slump (flow)

because the pressure force
in the non radial direction
is equal.



} column of mass to be
supported

Pressure at interface.

Pressure decreasing going upward
because less mass to support.

Note rigid solid bonds will break
or plastically deform if pressure
force gets too strong.

Plastic deformation = slow flow

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02-05047

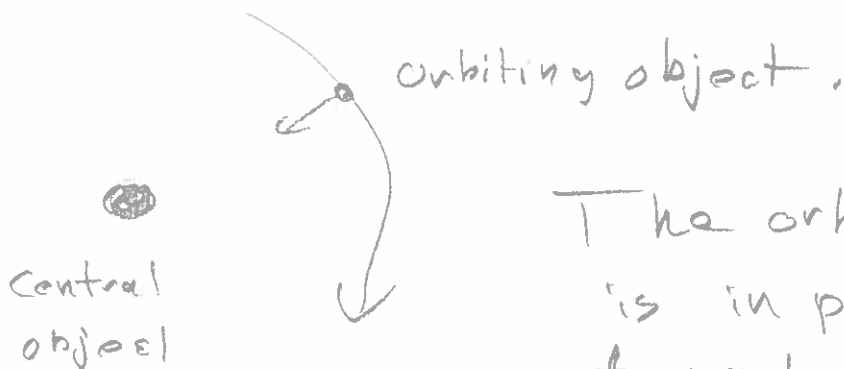
h) Large Scale Structure

Cannot be static

planetary systems
star clusters
galaxies
galaxy cluster

held up by
a combination
of
gravity pulling in
and 'macroscopic' or
kinetic energy
resisting.

Orbital systems with central objects



The orbiting object
is in perpetual free-fall
toward the center,
but keeps missing
because it has a
sideways motion
(i.e., angular momentum)

22-05078



Clusters of stars and galaxies tend to have chaotic orbits determined by interior mass to any radius R from the center of Mass

Observable Universe as a whole

Has the Expansion of the Universe



Distance between bound systems grow by a cosmic scale factor $a(t)$

$$r(t) = a(t) r_0$$

with $a(t = \text{cosmic present}) = 1$
by convention.

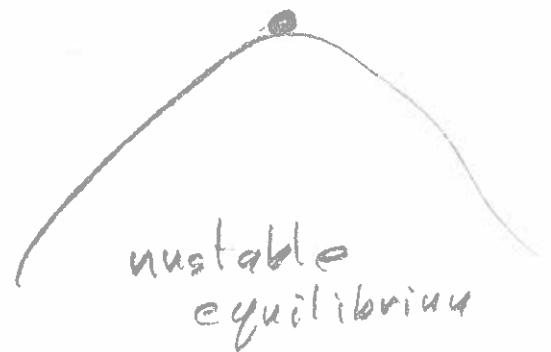
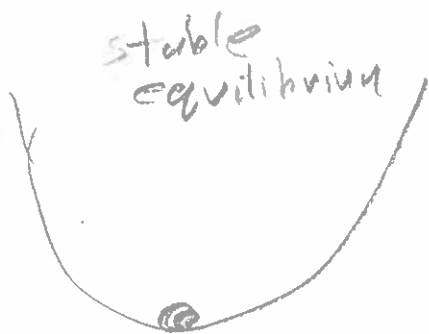
We live at cosmic present, and so we set $a(t = \text{cosmic present}) = 1$

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02-05049

The evolution of $a(t)$ is determined
by a combination of gravity, kinetic energy
and dark energy (as it usually called)
all in a GR sense.

$a(t)$ can only increase or decrease
except by fine-tuning to be static,
but that is an unstable equilibrium.

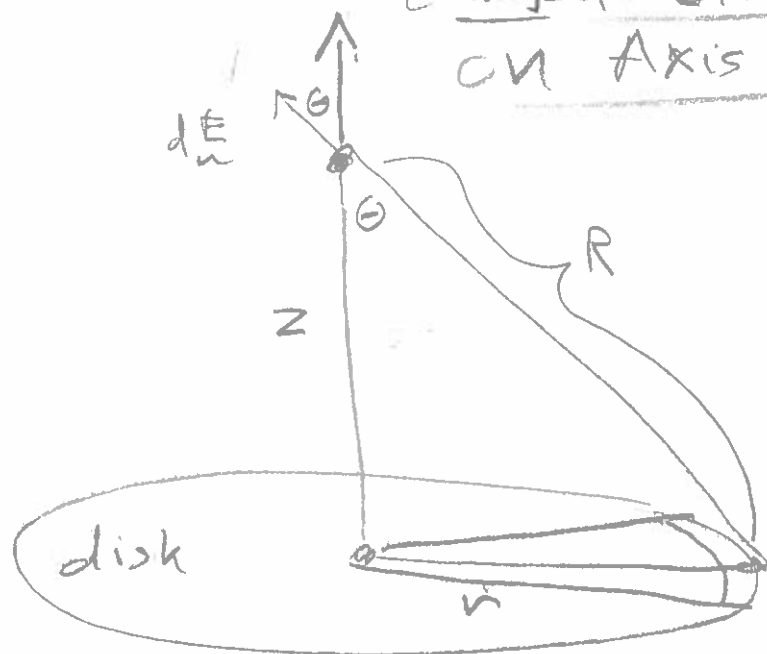


At present $a(t)$ is increasing
and rate of growth accelerating
due to dark energy
or so it seems.

22-05050

5.5 Analytic Calculation for

An Electric Field of a Uniformly
Charged Circular Disk
on Axis of Symmetry



In electrostatics,

$\Rightarrow \rho$ is used
for volume
charge density

$$\rho = \frac{Q}{\text{Volume}}$$

$\Rightarrow \sigma$ for
surface
charge density

$$\sigma = \frac{Q}{\text{Area}}$$

$$\lambda = \frac{Q}{\text{length}} \text{ : line charge density}$$

(Just conventions
that NOT everyone
obeys all
the time)

By symmetry
at z on axis,
only the z -component
of the electric field
is non zero

$$\therefore dE_z = dE \cos \theta$$

$$= \frac{k dQ \cos \theta}{R^2} = \frac{k (\sigma (r d\phi) dr \cos \theta)}{R^2}$$

dQ is differential charge

$r d\phi$ is differential azimuthal length

dr is differential radial length

$\left\{ \begin{array}{l} z\text{-component} \\ \text{of} \\ \text{electric} \\ \text{field} \end{array} \right.$

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02.0505

For complex charge distributions,
the differentials get replaced by
small bits

$dr \rightarrow \Delta r$ and $d\phi \rightarrow \Delta\phi$
and you do numerical integration
by sums OR some kind of
approximation (e.g., importantly
multipole expansion series)

which we might briefly describe later
(Sec 145),

But here, high symmetry, allows
analytic integration (antidifferentiation)

$$E_z = k\epsilon \int_0^{2\pi} d\phi \int_0^r \frac{r' dr'}{R^2} \underbrace{\frac{z}{R}}_{\cos\theta}$$

$$R = \sqrt{z^2 + r'^2}$$

$$= 2\pi k\epsilon z \int_0^r \frac{r' dr'}{(z^2 + r'^2)^{3/2}}$$

Students do the integral for
yourselves.

72-05052

$$\int_0^r \frac{r' dr'}{(z^2 + r'^2)^{3/2}} = \left. -\frac{1}{2} \frac{1}{2} (z^2 + r'^2)^{-1/2} \right|_0^r$$

$$= -\frac{1}{\sqrt{z^2 + r^2}} + \frac{1}{z}$$

$$\therefore E_z = 2\pi k\epsilon z \left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + r^2}} \right)$$

$$= 2\pi k\epsilon \left[1 - \frac{z}{\sqrt{z^2 + r^2}} \right] \text{ in general}$$

for $z \ll r$

$$\frac{1 - \frac{z}{r} + \frac{1}{2} \left(\frac{z}{r} \right)^3}{}$$

for $z = 0$

$$\frac{1}{}$$

for $z \gg r$

$$\frac{1}{2} \left(\frac{r}{z} \right)^2$$

and so

$$E_z = \frac{k \pi r^2 \epsilon}{z^2}$$

$$= \frac{kQ}{z^2}$$

where

$Q = \pi r^2 \epsilon$ is
the total charge
and this is like
a point charge

$$\frac{z}{\sqrt{z^2 + r^2}} = \frac{z/r}{\sqrt{1 + (z/r)^2}}$$

$$\approx \frac{z}{r} \left(1 - \frac{1}{2} \left(\frac{z}{r} \right)^2 \right)$$

$$\approx \frac{z}{r} - \frac{1}{2} \left(\frac{z}{r} \right)^3$$

to 3rd order

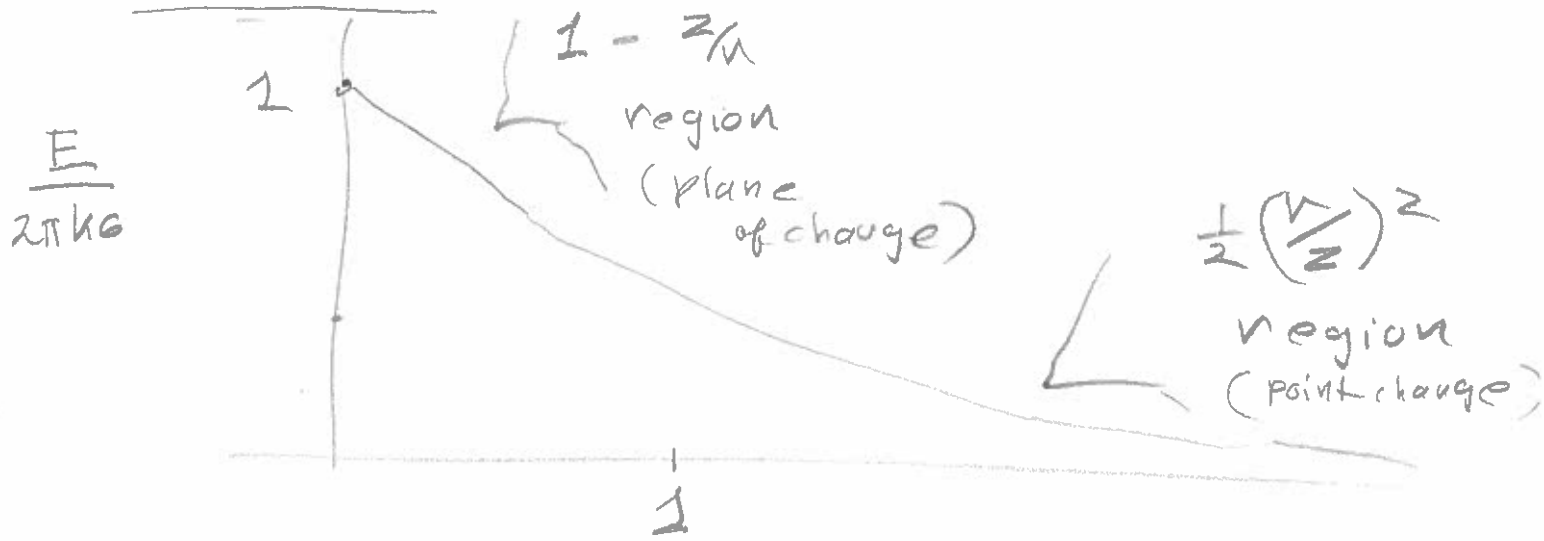
$$\frac{1}{\sqrt{1 + (r/z)^2}}$$

$$= 1 - \frac{1}{2} \left(\frac{r}{z} \right)^2$$

to 2nd order

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02-05053

Plot of E_z 

Actually for small z/r

$$E \approx 2\pi k\sigma \left[1 - \frac{z}{r} + \frac{1}{2} \left(\frac{z}{r} \right)^2 \right]$$

and as $r \rightarrow \infty$, one obtains the
E-field for an infinite plane
of charge



where $E = 2\pi k\sigma$

the E-field is independent
of distance z

and by symmetry there is
only a non zero z component

22-05054

5.6 Electric Field Lines

a) Any vector field can be visualized in terms of field lines.

At every point in space there is a vector value



← Vectors point in real space but their length is in their own abstract space; e.g., velocity space, E-field strength space.

You can only draw a representative set of vector since they form a continuum. The set is a visualization.

But you can a line through a continuum of points at every point in the direction of the field.

These are field lines.

The best known are
streamlines for fluid velocity field
E-field lines for the electric field
B-field lines for the magnetic field

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02-05059

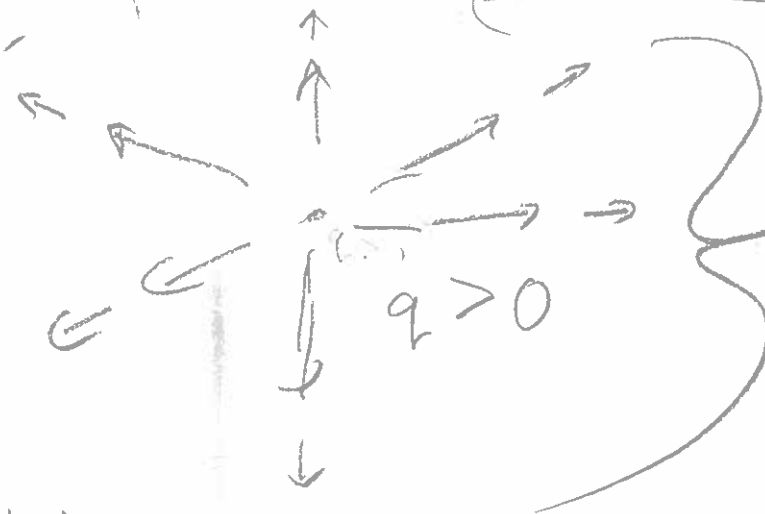
Mostly field lines are a very useful visualization tool, you can think of special cases where they are physically realized.

Michael Faraday (1791-1867) was invented them as a visualization tool.

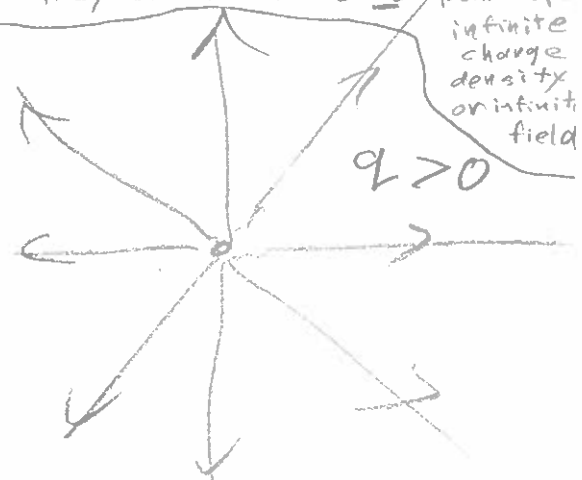
Now for some examples

b) Classical Point Particles

i) Classical Point Charges



For classical point charges $E = \infty$ at the charge. A limit for concentrated charge. Quantum mechanical point particles are in a continuum superposition of positions, and so even if they exist, there is no point of infinite charge density or infinite field.



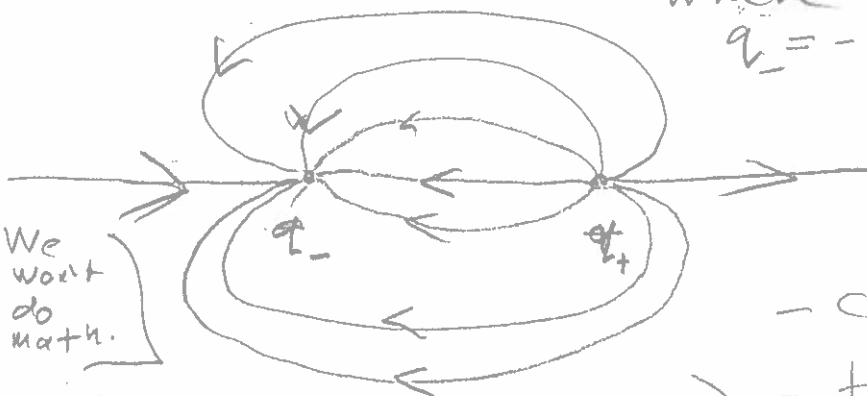
Note spherical symmetry is implied. The drawing is a cross-section.

the field lines extend to infinity

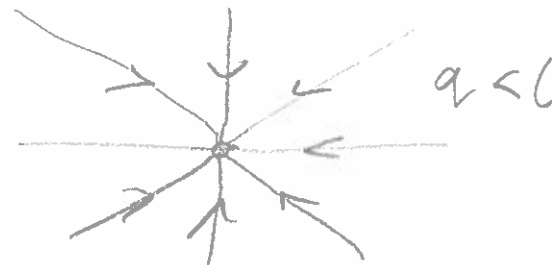
c) Electric dipole

made of point charges

where $q_- = -q_+$



For $q < 0$, the field lines point inward



In the far field limit, $E \sim \frac{1}{r^3}$ from the center not as $\sim \frac{1}{r^2}$ and E has directional dependence

- cylindrical symmetry implied
- the field lines must begin and end on a charge OR extend to infinity for electrostatics,

52_05056/

How do we know this — that field lines must start/end on charge or extend to infinity?

d) A tricky point

i) Note, E-field lines cannot intersect, except where $\vec{E} = 0$.

Why?

Because the field



would point

in multiple directions at one point

which violates a basic axiom

of classical EM that the E-field is a vector field.

This axiom is just an implied part of Maxwell's equations (see p. 05-05003).

ii) Note again classical point particles are an idealization.

Really we have regions of continuum of electric charge which the quantum mechanical view

So a point charge

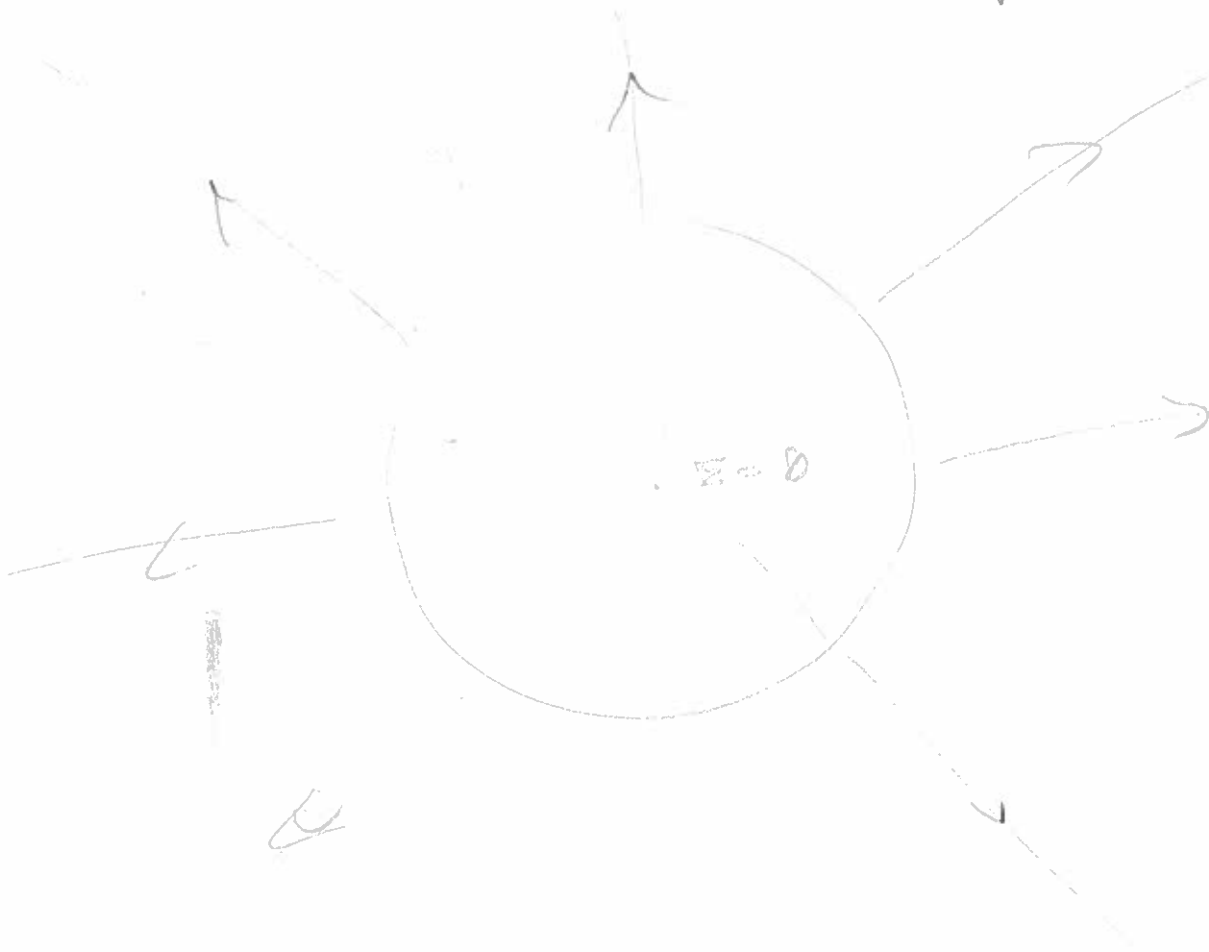
q • is really a clump of charge



2026 aug 01

02-05057

ii) Imagine a small spherically symmetric charge distribution of positive charge



from outside

the field lines begin
at a point-like source;

But inside the field lines
begin at the center
where $\underline{E} = 0$ by symmetry

Google AI
seems to
agree
if you
interrogate
it for
enough.

In the classical continuum picture, I think field lines always end on $\underline{E} \neq 0$ points.
For the classical point, line, plane of charge, they end on those.
The quantum electrodynamics is fuzzy to me, but probably it's
a fuzzy end.

2-05058]

iv) However the classical point charges
as 'start/end of $\vec{E}$ -field lines
is a useful picture,

v) Note, $\vec{E} = 0$ points can happen
where there is no start/end
of $\vec{E}$ -field lines, except in one direction.

Consider two positive point particles



In far field limit, the two point charges
asymptotically appear as a single
point charge of charge $2q_+$.

We won't do the math proof here, but it is
physically intuitively reasonable.

(2026 Aug 01)

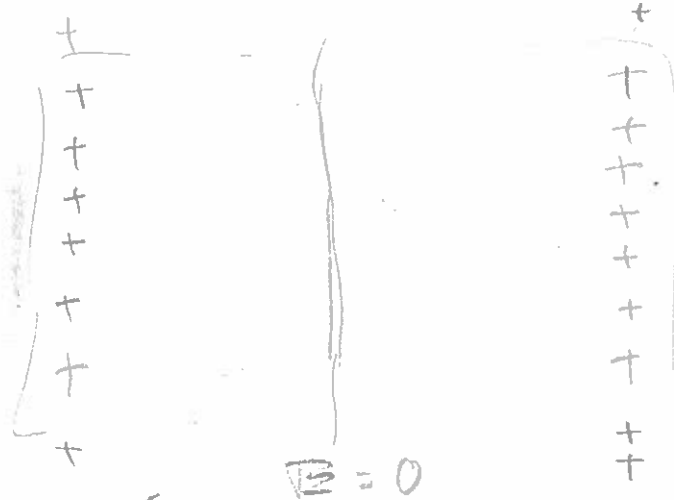
(02-0505)

vi) In general clump of continuum charge?

Not
a conductor



There might
be multiple
points of
 $\underline{E} = 0$



Note, this
description is
still at the
macroscopic scale
where we are
averaging over
field variations
at the atomic and
nuclear scale
quantum vacuum scale
(where the E-field
is never zero
and has no fixed direction)

NCT a conductor

At atomic scale maybe
there are $E=0$ points above
the vacuum level.

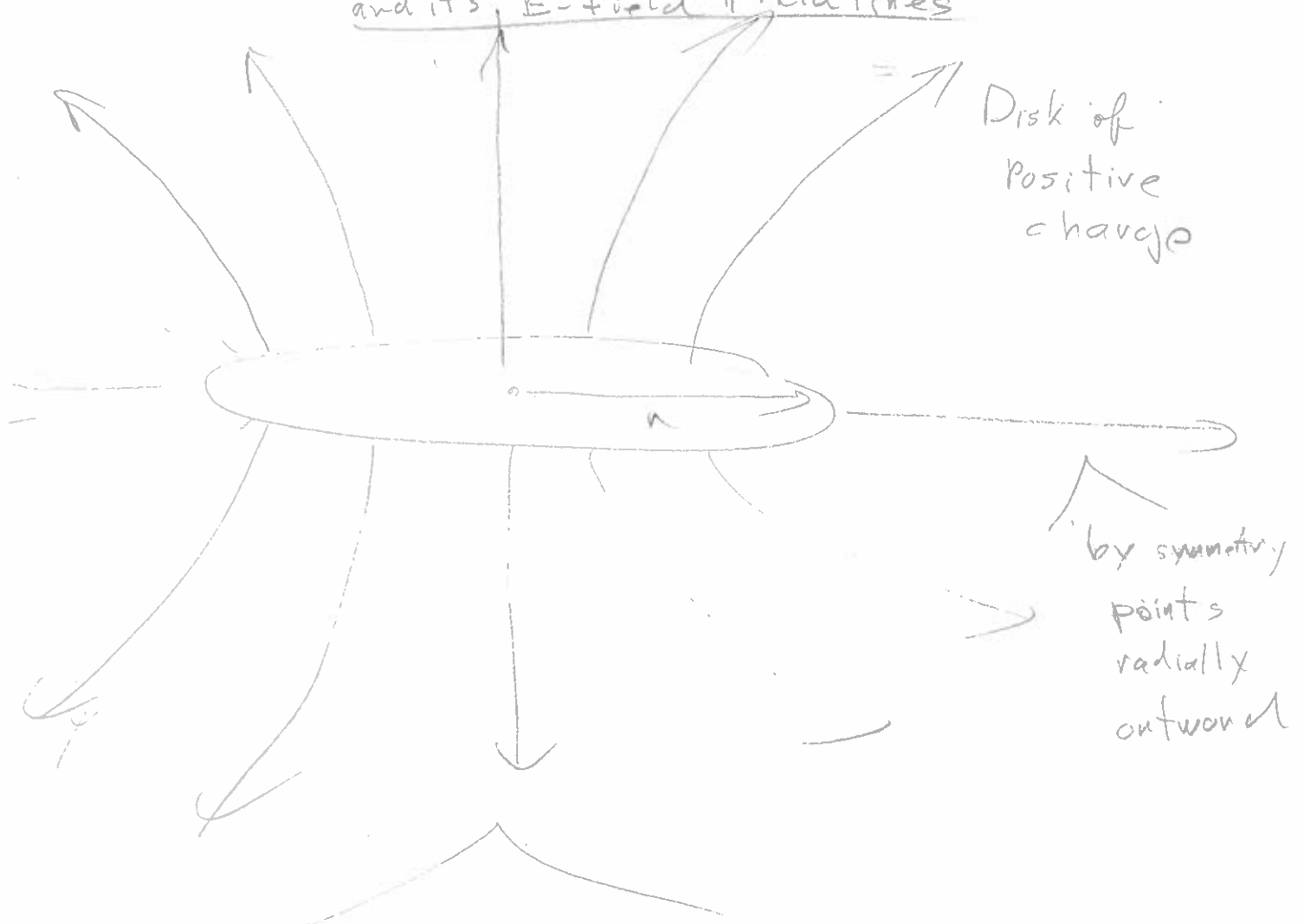
Conductors are important special case,
we consider in Chapter 6.
Oh Gauss' Law

vii) Actually at the quantum field level,
there is always a vacuum
Zero point (i.e., non zero)
 E & B field, but that is
well beyond our scope.

02-05060

02-05058

e) Disk of charge (see p. 02-05050)
and its E-field field lines



on axis of symmetry

$$\vec{E} = \left(2\pi k \epsilon \left[1 - \frac{z}{\sqrt{z^2 + r^2}} \right] \right) \hat{z} \quad (\text{see p. 02-05052})$$

$$\left\{ \begin{array}{l} 2\pi k \epsilon \hat{z} \quad \text{for } z/r \ll 1 \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{kQ}{z^2} \hat{z} \quad \text{for } z/r \gg 1 \end{array} \right. \quad \text{where } Q = \pi r^2 \epsilon$$

[2026aug01]

[02-05061]

In far field limit in all directions

$$\frac{\vec{E}}{r} = \frac{kQ^2}{r^2} \hat{r}$$

unit
vector
pointing
radially
outward
from
center of
disk.

Physically intuitively reasonable,
but to be sure it needs
a proof we will not
give here.

For an infinite plane of charge



$$\vec{E} = 2\pi k\sigma \hat{z} \text{ for all space,}$$

No distance dependence.

We prove this
ideal limit case
in chapter 6 on Gauss' law

f) Electric Field Line Loops

On p. 02-05055, we said E-field
lines extend to infinity or start/end
on classical pointcharge which must be
qualified to points of $\vec{E}=0$ in the true
classical continuum picture,

22-05062

But this is the electrostatic case
where $\mathbf{E}$ -fields are generated
by static electric charge
(static in the inertial frame
of observation),

However, $\mathbf{E}$ -fields can also
be created by time varying $\mathbf{B}$ -fields
as shown by the Maxwell equation
the Maxwell-Faraday equation

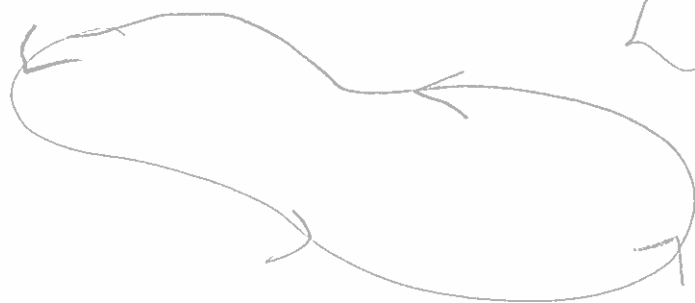
$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

or Faraday's law of induction
differential form

(see p. 05-05003)

curl
of $\mathbf{E}$
for those
familiar
with vector calculus

The equation can be read
as a time varying magnetic field
generates an electric with a
structure where the $\mathbf{E}$ -field lines
are loops



The $\mathbf{E}$ -field
lines in this
case form
loops

[2026aug01]

[02-05063]

And such E-fields
can drive currents and this
is what goes on in electric generators.

A time varying B-field drives
an electrical current.

This is the basics of
thermo, hydro, Nuclear electric power.

9) Visual Tool

E-field field lines can often be
estimated qualitatively using
only qualitative concepts,

And that's we've done in
this section (5.6 starting p 02-05054),

We get more practice with
them and B-field field lines too
as we cover more of classical E & M,

22-05064 }

5.7 Electric Dipoles

a) We already consider the point-charge electric dipole

on AG2-05055.

$q_- = -q_+$
and so overall
neutral



Asymptotically close to each point charge,
the E -field looks like a point charge E -field

$$\underline{E} = \frac{kq_-}{r^2} \hat{r}$$

where r is radius
from q_-

$$\underline{E} = \frac{kq_+}{r^2} \hat{r}$$

where r is radius
from q_+

In the far field (asymptotic) limit

$$\underline{E}(\underline{r}) = k \left[\frac{3(\underline{p} \cdot \hat{r})\hat{r} - \underline{p}}{r^3} \right]$$

where r is radius
from the center

$|\underline{r}| \gg |\underline{d}|$
 $\underline{q}_+ \rightarrow \underline{q}_-$
is the charge
separation vector

Wiki:
electric
dipole
moment

(2026aug01)

(02-05065)

and $\underline{P} = q_+ \underline{s}$ is the dipole moment
(vector)

just
the positive
charge



For a physical dipole,
 q_+ and $\underline{s}$ are
finite.

displacement vector
from negative
to positive charge

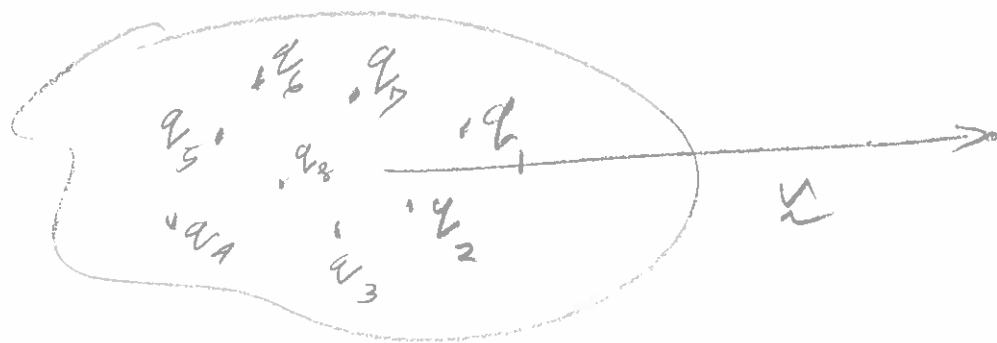
An ideal dipole has $q_+ \rightarrow \infty$ and $\underline{s} \rightarrow 0$
in such a way that $\underline{P}$ stays finite
(G-150)

For an ideal dipole, $\underline{E}(\underline{r})$ given on p.02-05064
is exact. (G-150)

b) Now our dipole formula $\underline{P} = q_+ \underline{s}$
can be to any set of localizable point charges
and any localizable continuum of charge.

12_05066

Localizable in this context means you can put the charge within a finite closed surface



) When you are sufficiently far away from the charge distribution, you can expand the field in a multipole expansion series (6-145)

$$\vec{E} = O\left(\frac{1}{r^2}\right) + O\left(\frac{1}{r^3}\right) + O\left(\frac{1}{r^4}\right) + O\left(\frac{1}{r^5}\right) + \dots$$

Monopole term
Zero if
total charge
is zero

dipole
term

quadrupole
term

octopole
term

etc.

It is well beyond our scope to cover multipole expansion but we should keep in mind that it can be done.

In fact quantum mechanical particles (e.g., electrons, protons) can be treated as classical point particles in many cases since only the monopole is important in those cases

2026 Aug 01

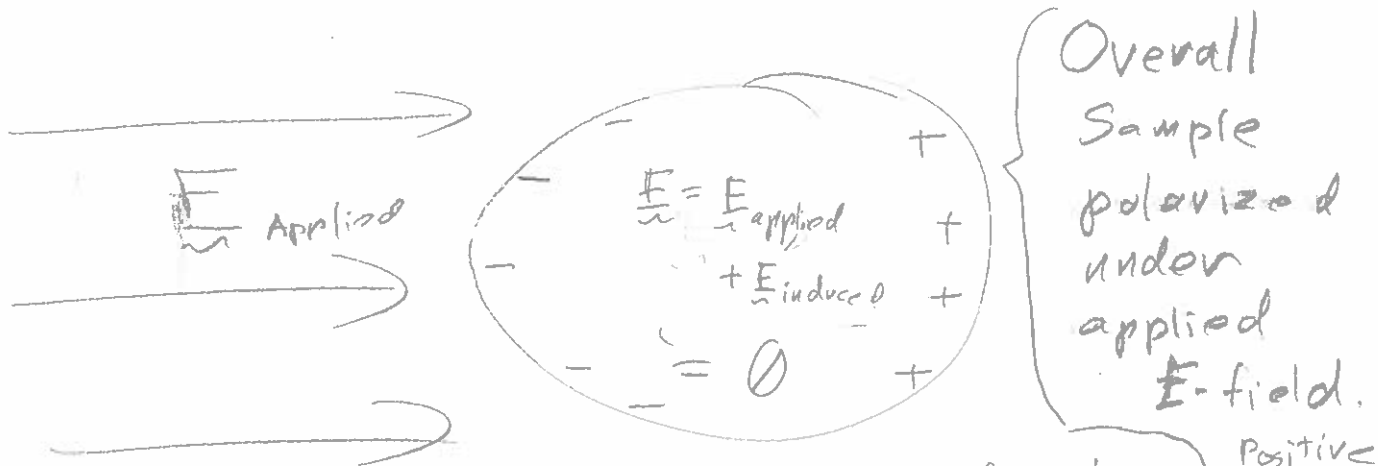
02_05067

5.8 Polarization of Bulk Materials

Mostly Thinking of Insulators

Leaving Conductors Mostly
for Chapter 6

a) Conductor (Thinking of Solid Metals)



The negative charge are attracted
The positive charge is repelled.

Positive charge repels in field direction

Negative charge attracts in the opposite direction

In metals the free electrons
(negative charge mobile charge carriers)

$$q = -e = -1.602176634 \times 10^{-19} \text{ C}$$

exact by definition

move

and the free positive charge is
absence of electrons,

All such free charge is on the surface

02-05068

Ideally all free charge
is on the surface in
electrostatic cases
and inside $\underline{E} = 0$

This is macroscopic $\underline{E}$

At the microscopic level
the $\underline{E}$ -field has all kinds
of variations over the
ions,

The free electrons are
the outermost 1 or 2 valence
electrons become unbound
to ions in metals, but
are still bound inside
the metal.

In Chapter 6 we go into a bit
more detail about metal conductors

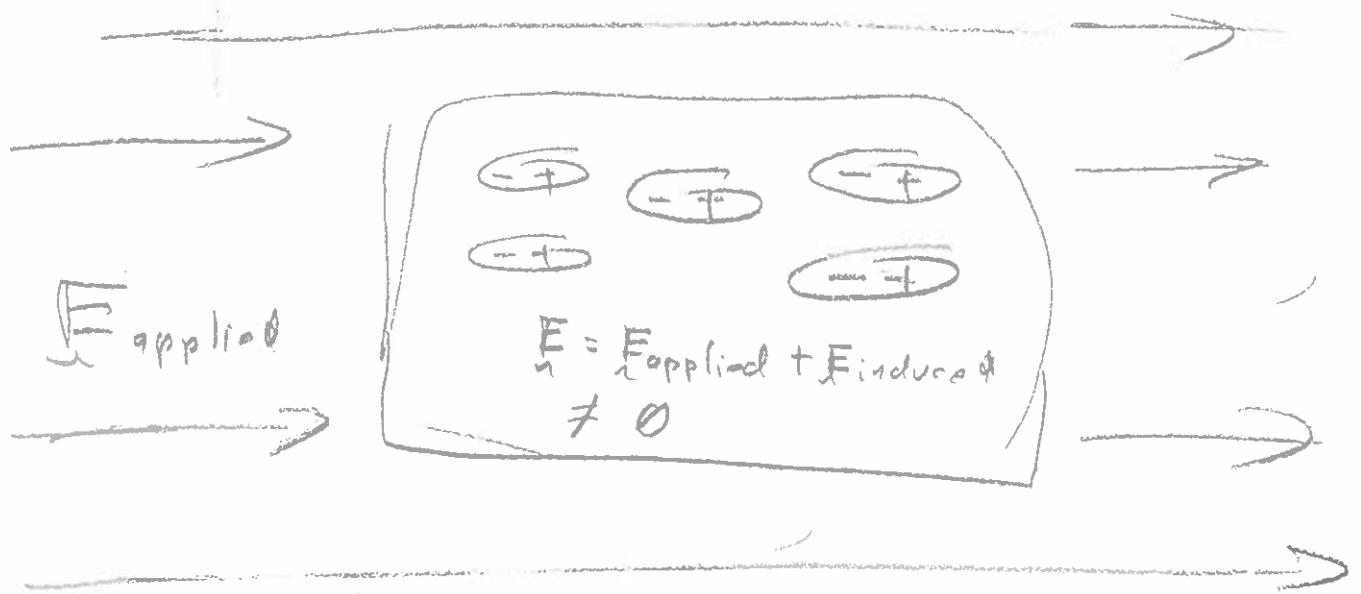
In fluid conductors, positive and
negative ions are the charge carriers.

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[02.05065]

b) Dielectrics

A dielectric is an insulator that can be polarized by an applied E-field and, in fact, all insulators must be dielectric to some degree.



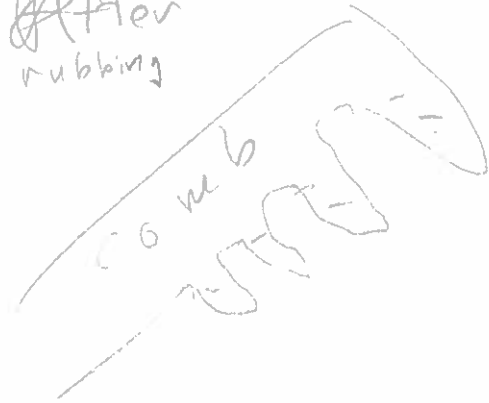
There is NO free charge to move macroscopic distances but the individual atoms or molecules polarize or if there are permanently polarized molecules they can align with the applied E-field if free to rotate.

Develop an induced dipole moment (see p. 02.05065.)

2:05070

c) Demonstration repeated from p. 02-05041.

After rubbing



There is an attraction of the polarized sample because the positive end of each dipole moment is in a stronger E-field than the negative end.



E_{applied} field of comb

Polarized paper made of individually polarized molecules and atoms.

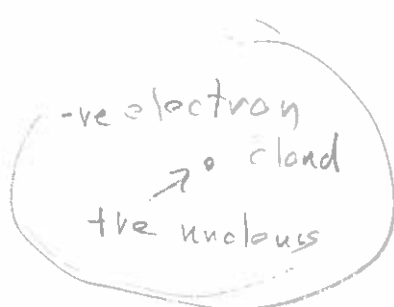
Probably mostly induced polarization since paper molecules NOT free to rotate much.

This is a highly simplified picture of a polarized atom or molecule.

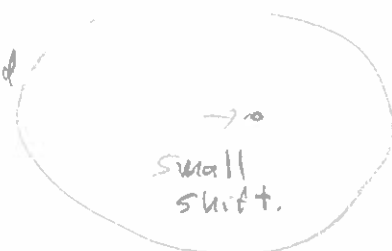


A bit more realistically

Atom & nucleus



E_{applied}



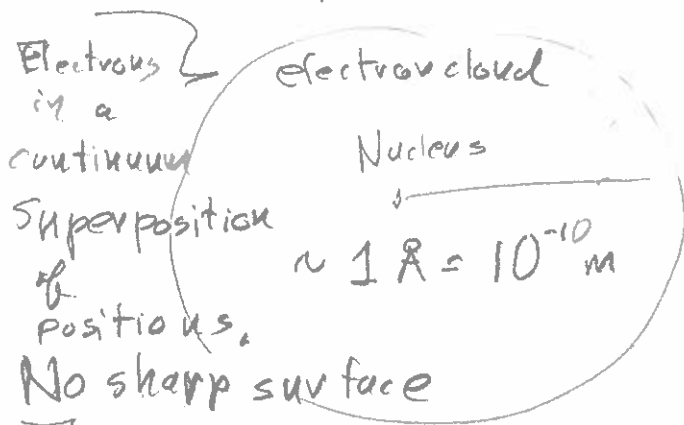
(G-159)

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d) Recall

Atom size scale



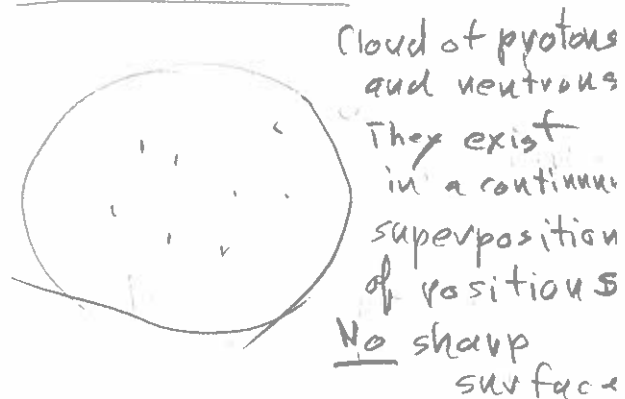
Nucleus bonded electron cloud by the long-range inverse square law Coulomb force

$$\propto \frac{1}{r^2}$$

which formally only goes to zero at infinity

Nucleus size scale

$$\sim 1 \text{ fm} = 10^{-15} \text{ m}$$



Nucleus of the protons and neutral neutron

bonded together by nuclear strong force. Very strong since it can overcome the Coulomb force repulsion of protons, but it extends only $\sim 1 \text{ fm}$ from

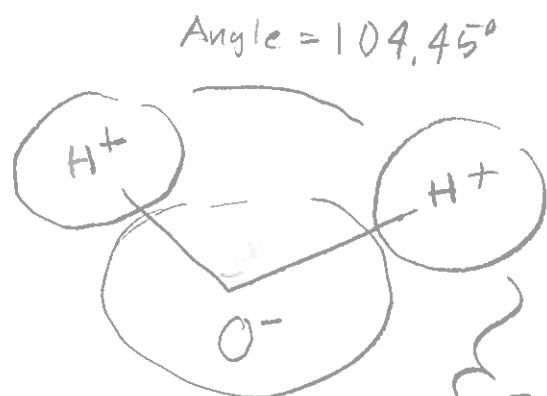
22-05072]

) According to Google AI,
ground state atoms
(atoms in their lowest
internal energy state)
have no intrinsic polarization.

Do NOT
have
permanent
dipole
moments

But there can be polar atoms
if the atom is an excited state
(I'm NOT totally convinced).

Molecules can be polar molecules
(i.e., have permanent dipole moments).
An all-important example is
the H_2O molecule of water



(Wik
G-161)

Inter-nuclear
separation
= $0.9584 \text{ \AA}$

An overall
small charge
separation,
and so a
permanent
dipole moment
= 1.8546 D

$D = \text{debye}$
 $\cong 3.33564 \times 10^{-30} \text{ C}\cdot\text{m}$
 $\cong 0.2081943 \text{ e}\cdot\text{\AA}$
unit of dipole moment (Wik)

[2026aug01]

[02-05073]

Why can a small charge separation exist?

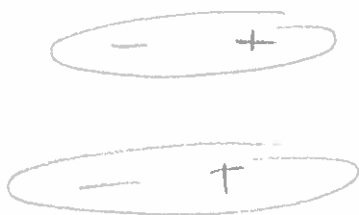
Short answer $\rightarrow$ Complex quantum mechanics
interaction
of dynamics and forces

We leave it to chemistry for
a longer answer.

All very
complex, but
it allows
complex materials
and
life as
we know it

f) In liquid water
the permanent dipole moments
are torqued to align
with an applied E -field
and some induced polarization
can exist too

E_{applied}



{ schematic
water molecules



torquing action of E_{applied} tends to align dipoles
(thermal motions try to randomize)
order

02-05074

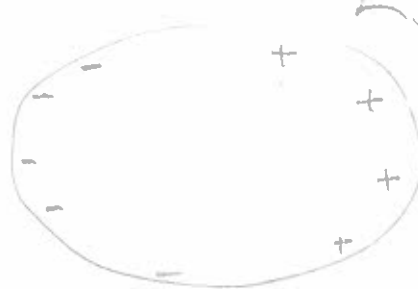


a) Just a short coverage

Positively charged object



E_{applied}



Isolated metal conductor
Polarized by E_{applied}
Recall the separated charge is on the surface



E_{applied}



Wire to ground

Ground symbol

Positive charge flows to ground.
Really electrons flow out of ground

[2026aug01]

[02-050]

If you disconnect the wire,
the conductor stays
negatively charged.

b) Ground is any huge
(effectively infinite) source
of charge.
Ground can just be the physical ground.

As an electrical safety feature
conductive parts of a device
that insulated from current
and charge are grounded.
Thus, if insulation fails
there is a very low resistance
(more generally impedance) path
for current to flow through
(and NOT flow through a human)
and the strong current should cause
the circuit to break by a blown fuse
or circuit breaker of some kind.

02-09076

c) Lightning

- very short coverage
of cloud-to-ground lightning

~ 99% cases

the atmospheric

E-field points
up and

so

electrons flow
down to ground

(Wiki: lightning)

If your hair stands
on end, negative
charge has been
repelled to Ground
and the hair strands
are mutually repulsive
and you are
in danger of
a lightning strike.

Get out of that place.
Take shelter in a closed
building or vehicle.

Don't touch conductor →

(Wiki: lightning strikes)

