2025jun01 | 14001

Ch14 Fluids, but primarily Liquids

Show Phase image

14.1 a) 3 main phase of matter

Solid | Liquid | gas

(Liquid and gas: Fluid)

Solid

fairly rigid chemical bonds between atoms and/or molecules

Terrestrial ones do not compress much under air pressure.
But a slab of iron in vaccum, very little de-strain

Liquid

No rigid bonds, but bonded enough for atoms and molecules to be touching. So strong resistance to compression

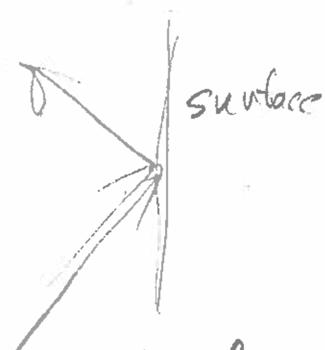
The particles slide over each other, and so little resistance to shear stress
- surface tension gives a little

Atoms and molecule strongly resist compression.

So both solids and liquids have significant bulk moduli

gas

The atoms are far apart only collide with each other and container to exert pressure



Presure is due to particle collisoons at micro level

(1-002)

Actually, ordinary terrestrial liquids are delicate states. They do not form if the pressure is too low. (show water phase diagram and dry ice image)

Except for planets, astrophysics seldom think of liquids.

Pressure too low and/or temperature too high,

For example, deep interior of stars. Density is very high, but temperature is so high the atom do not bond

or for that matter think of solids.

Stars are all too hot.

In stars, we have ideal gas law pressure for particles and photon gas for intense radiation: i.e.,

$$\underbrace{p = \frac{N}{V}kT = \overbrace{n}^{\text{particle density}}kT}_{\text{ideal gas law}} \qquad \underbrace{p = \frac{1}{3}aT^4}_{\text{photon gas}}$$

b) Density (2025 jun 01) (14003

Note: density $= \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$

A very few numbers

Densities in g/cm³ multiply by 10^3 for kg/m³

{ g/cm³ are natural unit, Right size for understanding

Solids ($0°C$, $P \doteq P_{air}$)		Liquids ($0°$, $P \doteq P_{atm}$)	Gases STP - pre 1982 $T = 0°$, $P = 101.325$ kPa ≡ 1 atm exactly
Ice	0.917	water 0.9998 (at 4° 1.000 densest)	Air 1.29×10^{-3} (so about 1000 times less dense than water)
Earth crust	3.30		
Iron (Fe)	7.86		
Gold (Au)	19.3	Sea Water 1.030	
Osmium	22.59	Mercury 13.6 only metal a liquid at room temperature but toxic	

c) Pressure

The magnitude of a normal force per unit area exerted on any surface in any direction by a substance.

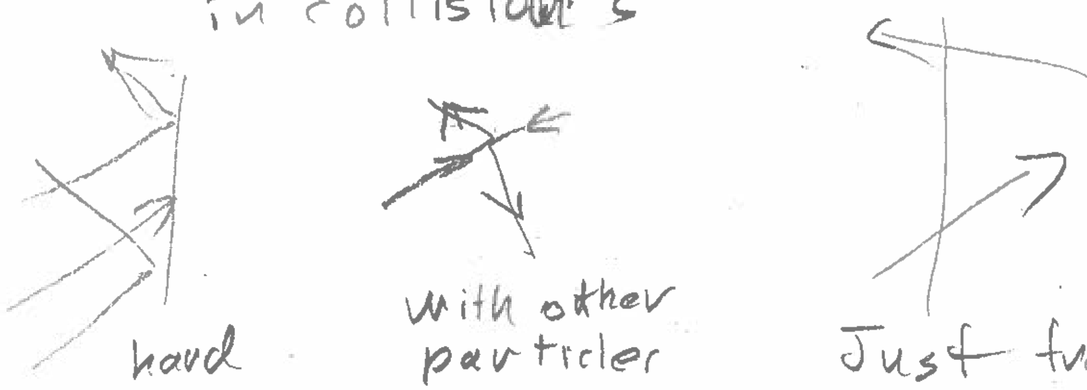
i.e., it's isotropic = same in all directions

$\boxed{14004}$

The surface can be hard or soft (e.g., the fluid pressure on itself)

<u>Pressure</u> as usually defined is isotropic

<u>Gases</u> exert pressure because they are made of free flying particles that transfer momentum in collisions



hard

with other particles

Just free flying thru an imagined surface

<u>Liquids</u> exert pressure because the atoms are being squeezed by containment

illustration

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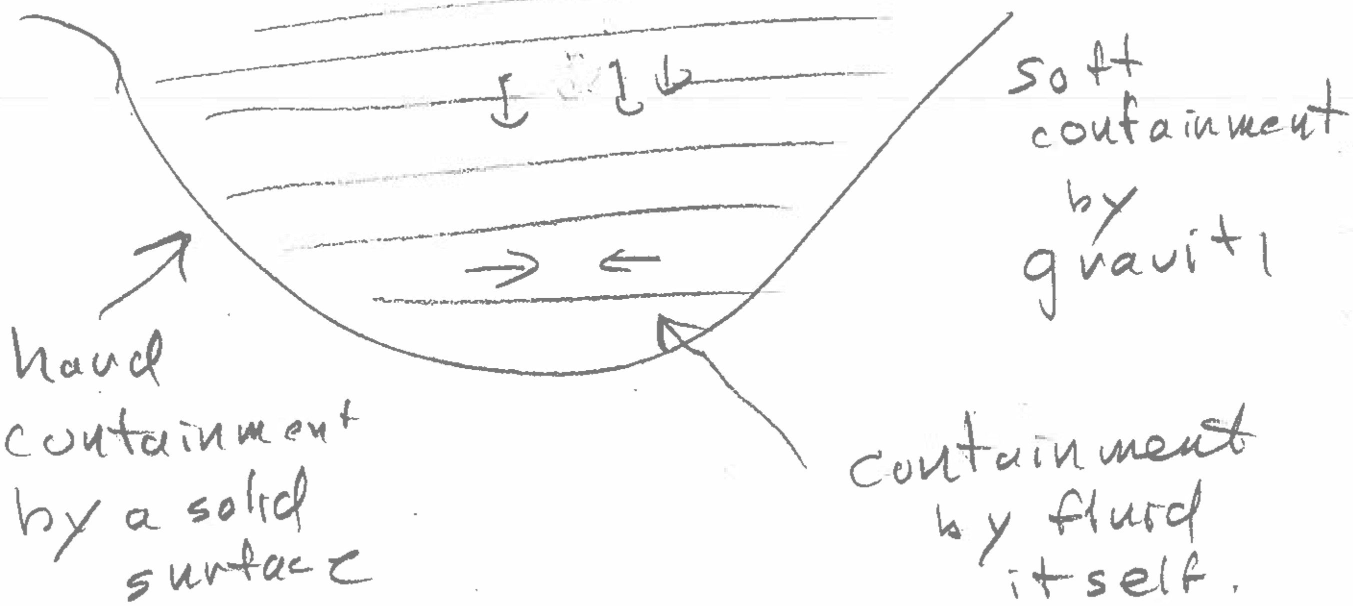
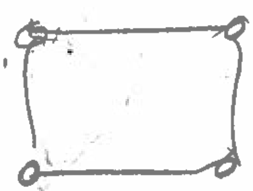


What if no pressure imposed on a liquid?

Usually, it evapourates and becomes a gas. Liquid bonding requires an external pressure except a little surface tension might hold it bond in some cases

Solids

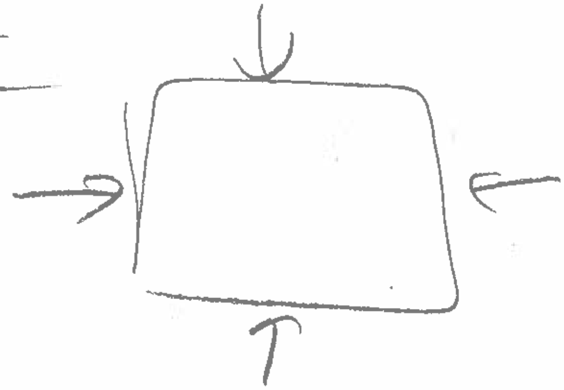


The rigid bonding of solid means they hold there shape without containment.

With no outside force squeezing them they have zero pressure

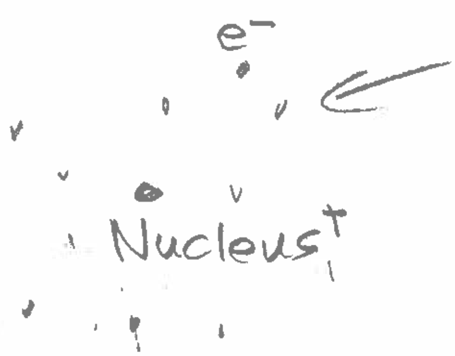
14006

but ambient pressure squeezes them



and so does gravity if they are NOT in free-fall.

d) Why Atoms Don't Like being Compressed



negative electrons bonded to positive nucleus by electrostatic force. { Inner ones very tightly bonded }

{ But qualified below } Quantum mechanics forbids them to collapse into nucleus and gives them a very strong

[14007]

containment pressure
called degeneracy pressure
which near independent
of temperature
and strongly
dependent on density
Strongly resists
more compression than
they already have by bonding.

~~Act~~ Actually under astrophysically
high pressures
the electrons will
interact with protons
to create an all neutron
object called a neutron star
held up against selfgravity
by neutron degenerate pressure.
But even neutron stars
will collapse under

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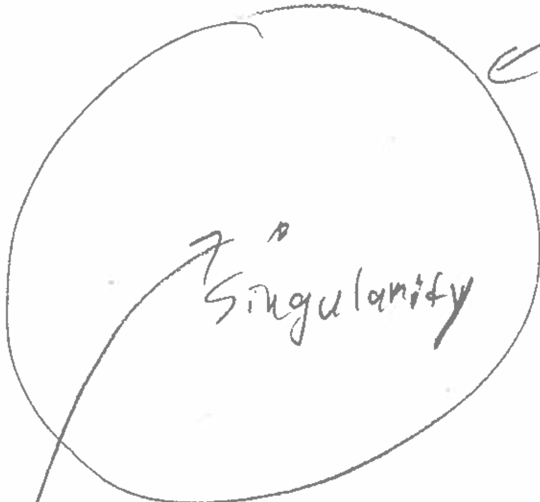
self-gravity become black holes

event horizon



according to generality nothing can get out by passing through the event horizon

(Quantum mechanics dictates mass-energy can get out slowly, but in an ordinary sense

all mass-energy collapse to a point of infinite density according to general relativity

But we believe quantum gravity must forbid this somehow but we don't know how.

We have no established theory of quantum gravity. There many but which is right if any we don't know.

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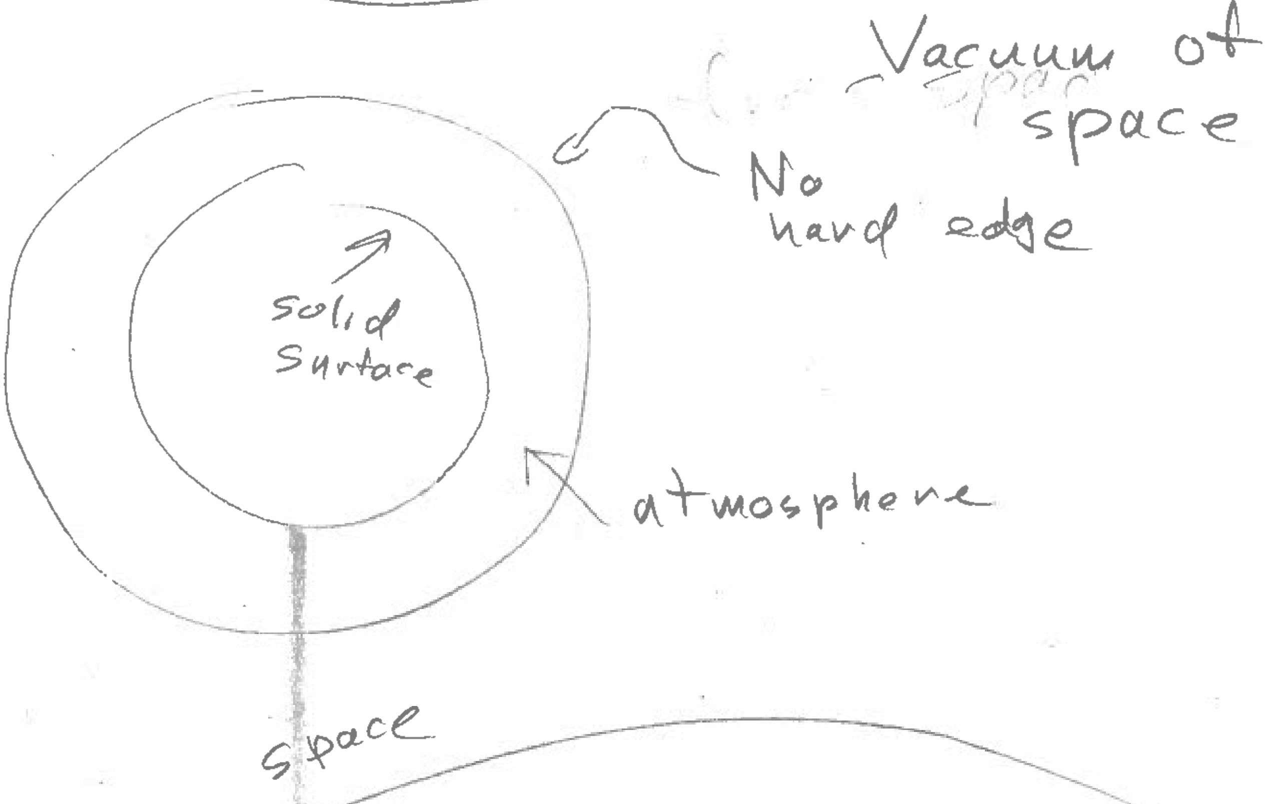
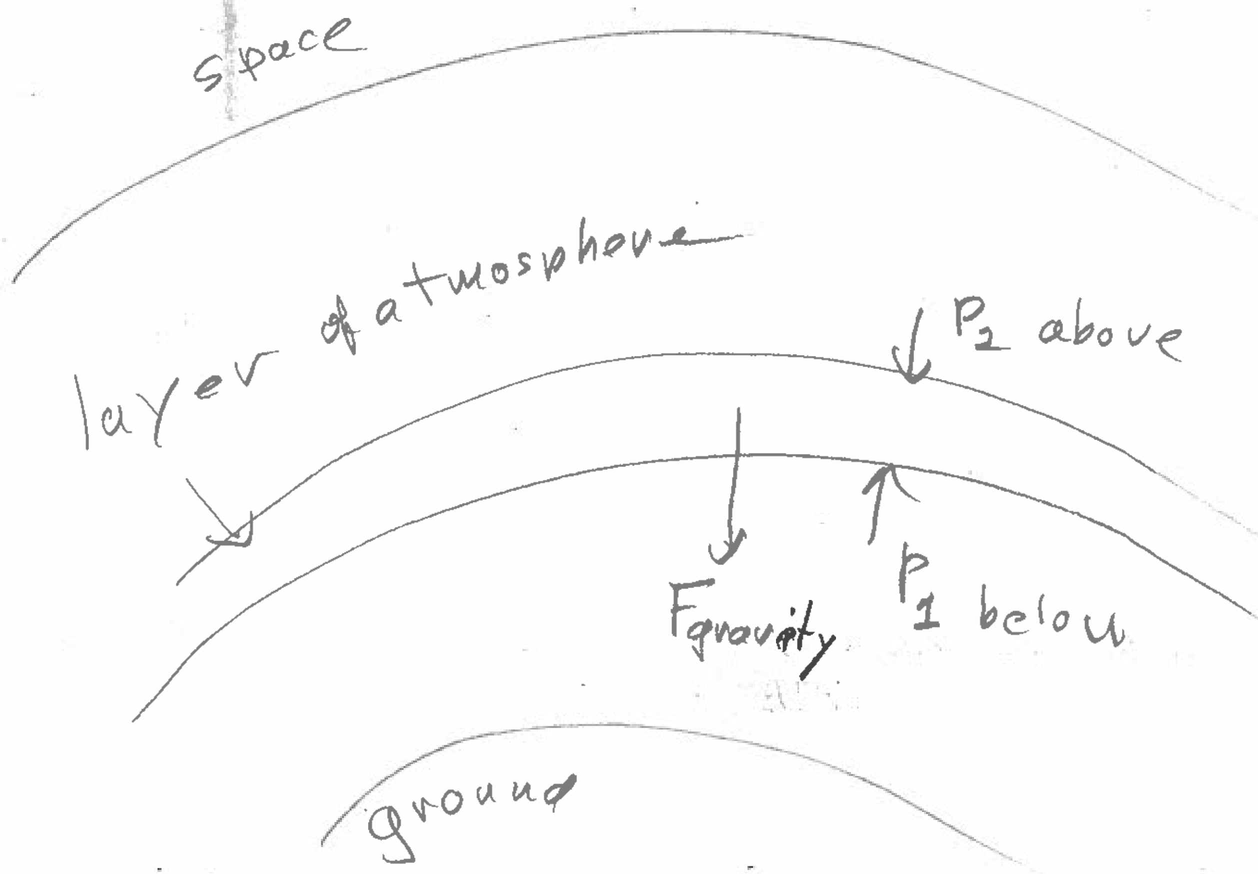
e) How does gravity Contain the Earth's Atmospher





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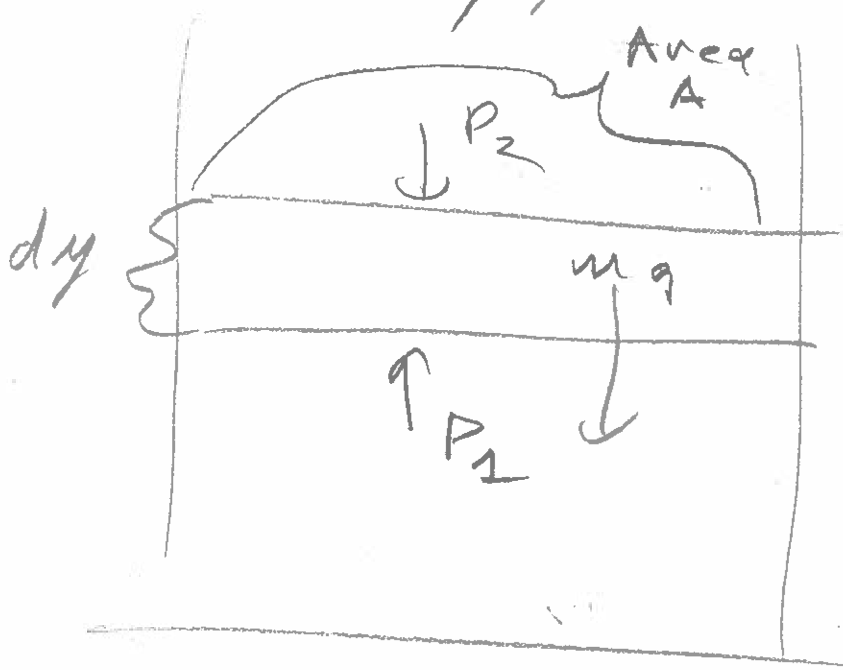
In tangential direction, the pressure is constant on it would air would flow tangentially to come into hydrostatic equilibrium where pressure is constant tangentially

hydrostatic equilibrium = fluid equilibrium

Vertical air column

Vertically, there is also gravity



The atmosphere is thin enough we can ignore curvature.

$F = ma$ applied vertically for hydrostatic equilibrium

$\therefore a = 0$

$$0 = (P_1 - P_2)A - mg$$

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$$m = \rho A\,dy$$

ρ is density

$$\therefore \quad 0 = (P_1 - P_2)A - \rho g A\,dy$$

$$(P_2 - P_1) = -\rho g\,dy$$

$$\frac{dp}{dy} = -\rho g$$

which is a differential equation for pressure

To solve we need an <u>equation of state</u> EOS

$$P = P(\rho, T, \text{composition})$$

Air closely approximates an ideal gas and so the ideal gas law is our equation of state

$$P = nkT$$

14012

$$n = \frac{\rho}{m}$$

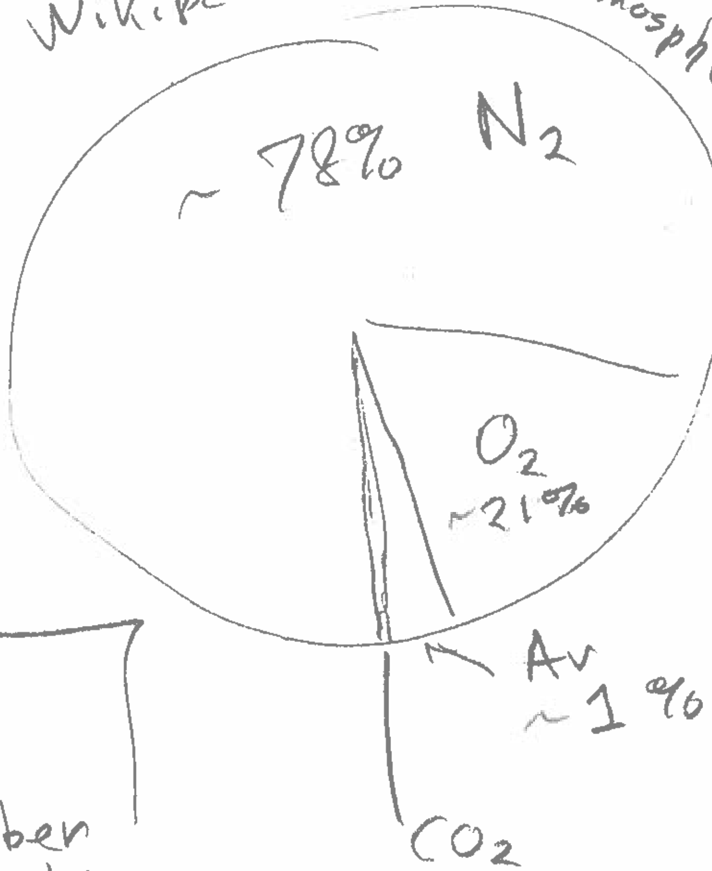
particle density

mass density

mass of particles

N_2 dominates, we can just

Wikipedia: Earth Atmosphere



by number count

Boltzmann constant

$k = 1.380\,649 \times 10^{-23}$ J/K

defined to be exact

T is kelvin Temperature where zero is absolute zero

$1\,\mathrm{K} = 1^\circ \mathrm{C}$

— only the zero point differs between Kelvin and Celsius scale

Water Vapor is excluded because it varies so strongly

0 – 3%

and typically ~ 1%

So $P = \frac{\rho}{m} \underbrace{kT}$

assume costant though it does vary significantly

Our DE recall is

$$\frac{dP}{dy} = -\rho g$$

$$\underbrace{\frac{dP}{d\rho}\frac{d\rho}{dy}} = -\rho g$$

using chain rule of calculus

$$\frac{kT}{m}\frac{d\rho}{dy} = -\rho g$$

$$\frac{1}{\rho}\frac{d\rho}{dy} = -\frac{mg}{kT} = -\frac{1}{y_s}$$

absorbing the constants into y_s. — a scale height

We can solve !!!

14014

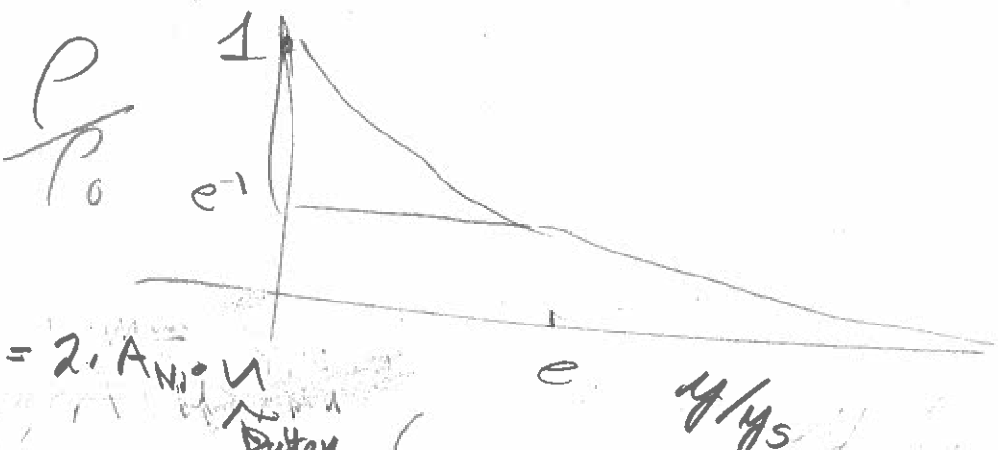
$$\frac{d\rho}{\rho} = -\frac{dy}{y_s}$$

$$\ln(\rho/\rho_0) = -\frac{y}{y_s}$$

ρ_0 is ground density

We take ground to be $y=0$

$$\rho = \rho_0 e^{-y/y_s}$$



$m = 2 \cdot A_N \cdot u$ (Dalton)

$$y_s = \frac{kT}{mg} \simeq \frac{1.4\times10^{-23}\cdot 250}{2\cdot 14\cdot 1.7\times10^{-27}\cdot 10} = \frac{2.5}{2\cdot 1.7}\times 10^{4}$$

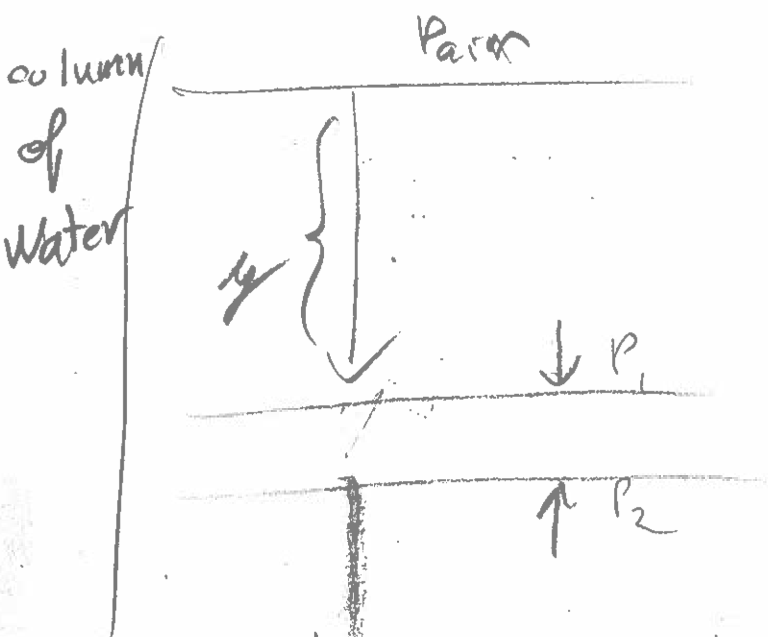
Maybe a good characteristic value

$\hat{=}\ 0.75\times10^{4}\,\mathrm{m} = 7.5\,\mathrm{km}$

Google AI says $y_s \hat{=} 8.5\,\mathrm{km}$ } Not a bad estimate

(2025 jun 01) 14015

f) Pressure with depth in water



Take down as +ve

$$0 = P_1 - P_2 + \frac{mg}{A}$$

$$m = \rho A\,dy$$

$$P_2 - P_1 = \rho g\,dy$$

$$\underbrace{dP = \rho g\,dy}_{\text{No minus since down is positive}}$$

First, assume water is incompressible. Usually a good assumption with liquids.

$\therefore\ \rho = \text{constant}$

$$\therefore\ P = \rho g y + \underbrace{P_{air}}_{\text{the boundary value}}$$

$$P = 10^{+3}\cdot 10\,y + 10^5\,Pa$$

$$= \Big(10^4 \underbrace{\frac{Pa}{m}}_{\text{water}}\Big)\,y + 10^5\,Pa$$

14016

So every 10 m further down, the pressure increase by ~1 atm.

This is why you can't snorkel <u>breathing</u> deeper than ~2 m
Hard to inhale (Google AI)
Snorkelers can go deeper holding their breath as free divers do.

Omit in class Optional Reading

<u>But</u> water does actually compress with pressure.
<u>Can</u> we do a phony estimate?

We need an equation of state EOS
Consider the <u>Bulk modulus formula</u>

$$\frac{dV}{V} = \frac{P}{-B}$$

Now $\rho \propto \frac{1}{V}$

$$d\rho \propto \frac{-1}{V^2}dV$$

$$\frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\frac{d\rho}{\rho} = \frac{P}{B}$$

Can P be really a differential?

$$\frac{d\rho}{\rho} = \frac{dP}{B}$$

$$\ln(\rho/\rho_0) = P/B$$

$$P = B\ln(\rho/\rho_0)$$

$$\frac{dP}{d\rho} = B/\rho$$

$$\frac{B}{\rho}\frac{d\rho}{dy} = \rho g$$

$$\frac{1}{\rho^2}d\rho = \frac{g}{B}dy$$

$$\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right) = \frac{gy}{B}$$

$$\frac{1}{\rho} = \frac{1}{\rho_0} - gy/B$$

$$\rho = \frac{\rho_0}{1 - \rho_0 g y/B} \begin{cases} \rho_0 & \text{for } y = 0 \\ \infty & \text{for } \rho_0 g y/B = 1 \end{cases}$$

Kind of makes $P \to \infty$ as $\rho \to \infty$

$$y_\infty = \frac{B}{\rho_0 g} \quad \left\{ \frac{N/m^2}{N/m^3} \;\; m \text{ units correct} \right.$$

$$= \frac{0.22 \times 10^{10}}{10^3 \cdot 10}$$

$$= 0.22 \times 10^6 \text{ m}$$

$$= 220 \text{ km}.$$

All well beyond Bulk modulus range of validity

14.3 Pascal Principle

When a change in pressure is applied to a rigidly contained incompressible ~~fluid~~ the change is transmitted to all parts of the incompressible ~~fluid~~ and the container

↳ incompressible fluid means it has to be an ideal incompressible liquid

↳ All liquids do compress a bit, but by small amounts

e.g., water

$$\underbrace{\frac{\Delta V}{V}}_{\text{strain}} = - \underbrace{\frac{dP}{B}}_{\text{stress}}$$

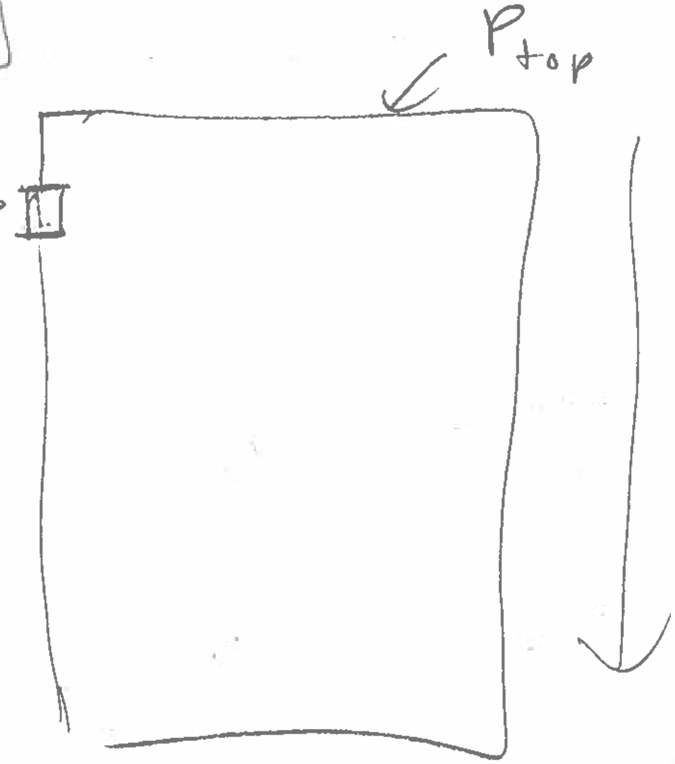
I think the pressure must be differential pressure

Bulk modulus for water $B = 0.22 \times 10^{10}$ Pa

$\therefore dP = P_{air} = 10^5$ Pa

cause $\frac{\Delta V}{V} \doteq -5 \times 10^{-5}$ relative compression

Piston force F_{piston} on a one-way piston



$$P = \rho g y_i + P_{top}$$

(see p. 14015)

and

$$\Delta P = \frac{F}{A_{piston}}$$

$$\therefore$$

$$P_{new} = P + \Delta P = \rho g y + P_{top} + \Delta P$$

The change must be communicated at a finite speed

We are NOT proving. We just accept.

Of order speed of sound in fluid (Sound is a pressure wave)

Quasi instantaneous to casual observation

$$v_{\substack{sound\\ water}} \triangleq 1500\,m/s = 1.5\,km/s$$

($T \sim 25°C$) (Google AI)

The compression probably reflect around quite a bit before dissipating to waste heat.

14019

Actually there must be a tiny change in Volume

In thermodynamics terms

$$dE = -P\,dV + dQ$$ < waste heat

(1st law of thermodynamics)

dE: change in fluid internal energy

P: Pressure Not Δp

dV: differential change in Volume

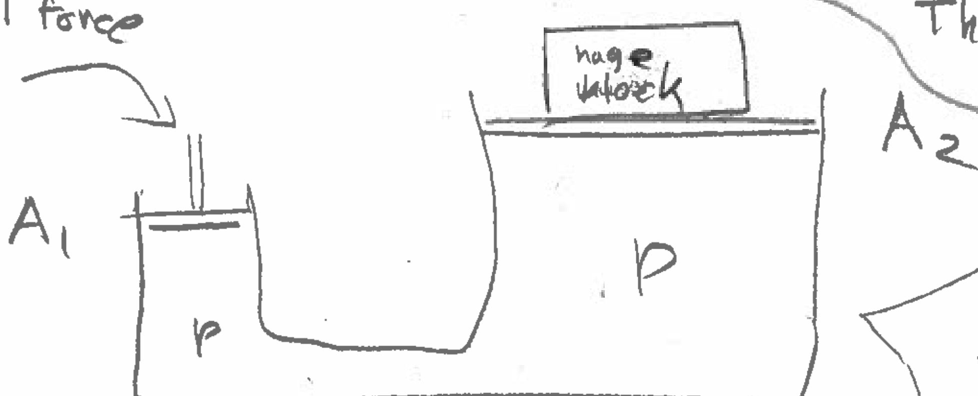
$-PdV$ is called P-D-V work

But these changes are probably below ordinary observation

<u>But</u> for hydraulics

The container is NOT rigid.

There are two piston



Pressure, ~~but let's~~ let's say negligible for simplicity

The effect is the fast transmission of fluid momentum by pressure waves. <u>not</u> pressure change per se

14020

So NOT exactly exploiting Pascal's Principle, but one of bases that is exploited

Quasi-instantaneous transmission by pressure waves of momentum

So same $P_1 = P_2$ same

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

$F_2 = m_2 g$

Say 10^3 kg

$F_2 \triangleq 10^4$ N

Say $F_1 = 1$ N

≈ 0.225 lb

(Wik)

Ratio of $\frac{A_2}{A_1} \triangleq 10^4$

If $A_1 = (0.01\,m)^2$

$= 10^{-4}\,m^2$

$A_2 = 1\,m^2$

So you, yes you with one hand can hold up a car — but really you are just part of the container wall

14021

Every $10^{-4}\,m^2$ bit of it is helping out

<u>But</u> say you do work

$$dW_1 = F_1\, ds_1$$

$$= \frac{F_1}{A_1} A_1 ds_1 = \frac{F_1}{A_1} \underbrace{dV_1}_{=dV_2}$$

$= dV_2$ since fluid is incompressible

$$= \frac{F_2}{A_2} dV_2$$

$$= dW_2$$

Ideally (w Negligible, dE, dV, dQ)

internal energy

waste heat

change in volume

Your energy source has lifted the block and transferred energy to its gravitational potential.

14Q22

But maybe not lifted the block very high

Say $ds_1 = 1m$

$$V_2 = V_1$$

$$A_2 ds_2 = A_1 ds_1$$

$$ds_2 = \frac{A_1}{A_2} ds$$

Example value

$$= \frac{10^{-4} m^2}{1 m^2} \cdot 1m$$

$$= 10^{-4} m$$

$$ds_2 = 10^{-1} mm$$

So a lot of mechanical advantage (Wik) — Force amplification, but energy is still conserved.

14.4 Archimedes Principle & Buoyancy

L 14023

3 cases

a) Body surrounded by static fluid with a downward uniform gravity field

$\therefore \underset{\sim}{F_g} = mg(-\hat{y})$

g is constant

But the body displaced at static fluid Volume of same shape

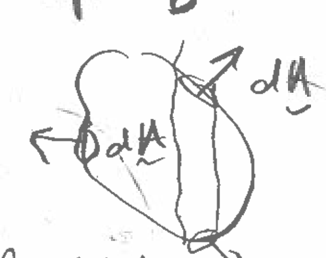
for this volume

$ma = 0 = F_B - m_{fluid}\, g$

$= F_B - \rho_f V g$

$\therefore F_B = \rho_f V g$

Cause of F_b



$F_B = -\oint P(y)\,\hat{y}\cdot d\underset{\sim}{A}$

if $P(y) = $ constant

$F_B = -P\oint \hat{y}\cdot d\underset{\sim}{A} = 0$

all entries and exits canceling

14024

minus the force vectors point inward

$$F_B = \oint P(y)\,\hat{y}\cdot(-d\underline{A})$$

Exits and entries do Not cancel since P varies with y

P(y) increases with depth e.g., water

$P(y) = \rho g y + P_{air}$

$$F_x = \oint P(y)\,\hat{x}\cdot(-d\underline{A}) = 0$$ all entries and exits cancel

$$F_B = \rho_f V g$$ ρ_f density of fluid and volume displaced by body

Now for the submerged body in the y direction

$$ma = F_B - mg$$

but this must be the same since fluid pressure is the same on

mass of body

L14025

$$ma = (\rho_f V - \rho V)g$$

$$= (\rho_f - \rho)Vg$$

average density of body

Average is a key word since steel boats float

$\therefore$ Archimedes principle

$$F_B = \rho_f V$$

density of fluid

Volume of Fluid displaced

In words, the buoyant force is equal to the weight of the displaced fluid (displaced by body)

Note

$\rho_f > \rho$ $a > 0$ & body floats up

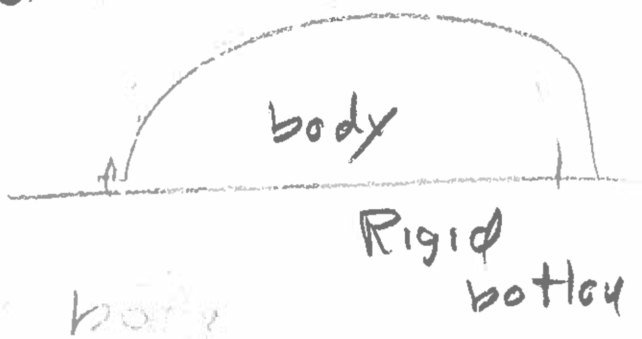
$\rho_f = \rho$, $a = 0$ and body floats neutrally

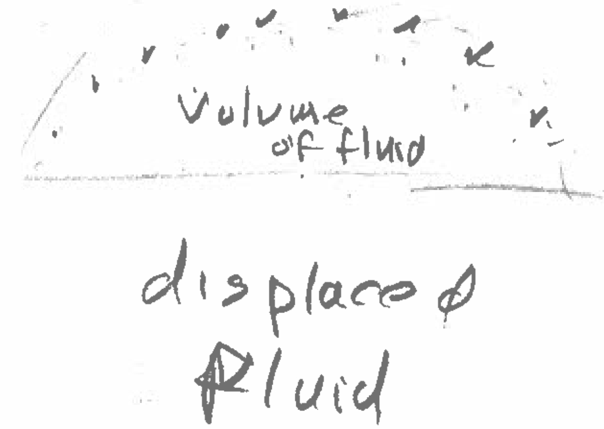
$\rho_f < \rho$, $a < 0$ and body sinks

$\boxed{14026}$

Not a thrilling case

b) Body on rigid Bottom





$$0 = F_B + F_N - \underbrace{mg}_{\rho V}$$

ρ: body density

$$0 = F_B + F_{N_f} - \underbrace{m_f g}_{\rho_f V}$$

ρ_f: fluid density

Buoyant force Must the same, but it can even be negative since less pressure from below.

But F_N is not in general equal to F_{N_f}, and so you cannot in general

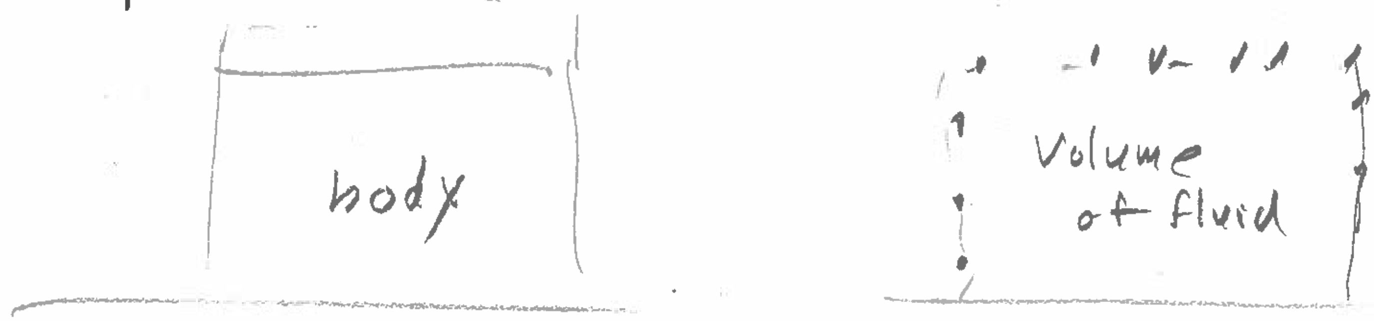
14027

determine F_B

Nor F_N or F_{Nf}.

In special cases, you can, but they are not very interesting.

Example



$$0 = F_B + F_N - mg$$

$F_B = -m_{\text{fluid}}\, g < 0$ fluid above body in the fluid column

$\therefore\ F_N = m_{\text{fluid}} + mg$ ~~above~~ body in the fluid column

$$0 = F_B + F_{Nf} - m_f g$$

$\therefore\ F_{Nf} = -F_B + m_f g = m_{\text{fluid}\downarrow\text{bottom}}\, g$

mass of all fluid above bottom in fulid column

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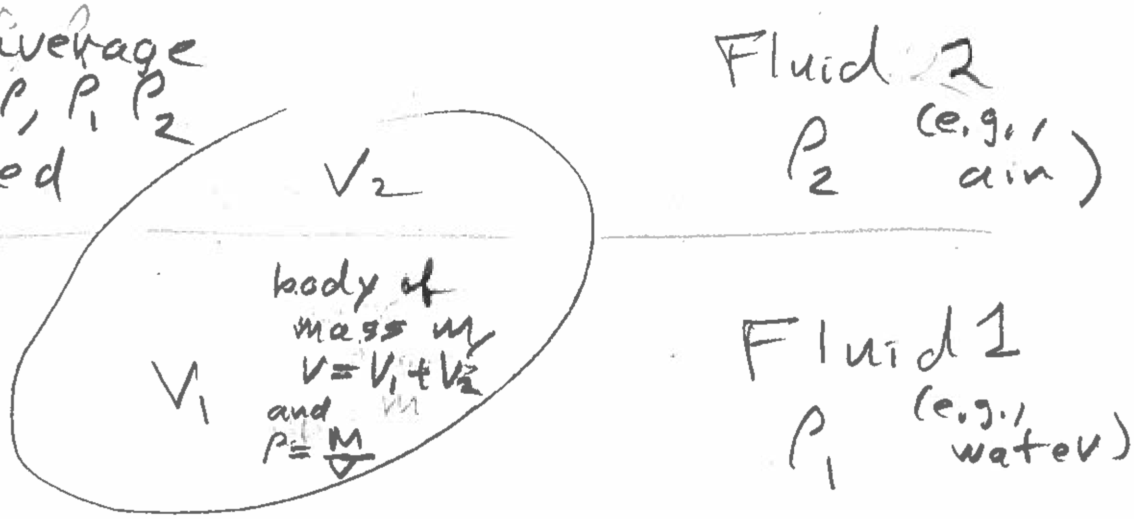
c) Surface Floating Body

Note only average values of ρ, ρ_1, ρ_2 are needed in the derived formula



In this case it is a given that

$$0 = ma \text{ for body}$$

$$0 = F_B - mg = F_B - \rho V g$$

But also

$$0 = F_B - m_{f1+f2}\, g = F_B - m_{f1} g - m_{f2} g = F_B - \rho_1 V_1 g - \rho_2 V_2 g$$

$F_B = F_{B1} + F_{B2}$ but you don't need them individually and only a detailed calculation would give them individually I think

$$0 = -\rho V g + \rho_1 V_1 g + \rho_2 V_2 g$$

1 equation

$$0 = \rho_1 V_1 + \rho_2 V_2 - \rho V$$

2 equation

$$V = V_1 + V_2$$

We have two unknowns V_1 and V_2 and two equations. We can solve

114029

$$V_2 = V - V_1$$

$$\therefore \quad 0 = \rho_1 V_1 + \rho_2 (V - V_1) - \rho V$$

$$= (\rho_1 - \rho_2) V_1 - (\rho - \rho_2) V$$

$$\therefore \quad (\rho_1 - \rho_2) V_1 = (\rho - \rho_2) V$$

$$\frac{V_1}{V} = \begin{cases} \dfrac{\rho - \rho_2}{\rho_1 - \rho_2}, & \text{in general} \\ \dfrac{\rho}{\rho_1} & \text{if } \rho_2 = 0 \text{ or is negligible} \\ 0 & \text{if } \rho = \rho_2 \\ 1 & \text{if } \rho = \rho_1 \end{cases}$$

fluid 2

fluid 1

body all above interface

body all below interface

Note the solution V_1 and V_2 do NOT know about physical limitations:
$V_1/V \in [0, 1]$
and $\rho \in [\rho_1, \rho_2]$
In fact if $\rho_1 = \rho_2$,
V_1 and V_2 are indeterminate without more conditions since the body is floating in effectively 1 fluid.
You could impose
$\rho = \frac{1}{2}(\rho_1 + \rho_2)$
then $\frac{V_1}{V} = \frac{1}{2}$ and
this would the same at $\rho_1 = \rho_2$

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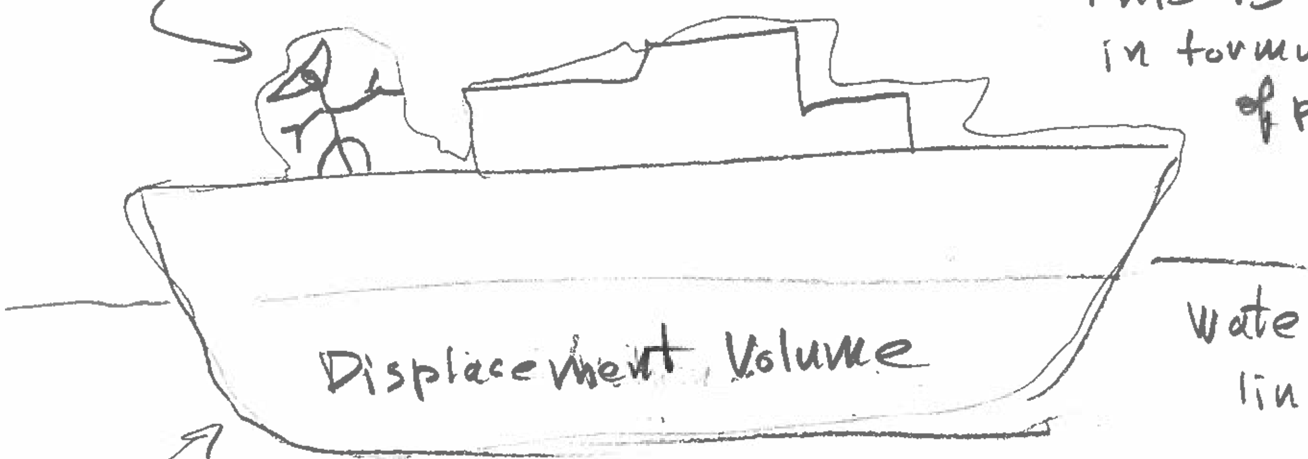
So why do steel ships float?

To give an cogent answer, I think you must say what is a ship's volume.

There are various conventional ways specify a ship's size and they have their uses.

But for my purposes, anything within the recognizable 'perimeter' of the ship.

This is V in formula of p. 14029.



water line

Displacement Volume is within 'perimeter' $= V_1$ in the formula of p. 14029

Formula of p. 14029

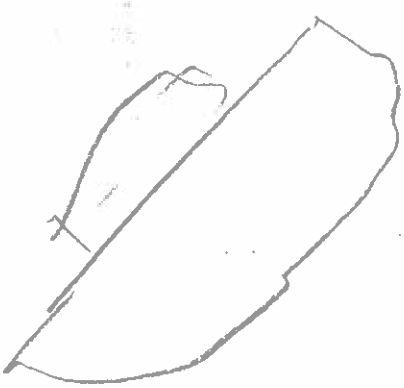
$$\frac{V_1}{V} = \frac{\rho - \rho_2}{\rho_1 - \rho_2}$$

When Displacement Volume is full of air, $\rho_{ship} < \rho_{water}$ is generally met.

Any water flowing to the ship perimeter adds to ship mass and density and the ship sinks.

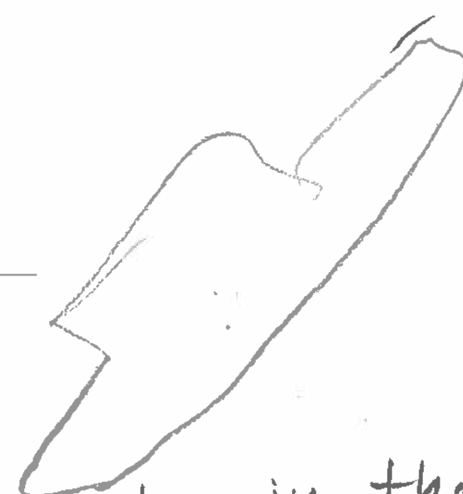
Two perspectives on sinking ship

(1) water line



water in ship is part of ship and (~~buoyant~~ force F_B is still on ship perimeter

(2) water line



water in the ship is now part of medium since it shares medium pressure structure

Either way the ship

14032

the ship sinks
since $\rho_{ship} > \rho_{water}$

I prefer perspective 1
since you don't have
to change your
valuation of what
is ship and what
is non-ship