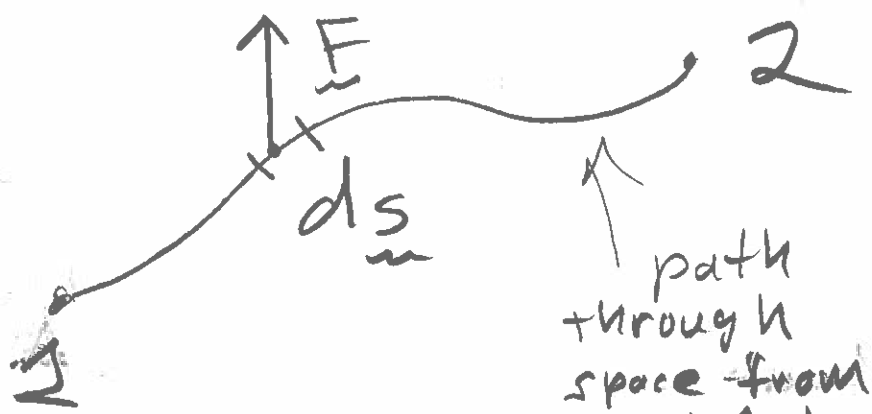
7.1) Work

a) In a physics sense

$$dW = \underset{\sim}{F} \cdot d\underset{\sim}{s}$$

- dot product and so work is a scalar
- dW: differential work done on a body (which has a center of mass)
- $\underset{\sim}{F}$: Force of any kind. Not necessarily net force done on body — external force
- $d\underset{\sim}{s}$: differential displacement



path through space from point 1 to 2.

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Note if

$$\underset{\sim}{F} \cdot d\underset{\sim}{s} = |F||ds|\cos\theta$$

because $\underset{\sim}{F} = 0$

or $\cos\theta = 0$ meaning
$\theta = 90° = \frac{\pi}{2}$

Work is
an <u>energy</u> transfer
process
and if object
has not moved
no energy is transferred
by the
force from
the force source

<u>Units</u>
of work in <u>MKS</u>

joule (J) = $N \cdot m = kg \frac{m^2}{s^2}$

the unit of energy

b) Energy is a rather hard thing to define

One aspect is easy from $E = mc^2$
It is the quantity that resists acceleration and has gravitational effect.

But that aspect is not everything and NOT what people in the 19th century thought of energy when the concept was first formulated.

Just my own perspective.
It is the quantitized and conserved elementary stuff of structure and capability of change

2004

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All structure have
an associated
energy
(e.g., chemical
bonds)
and if you have
energy. you can
do things with it
(e.g., convert chemical
energy
into energy of
motion
and heat energy
and energy of a
field of force)
But if you don't have the energy
to do something, you can't
do.

But forms of energy are energy and ~~any~~ form can be converted to any other form in principle though practice some conversion are hard for us.

e.g. converting intrinsic rest mass energy in large quantities into electrical energy.

If we could do it easily, we'd have no energy worries

Photosynthe for instance is immeasely complex chemistry but by knowing energy limits you can know without knowing what ATP is

<u>Importance of Energy</u> It's a limit on what you or any else can do because it's conserved. But by energy calculations you often get partial information easily where detailed information

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Two additios about energy that come to my mind

i) Except by measuring mass, I don't think there is any other direct way of measuring energy. You always measure other things and calculate it from a formula as we will see.

ii) General Relativity actually does guarantee conservation of energy. But there a general of conservation of energy that is obeyed. We will go into tricky aspect. It is important cosmology and gravitational waves.

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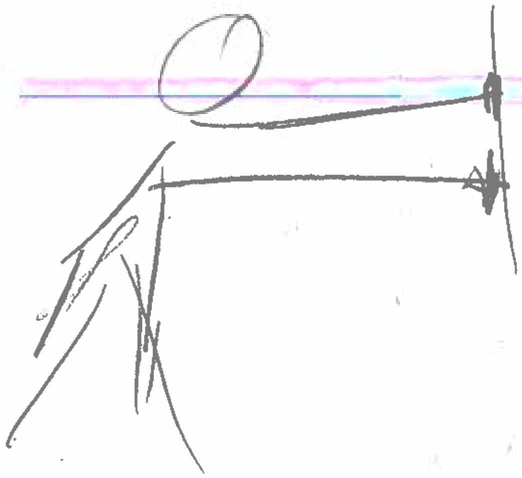
c) Work & No Work



What if you pressing as hard as you can on a wall.

It doesn't move.
You've done no work macroscopically.
At the microscopic level the wall has been flexing
So microscopically, you are do work on the wall and the transferred energy from your chemical energy to heat energy in the wall and sometimes interesting, but

7008

no macroscopic
energy is
transferred
and no macrosopic
work is done
and the Wall hasn't
moved

7.2 Kinetic Energy

Talking about work
is only interesting
I think when you
talk about where it
goes and comes from
One place it goes is
into kinetic energy

Kinetic energy
arise from Newton's 2nd Law

$$dW = \underset{\sim}{F} \cdot d\underset{\sim}{s}$$

general force

but say $\underset{\sim}{F} = \underset{\sim}{F}_{net}$

the net force on a body

$$dW_{net} = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s} = m\underset{\sim}{a} \cdot d\underset{\sim}{s}$$

by 2nd law

and body moves

but movement takes time

$$d\underset{\sim}{s} = \underset{\sim}{v}\, dt$$

and so there is velocity $\underset{\sim}{v}$

$$dW = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s} = m\underset{\sim}{a} \cdot \underset{\sim}{v}\, dt$$

$$= m \frac{d\underset{\sim}{v}}{dt} \cdot \underset{\sim}{v}\, dt$$

Reversing the product rule

$$= m \left(\frac{1}{2}\right) \frac{d\, \underset{\sim}{v} \cdot \underset{\sim}{v}}{dt}\, dt$$

$$= \frac{1}{2} m \frac{d v^2}{dt} dt = \frac{1}{2} m\, dv^2$$

$$dW = d\left(\tfrac{1}{2} m v^2\right) = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s}$$

We define this quantity $\left(\tfrac{1}{2} m v^2\right)$ to be kinetic energy the energy of motion.

$$K = \tfrac{1}{2} m v^2$$

or $KE = \tfrac{1}{2} m v^2$

It's a scalar unlike $\underset{\sim}{v}$ and momentum $\underset{\sim}{p} = m\underset{\sim}{v}$

$$dK = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s}$$

Integrate

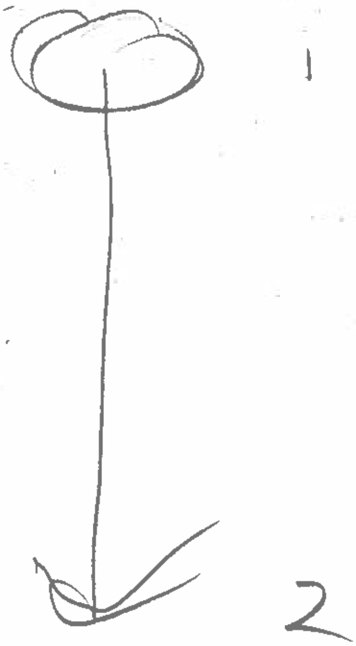
$$\Delta KE = \int \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s}$$

The net force creates a change in kinetic energy.

Note $\underset{\sim}{F}_{net} \cdot d\underset{\sim}{s}$ can be positive or negative and so ΔKE can be positive or negative

Example

Object falling under gravity



$$\underset{\sim}{F}_{net} = \underset{\sim}{F}_{gra} = mg(-\hat{y})$$

$$\underset{\sim}{ds} = \Delta y \, \hat{y} \quad \text{where } \Delta y < 0$$

$$\Delta KE = -mg\Delta y$$

$$\Delta E = mg|\Delta y|$$

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = mg|\Delta y|$$

since you always fall down

$$v_2^2 - v_1^2 = 2g|\Delta y| = 2(-g)\Delta y$$

$$v_2^2 - v_1^2 = 2(-g)\Delta y$$

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which just recovers
The timeless equation
of 1-d kinematics
again.
The essential reason
why is kinetic
energy $\propto m$
and gravity $\propto m$
and so mass
cancels out as
so often in
gravity problems

So energy change Example

$$\Delta KE = mg|\Delta y|$$

$$= 1\,kg \cdot 10\,\frac{m}{s^2} \cdot 1\,m$$

$$= 10\,J$$

So a kilogram falling is 10 J

To give context What is a kilowatt-hour

$$1\,kWh = 1000\,(J/s) \cdot 1\,hour \times \left(\frac{3600\,s}{1\,h}\right)$$
$$= 3.6 \times 10^6\,J = 3.6\,MJ$$

The electric company could bill us in MJ but they don't

Another context

1 food calorie = 4.184 kJ

Google AI: Minimum per day — 1200 foodcalorie women; 1500 foodcal men

$1500 (\text{food Cal}) * \left(\frac{4.184\ kJ}{1\ \text{foodcal}}\right)$

$\approx 6000\ kJ$

$= 6\ MJ$

This is a starvation diet

This is very minimum.

BMR = 80 W

Minimum energy just to live doing nothing

Energy per day $= 80\ W \cdot 10^5\ s$

$= 8\ MJ$

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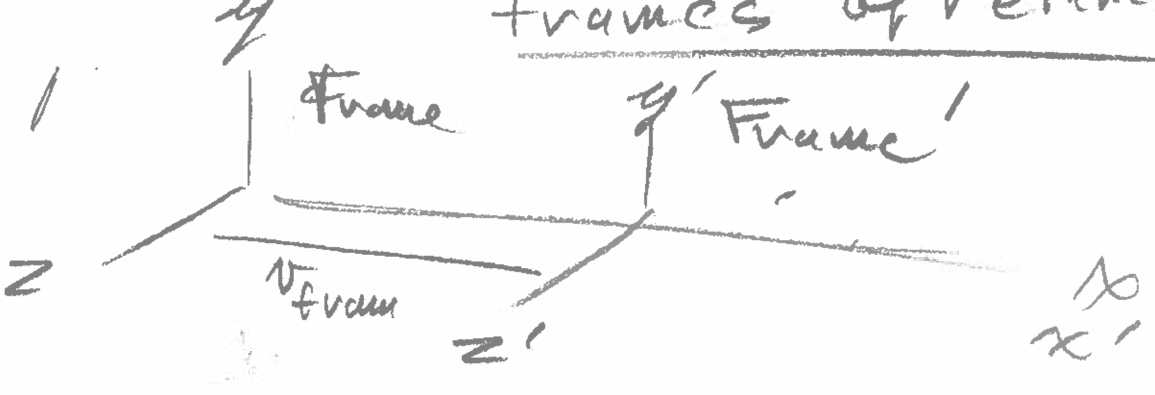
7.3 Work Energy Theorem

$$dW_{net} = \underset{\sim}{F}_{net} \cdot d\underset{\sim}{s} = \Delta K$$

Already derived it.

7.2 Kinetic Energy

In different frames of reference



$$x' = x - v_f t$$
$$y' = y$$
$$z' = z$$

Recall

$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \underset{\sim}{v} \cdot \underset{\sim}{v}$$

$$= \frac{1}{2} m \sum_i v_i^2$$

some over the component

Say there is an object moving the x direction

In frame $K = \frac{1}{2} m v_x^2$

In Frame' $K' = \frac{1}{2} m (v_x')^2$

$$= \frac{1}{2} m (v_x - v_{frame} t)^2$$

$$= \frac{1}{2} m v_x + m v_x v_{frame} t + \frac{1}{2} m v_{frame}^2 t^2$$

This is quite annoying.

If you want to do energy calculations consistently you may want to stick to one frame of reference.

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In practice I don't think there are great difficulties.

In astro realm, if you want account for more and additions to K, you just use a larger reference frame and on a large enough scale, general relativity tells ~~us~~ you how to account for kinetic energy (where in fact energy conservation in an ordinary sense fails)