2025 jun 01

6001

Ch 6 Applying Newton's Laws of Motion

Just More Cases

6.1) Solving Problems (Not an ordered list)

What we did for Atwood's machine in Chapt. 5

1) - Apply $F = ma$ to all objects needed in your system and to whatever directions needed
- Apply constraints
- exploit symmetries
- draw free body diagrams
- Solve for accelerations and/or Forces
- use accelerations to calculate Kinematics if asked

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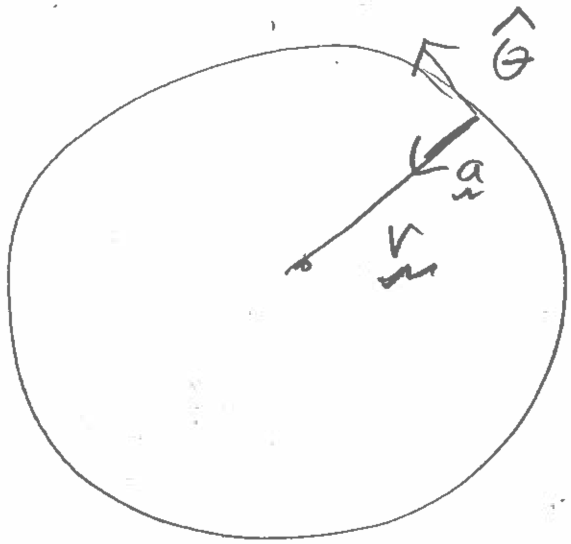
6.3) Centripetal Force

A bit of recapitulation



From kinematics if you have uniform circular Motion

$$\underset{\sim}{a} = -\omega^2 r \hat{r}$$

— $\omega = \frac{d\theta}{dt} = \dot{\theta}$ is angular velocity

— r is radius

— $\hat{r}$ is radial unit vector

— ω and r are constants

— $\hat{r}$ is always changing direction

— a is constant, but $\underset{\sim}{a}$ is always changing direction. It points to the center.

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L6003

So $\underset{\sim}{a}$ is the centripetal acceleration {center pointing}

$a = \omega r^2$ is it's ~~ma~~magnitude

But $\underset{\sim}{v} = \omega r \hat{\theta}$ {unit tangential to circle and point counterclockwise by convention}

$v = \omega r$

$\therefore \underset{\sim}{a} = \frac{v^2}{r}(-\hat{r})$, $a = \frac{v^2}{r}$

This is just from kinematics.

But Newton's 2nd law tell us an acceleration (relative to an inertial frame and any frame is an inertial frame if you set it up that way)

If there is a centripetal there is a centripetal force

$\boxed{6004}$ $\boxed{2025 \text{ jun } 01}$

$$\underbrace{\vec{F}_{centripetal}}_{\text{net force on object}} = m\,\vec{a}_{center\ of\ mass}$$

$$= m\frac{v^2}{r}(-\hat{r})$$

$$= -\frac{mv^2}{r}\hat{r}$$

$$F_c = \frac{mv^2}{r} \quad \text{magnitude}$$

The Centripetal Force is a force named for what it does not for it's intrinsic Nature

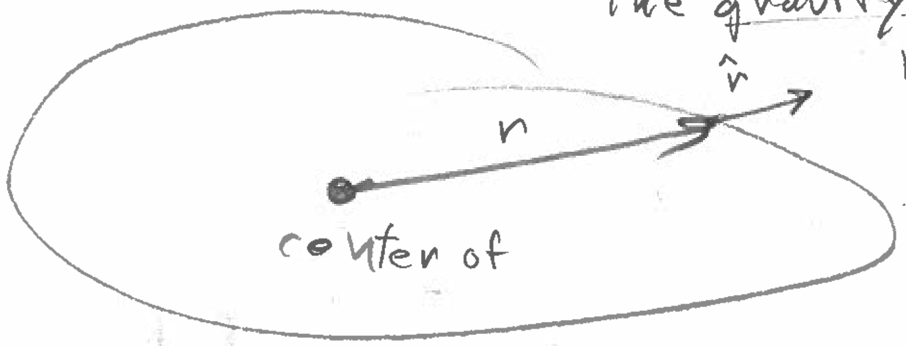
sling

Planet

By the Way any force points radially (positively or negatively) from a source of force and whose magnitude only depends on distance from the source is called a central force.

As a formula $\underset{\sim}{F}_{central} = f(r)\,\hat{r}$

The gravity of stars and planets are prime examples of central forces.

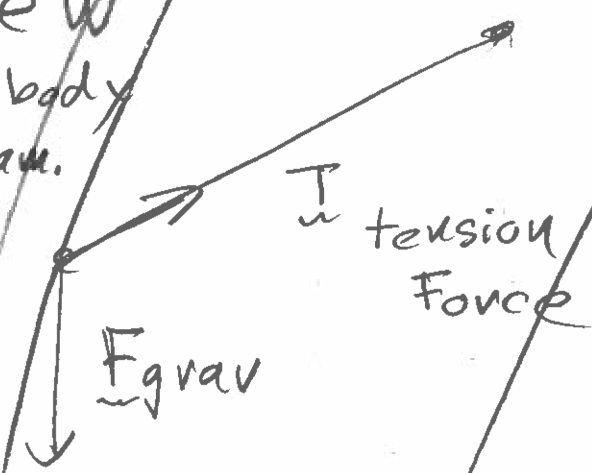


If could push or pull

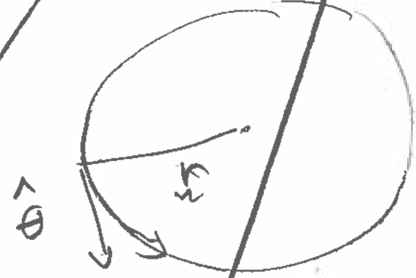
Example Sling No air drag

Cross section view

Free body diagram.



Top view



Ideal sling

- Massless
- can't resist shear or compression
- it only pulls,

and no intrinsic force Law

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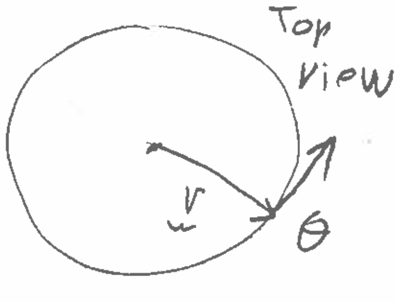
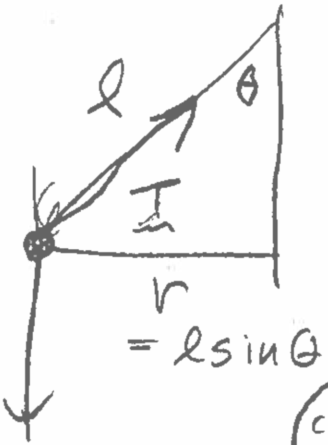
Perpetual motion. No friction or drag or dissipation in general.

Example Conical Pendulum

With Sling : Ideal point mass bob ; Ideal rope

- Massless,
- Infinitely thin
- No air drag
- can only exert tension force and only when Taut
- The tension force is constant along length





An ordinary swinger directly control parameters m, ℓ, and crudely period p. Must solve for θ, T, r, v

Newton's 2nd law always work and it always work component by component

Vertical

① $T\cos\theta = mg$

Tension component cancels gravity

Horizontal

$ma = \frac{mv^2}{r} = T\sin\theta$ ②

$v = \frac{2\pi r}{p}$

Tension component supplies centripetal force

a) Find θ, ②/①

$\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

argument is ratio of centripetal acceleration to gravity

But we still r and v

b) $T = \frac{mg}{\cos\theta}$

c) $\tan\theta = \frac{v^2}{rg} = \frac{(2\pi)^2 r^2}{rgp^2} = \frac{(2\pi)^2 \ell \sin\theta}{gp^2}$

$\frac{1}{\cos\theta} = \frac{(2\pi)^2 \ell}{gp^2}$

$\cos\theta = \frac{gp^2}{(2\pi)^2 \ell}$

$\theta = \cos^{-1}\boxed{\frac{gp^2}{(2\pi)^2\ell}}$

Example demonstration
$\ell = 0.5\,m$, $P = 1\,s$
$\cos\theta = \frac{10}{40}\frac{1}{0.5} = \frac{1}{2}$
$\theta \approx 60°$

As $P \to 0$
$\theta = 90°$,

But as $P\uparrow$, there is an upper limit when $\theta = 0$

$1 = \frac{gp^2}{(2\pi)^2 \ell}$

$P_{limit} = 2\pi\sqrt{\frac{\ell}{g}}$

d) $T = \frac{mg}{\cos\theta} = mg\frac{(2\pi)^2\ell}{gp^2}$

$\underline{T} = m(2\pi)^2 \ell / p^2$

$[m\ell/p^2] = ML/T^2$ dimensionally correct

e) $r = \ell\sin\theta = \ell\sqrt{1-\cos^2\theta}$

$\underline{r} = \ell\sqrt{1-\left(\frac{gp^2}{(2\pi)^2\ell}\right)^2}$

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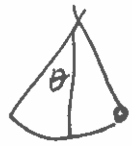
f) $\underline{v} = \frac{2\pi r}{p} = \frac{2\pi \ell}{p}\sqrt{1 - \left(\frac{g\,p^2}{(2\pi)^2 \ell}\right)^2}$

∴ Now we have θ, T, r, v all in terms of control parameter

It may seem odd that p has upper limit $p = 2\pi\sqrt{\frac{\ell}{g}}$ but note

This is the period of a simple oscillating pendulum for θ Amplitde $<< 1$ rad.



$p = \frac{2\pi r}{v}$

and as $r \to 0$ and $v \to 0$ the period has three options for it limit $p_{limit} = \begin{Bmatrix} \infty \\ \text{finite} \\ 0 \end{Bmatrix}$. It chooses the middle door

Omit in class

Why is $p_{limit} = 2\pi\sqrt{\frac{\ell}{g}} = p_{simple\ pendulum}$?

Consider when $\theta << 1$.

The centripetal force in this case is $F_c = T\sin\theta \approx T\frac{r}{\ell}$

$= (T/\ell)r$

This is a linear central force.

Linear in theta.

$\approx \frac{mg}{\ell\cos\theta} r$ for $\theta << 1$,

$\approx \frac{mg}{\ell} r$

A linear force ~~linear~~ in r

Under a linear central force, it can be shown that an object moving in a plane has solution

$\underset{\sim}{r} = (r_{max}\cos\omega t,\ r_{min}\sin\omega t)$ (cosmol. notes 3053-3055)

where $\omega = \sqrt{\frac{mg/\ell}{m}} = \sqrt{\frac{g}{\ell}}$ independent of r_{max} and r_{min}.

It's a unique frequency.

∴ Unique period $p_{unique} = f^{-1} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$.

for shape of swing for a small angle.

The shape ranges from circle to line

6008 [2024 jun 01]

6.4 Drag + Terminal Speed

(Terminal Velocity)

a) Drag Equation (Wik)

$$F_d = \frac{1}{2} \rho v^2 C A$$

density of fluid

relative flow velocity

drag coeffient dimensionless and often of order $\lesssim 1$ This can be estimated, but often just measured

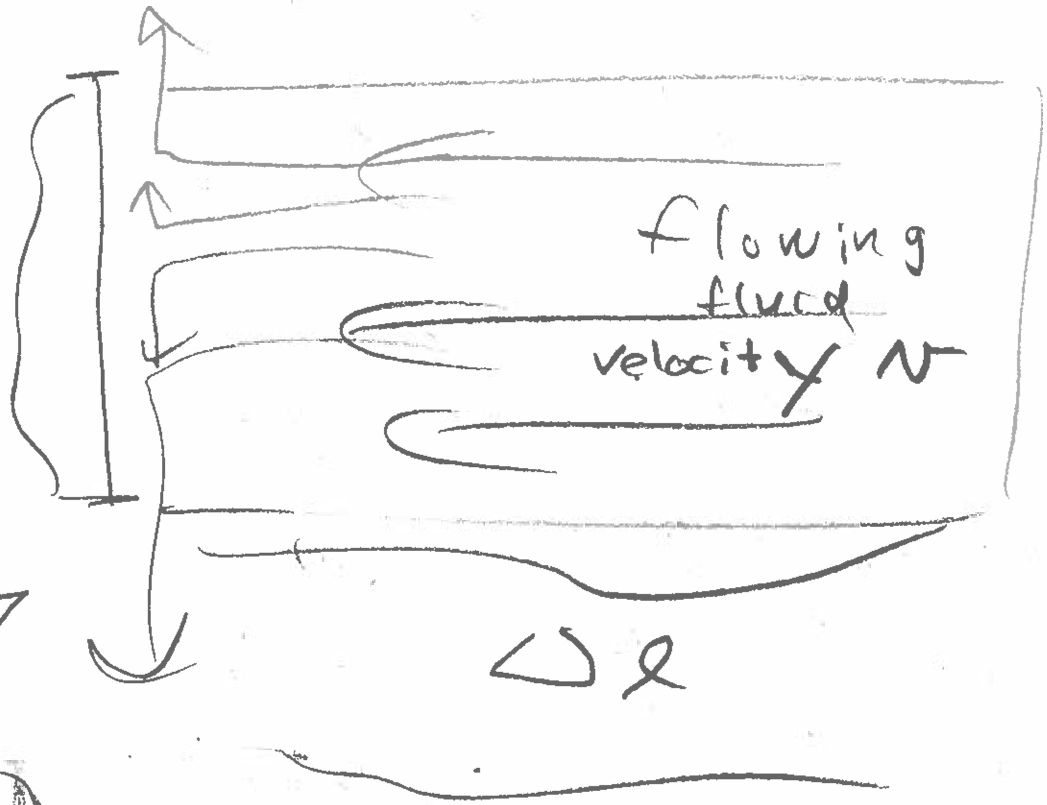
Cross section area facing fluid

Like many people I tend to use 'velocity' as a synonym for speed as well as for velocity

Derived first by Lord Rayleigh in 19th century

Not an exact law of nature but exact for idealized assumptions

Area A of a stopping surface



fluid can't penetrate an so must flow off direction of main flow

Momentum in volume $A\Delta l$ is

$$P = mv = \underbrace{\underbrace{A\Delta l\,\rho}_{\text{fluid density}}}_{\text{fluid mass}}\ \underbrace{v}_{\text{fluid}}$$

We investigate conservation of momentum later /

Fluid momentum goes to zero and to be <u>conserved</u> is transferred to stopping surface

6U1U | 2025jun01

From time zero to Δt
all momentum in volume
is tranferred to
stopping area

i.e. momentum transfer of mometum to surface which is a force

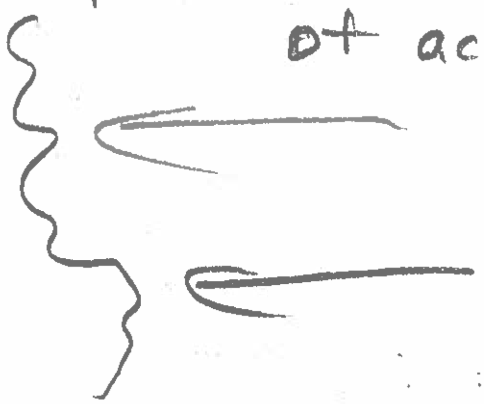
$$\left(\frac{dp}{dt}\right)_{\text{fluid to surface}} = \frac{A \Delta l \rho v}{\Delta t}$$

but $\frac{\Delta l}{\Delta t} = v$

$$= A \rho v^2$$

A drag force

But our calculation ignores
- fluid viscosity
- turbulence
- flow around edges of stopping surface
- complicated shape of actual surfaces



So a fudge factor

$\frac{1}{2}C$ is introduced

$$F_{Drag} = \frac{1}{2}\rho v^2 C A = b v^2$$

$b \equiv \frac{1}{2}\rho C A$

The $\frac{1}{2}$ is introduced so that we have

$\frac{1}{2}\rho v^2$: the kinetic energy per unit volume of fluid - relative to stopping surface

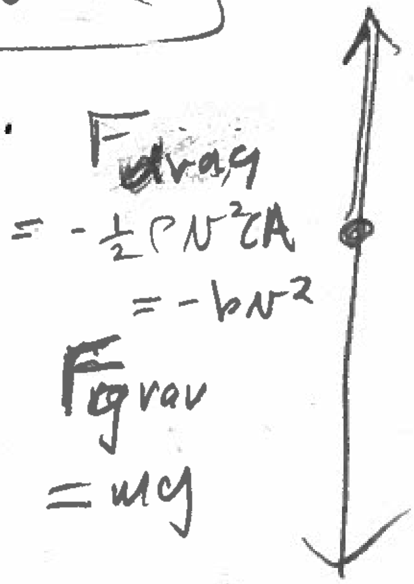
a useful quantity to know

b) Solution and Terminal of an object falling under gravity

With Drag Equation Drag

6012

2025 jan 01



We take down as positive

Apply 2nd Law

$$ma = F_{net} = mg - bv^2$$

$$a = g - \frac{b}{m}v^2$$

$$\frac{dv}{dt} = g - \frac{b}{m}v^2 = g\left[1 - \left(\frac{v}{\sqrt{\frac{mg}{b}}}\right)^2\right]$$

$v_{ter} = \sqrt{\frac{mg}{b}}$

$v < v_{ter}$, $a > 0$

$v = v_{ter}$, $a = 0$

$v > v_{ter}$, $a < 0$

A differential equation (DE)

for – 1st order DE since only a 1st order derivative occurs

— Non-linear since v is squared

<u>But</u> it can be solved exactly

$\boxed{\text{2025 jun 01}}$ 6013

$$\frac{dv}{g - \frac{b}{m} v^2} = dt$$

Let $\alpha^2 = g$

Let $z = \sqrt{\frac{b}{m}}\, v$

$dz = \sqrt{\frac{b}{m}}\, dv$

$$\frac{1}{\sqrt{\frac{b}{m}}} \frac{dz}{\alpha^2 - z^2} = dt$$

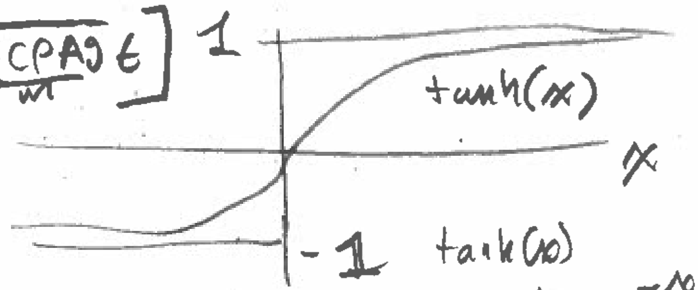
Table integral

$$\frac{1}{\sqrt{\frac{b}{m}}} \frac{1}{\alpha} \operatorname{artanh}\left(\frac{z}{\alpha}\right) = t$$

$$z = \alpha \tanh\left(\sqrt{\frac{b}{m}}\, \alpha t\right)$$

$$v = \frac{g}{\sqrt{b/m}} \tanh\left(\sqrt{\frac{b}{m}}\, \cancel{\alpha t}\right)$$

$$= \sqrt{\frac{mg}{\frac{1}{2} C \rho A}} \tanh\left[\sqrt{\frac{\frac{1}{2} C \rho A g}{m}}\, t\right]$$



$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Dimensional analysis

$$\left[\sqrt{\frac{mg}{\frac{1}{2} C \rho A}}\right] = \left[\sqrt{\frac{ML/T^2}{M/L^3 \, L^2}}\right]$$

$$= \left[\sqrt{L^2/T^2}\right]$$

$$= L/T$$

correct

$$\left[\sqrt{\frac{\frac{1}{2} C \rho A g}{m}}\right]$$

$$= \left[\sqrt{\frac{(M/L^3) L^2 \, L/T^2}{M}}\right]$$

$$= \left[\sqrt{\frac{1}{T^2}}\right] = \frac{1}{T}$$

correct.

6014

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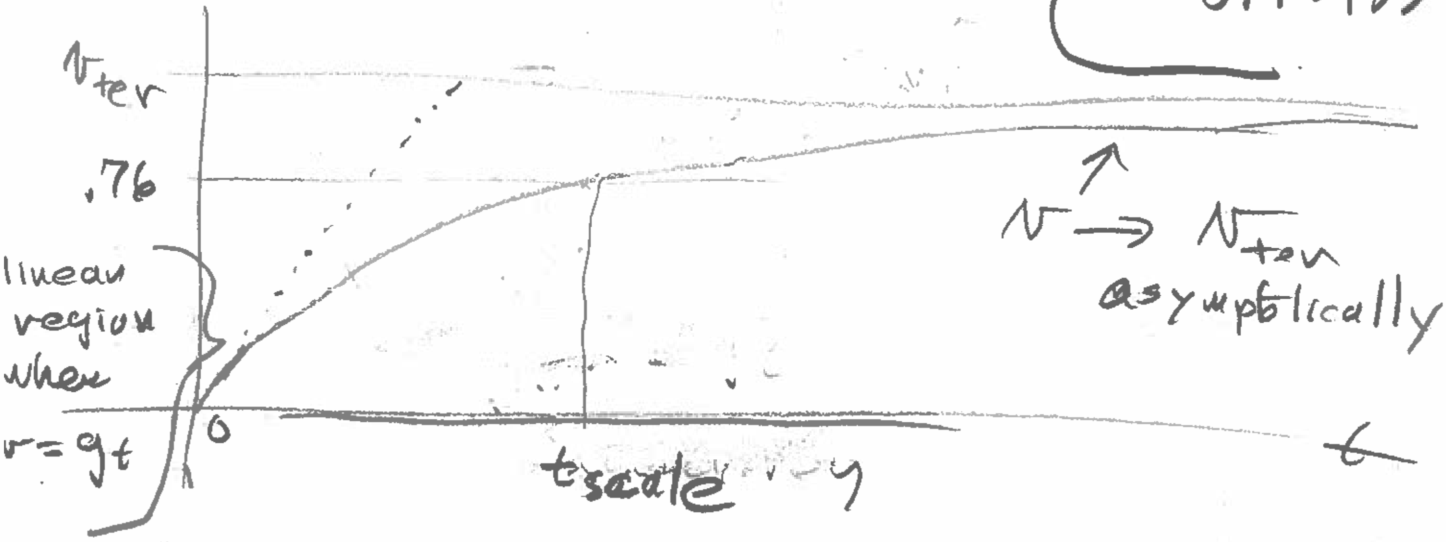
$$v_{term} = \sqrt{\frac{mg}{\frac{1}{2}C\rho A}}$$

$$t_{scale} = \sqrt{\frac{m}{\frac{1}{2}C\rho A g}}$$

$$\tanh\left(\frac{t_{scale}}{t_{scale}}\right) = \tanh(1) = 0.761594\ldots$$



Ideally v_{ter} is only reached as $t \to \infty$

but once v is very close to v_{ter} fluid fluctuations will cause it to fluctuate around v_{ter}.

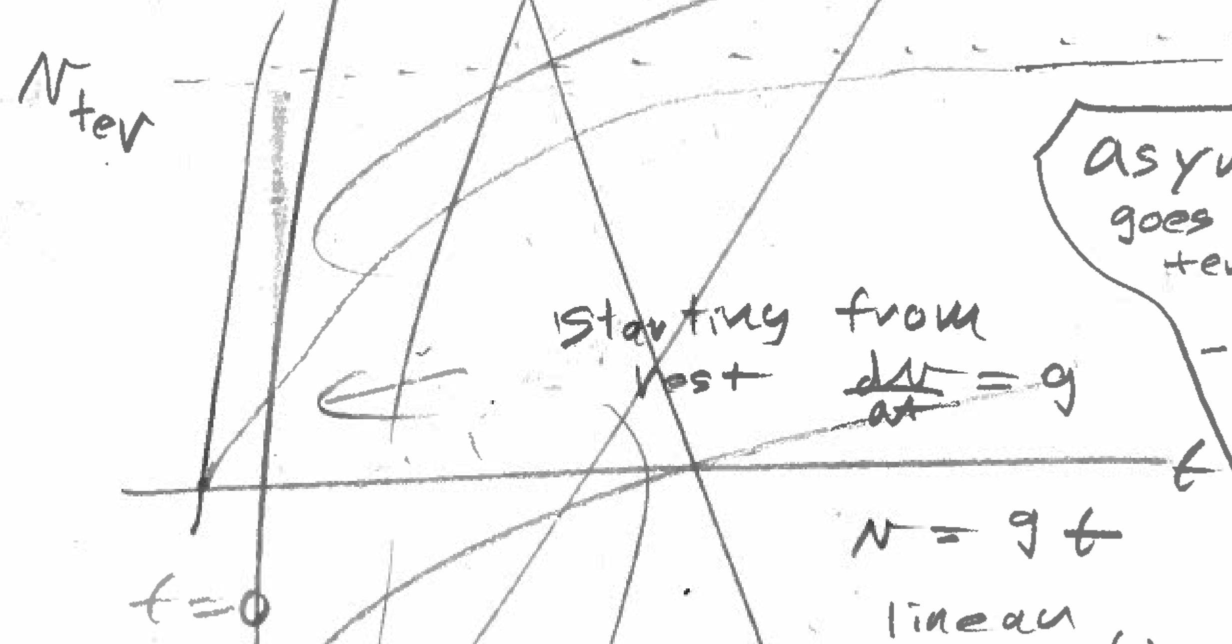
$$a = g - \frac{b}{m}v^2 = g\left[1 - \left(\frac{v}{\sqrt{\frac{mg}{b}}}\right)^2\right] \begin{cases} v < v_t, & a > 0 \\ v = v_t, & a = 0 \\ v > v_t, & a < 0 \end{cases}$$

slow down in this case

Note you can solve for v_t without solving the DE. Just set $a = 0$

This ~~innocent~~ little DE
has ~~no exact~~ solution
like so many non-linear DEs.

But ~~we can investigate~~ how
$v(t)$ evolves



Starting from rest $\frac{dv}{dt} = g$

$v = gt$

linear growth

asymptotically goes to terminal velocity - a 1st order behavior we will NOT prove here

but as $v \uparrow$ $\frac{b}{m} v^2 \uparrow$

~~and~~ eventually ~~the~~ two forces cancel

$$0 = \frac{dv}{dt} = g - \frac{b}{m} v_{ter}^2$$

$$v_{ter} = \sqrt{\frac{mg}{b}}$$

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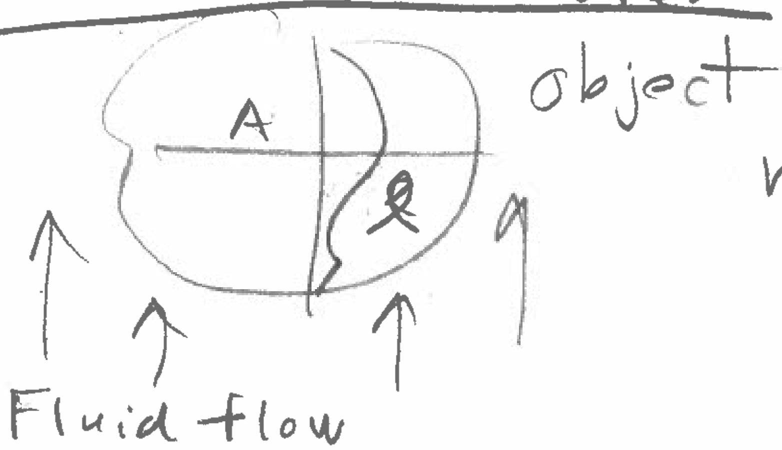
$$v_{ter} = \sqrt{\frac{mg}{\frac{1}{2}\rho C A}}$$

Let's analyze v_{ter}



$$m = \rho_{ob} V$$

$$= \rho_{ob} A \underbrace{\ell}_{\text{length scale of object}}$$

$$v_{ter} = \sqrt{\ell}\sqrt{\frac{\rho_{ob}}{\rho}}\sqrt{\frac{2g}{C}}$$

$$= \sqrt{\ell}\sqrt{\frac{1000\,(\rho_{ob}/\rho_{water})}{1.2\,(\rho/\rho_{air})}}\sqrt{\frac{2\cdot 10}{C}}$$

fiducial values

$$= \sqrt{\frac{\ell}{C}}\ \underbrace{1.3 * 10^2}_{(130\ m/s)}\ \sqrt{\frac{\rho_{ob}/\rho_{water}}{\rho/\rho_{air}}}$$

$$130\ m/s * \left(\frac{1\ km}{1000\ m}\right) * \left(\frac{3600\ s}{1\ h}\right) \cong 500\ \frac{km}{h}$$

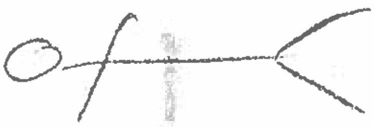
$\boxed{\text{2025 jun 01}}$ 6017

$$v_{ter} \doteq (500\ \mathrm{km/h})\sqrt{\frac{l}{C}}\ \underbrace{\sqrt{\frac{\rho_{ob}/\rho_{water}}{\rho/\rho_{air}}}}_{=1}$$

Let's consider a sky diver

$C \doteq 1$ if horizontal

$C = 0.7$ if diving



$l = 0.2\ \mathrm{m}$

$l = 2\ \mathrm{m}$

$v_{ter} = 500 \cdot 0.4 = 200\ \mathrm{km/h}$

$v_{ter} = 500 \cdot 1.7 = 800\ \mathrm{km/h}$

Wik says $\sim 200\ \mathrm{km/h}$ and so not bad

But Google AI give $300\ \frac{\mathrm{mi}}{\mathrm{h}} \left(\frac{1.6\ \mathrm{km}}{1\ \mathrm{mi}}\right) = 500\ \mathrm{km/s}$ as highish limit

Ling 292 says $350\ \mathrm{km/h}$ - so bad over estimate

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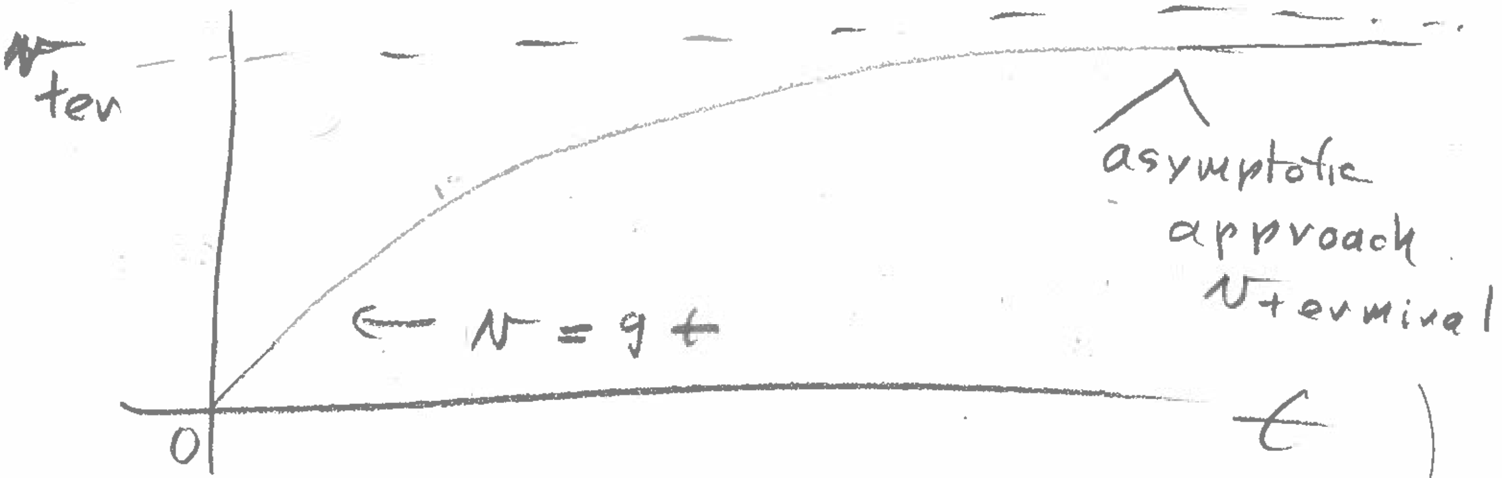
Remember my curve



To recapitulate

The DE formally dictates that terminal velocity is only reached at $t = \infty$

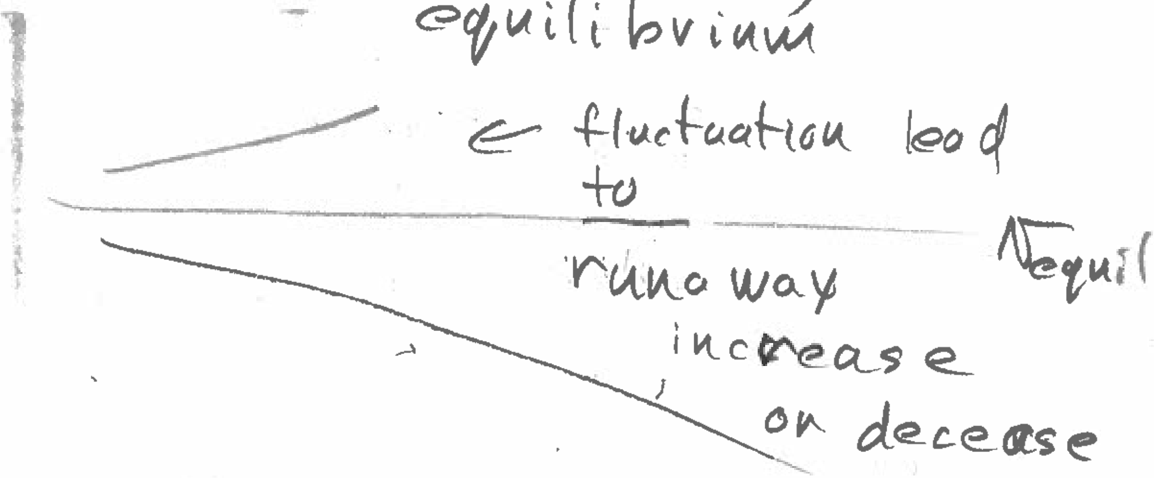
But in fact eventually fluid perturbations will cause $v(t)$ to fluctuated about $v_{terminal}$ but always being driven back to it

$$\frac{dv}{dt} = mg - \frac{b}{m}v^2 \begin{cases} > 0 & \text{if } v < v_{ter} \\ < 0 & \text{if } v > v_{ter} \end{cases}$$

So $v_{terminal}$ is a stable velocity equilibrium

fluctuations damp out

An unsbable velocity equilibrium



fluctuation lead to runaway increase or decease

The drag equation is valid (insofar as it is) for longish objects at high relative fluid velocity v and low viscosity:

ie., high Reynolds Number $= Re = \dfrac{v\,\ell}{\nu} \gg 1$

ℓ: length scale of object

ν: kinematic viscosity

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The Other extreme case is low Reynolds numbe

$$= \frac{v l}{\nu} << 1$$

In ~~this~~ case there is Stokes Drag

b) <u>Stokes Drag</u>

$$F_{stokes} = 6\pi r \eta v$$

flow velocity

radius of a sphere

dynamical viscosity

$$= 6\pi r \nu \rho v$$

$$F_{stokes} = bv$$

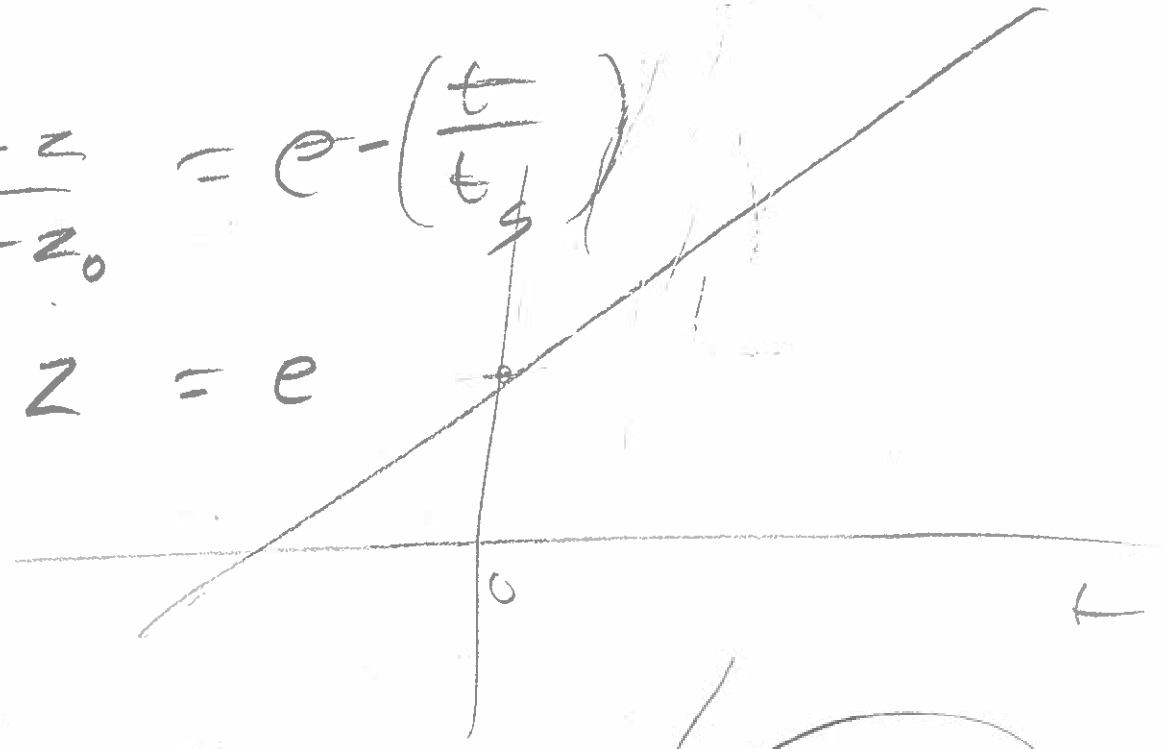
Not bv^2

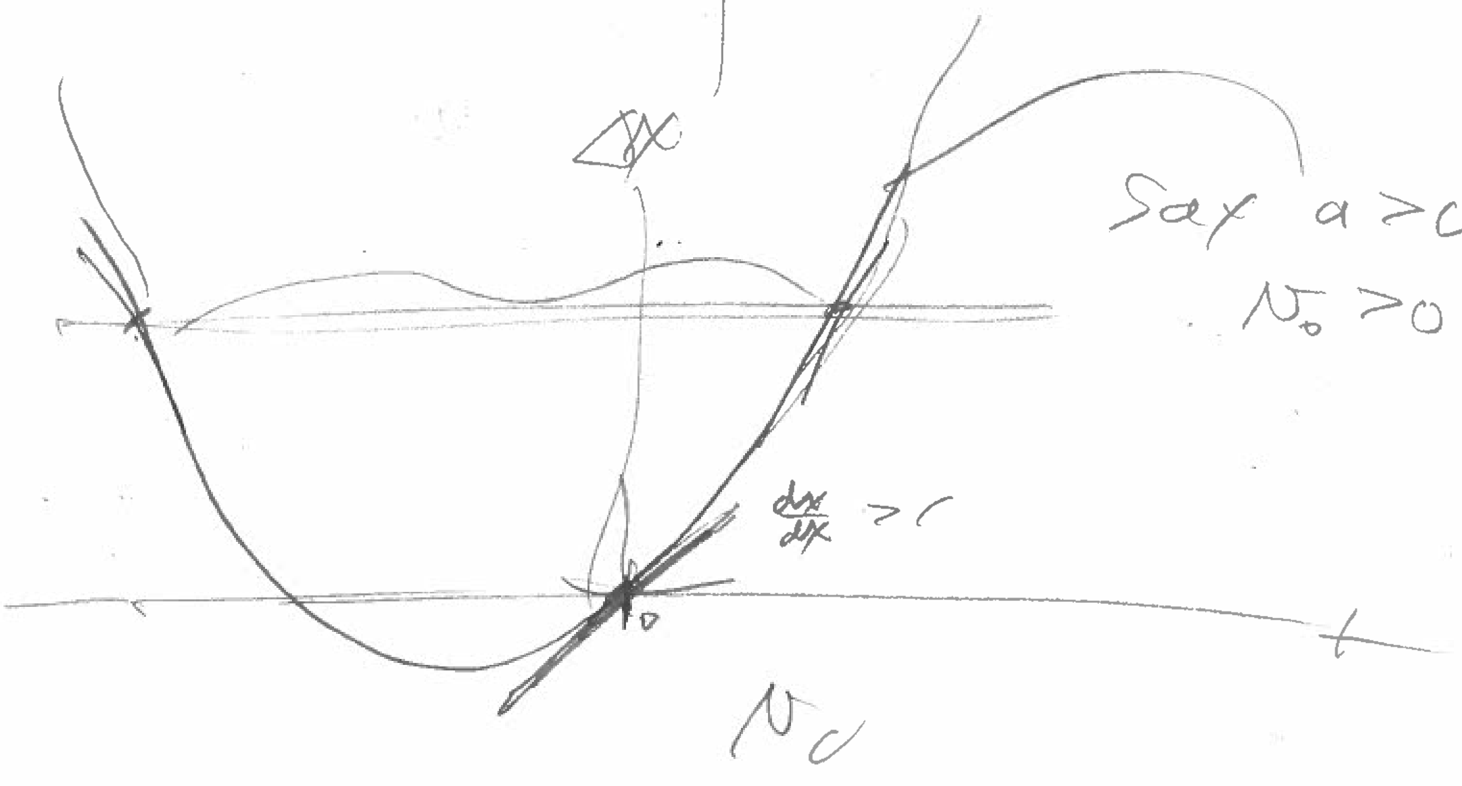
$\therefore$ $ma = F_{net} = mg - bv$, for an object falling in this case.

3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	UGRD
1	3 GRD	UGRD
3	3 GRD	UGRD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	UGRD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
3	3 GRD	GRAD
1	6 GRD	GRAD
1	1 GRD	GRAD

$$\frac{1-z}{1-z_0} = e^{-\left(\frac{t}{t_s}\right)}$$

$$1 - Z = e$$





Say $a > 0$
$N_0 > 0$

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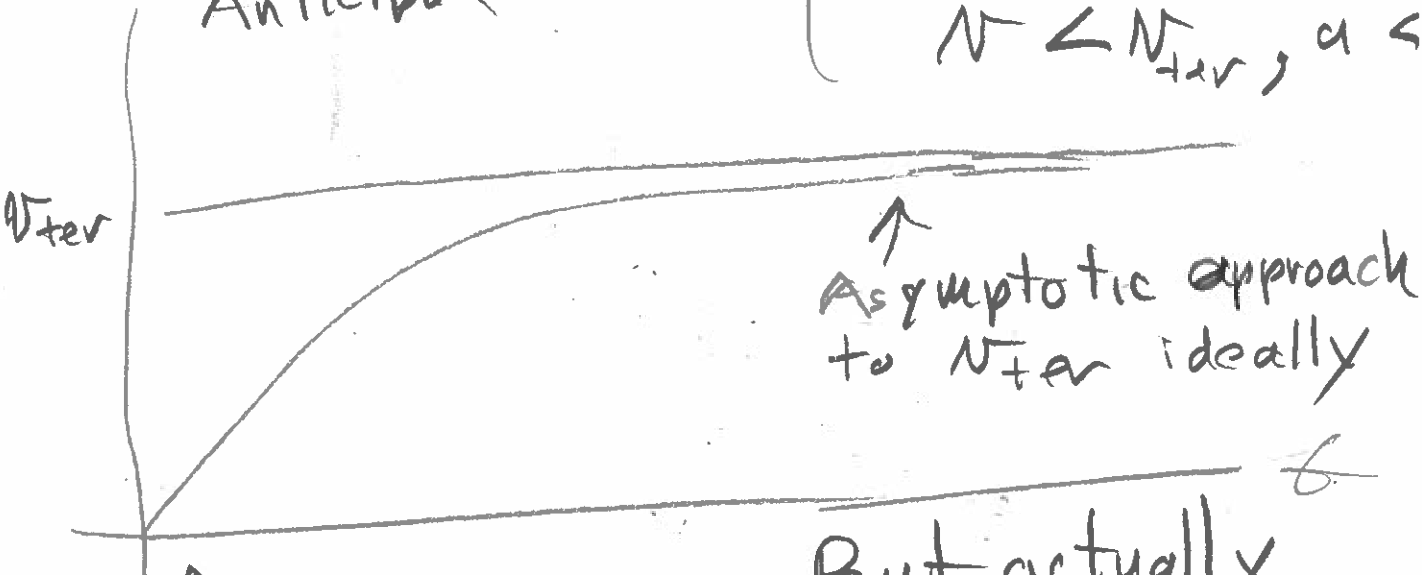
$$\frac{dv}{dt} = g - \frac{b}{m} v = g\left[1 - \left(\frac{v}{\frac{mg}{b}}\right)\right]$$

is our I order differential equation

$$v_{ter} = \frac{mg}{b} \quad \begin{cases} v > v_{ter}, & a > 0 \\ v = v_{ter}, & a = 0 \\ v < v_{ter}, & a < 0 \end{cases}$$

Anticipate



But actually again, medium perturbations cause v to fluctuate around v_{ter} which is a stable velocity equilibrium.

6022 [2025 jun 01]

An easy differential (DE) to solve:

1^{st} order and linear

$$\frac{dv}{g\left[1-\left(\frac{v}{v_{ter}}\right)\right]} = dt$$

Let $z = \frac{v}{v_{ter}}$

$dz = \frac{dv}{v_{te}}$

$$\frac{v_{ter}}{g}\,\frac{dz}{1-z} = dt$$

$$\frac{v_{ter}}{g}\left[-\ln(1-z)\right]\Big|_{z_0=0}^{z} = t$$

$$\ln(1-z) = -\frac{t}{t_{scale}}$$

$$1 - z = e^{-t/t_{scale}}$$

$$z = 1 - e^{-t/t_{scale}}$$

$$v = v_{ter}(1 - e^{-t/t_{scale}})$$

$t_{scale} = \frac{v_{ter}}{g}$

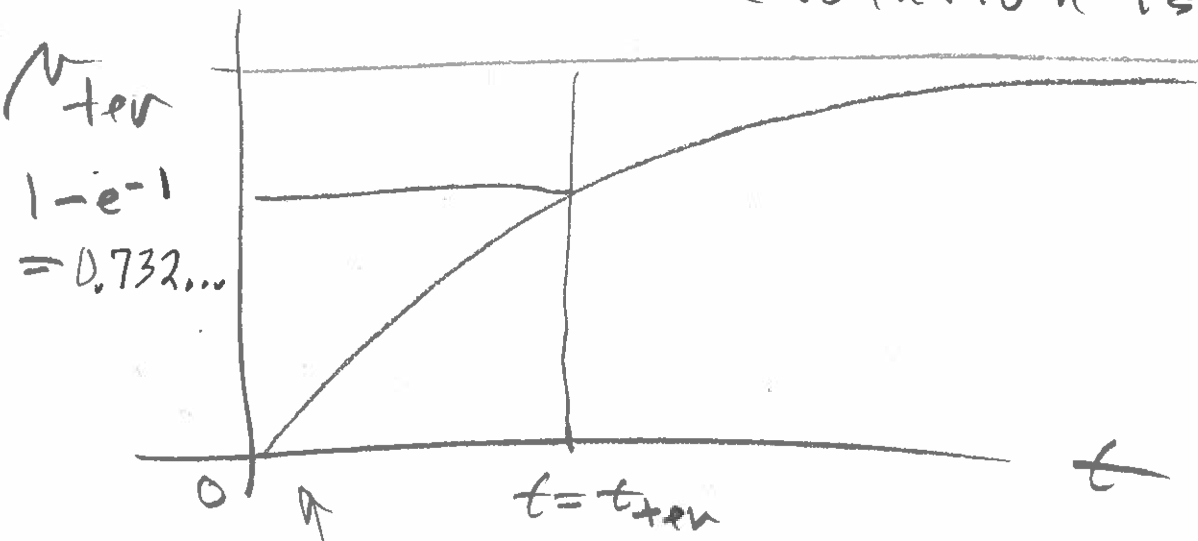
$[t_{scale}]$

$= \frac{L/T}{L/T^2}$

$= T$

dimensional analysis

As anticipated the velocity evolution is



linear region

$$v = v_{ter}\frac{t}{t_{scale}}$$

The simple theory Amontons Laws (Wik)

Guillaume Amontons 1663 – 1705 discovered effectively

6.2) Friction

For interface of two dry surfaces which are smooth at the macroscopic scale.

It is a force of resistance parallel to the surface

The Normal Force recall is perpendicular (i.e., normal) to surfaces.

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Static Friction between surface at rest with respect to each other

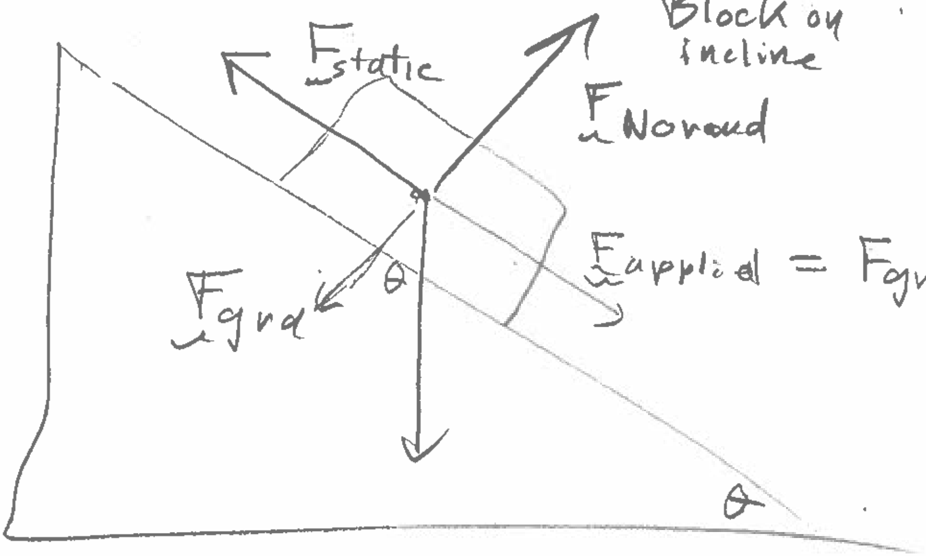
$$F_{static} = -F_{applied}$$ force parallel to surface

but and $F_{static} + F_{applied} = 0$ and there is no acceleration from rest,

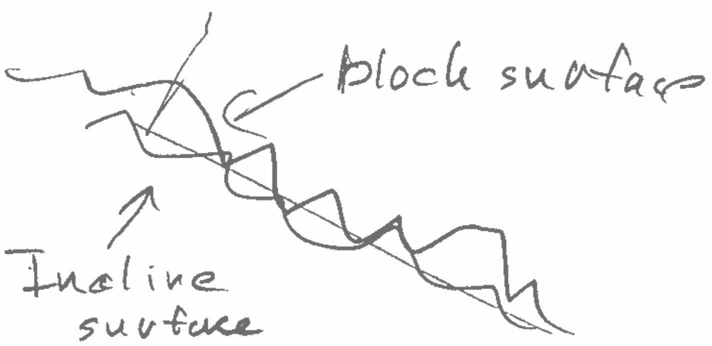
$$F_{static} = \mathrm{Min}[F_{applied}, \mu_s F_N]$$

μ_s dimensionless coefficient of static friction

example to clarify



$F_{applied} = F_{grav} \sin\theta$ component gravity downhill.



Gravity is long range an pulls atom by atom but internal forces communicate the gravity to actual surface to oppose friction.

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— microscopically friction and applied force are partially just microscopic normal forces

and partially chemical bonding which can be very strong sometimes

[cold welding happens between very clean (Not oxidized) metal surfaces of same metal are brought in contact.
The atoms have no way to know they belong to different pieces of metal]

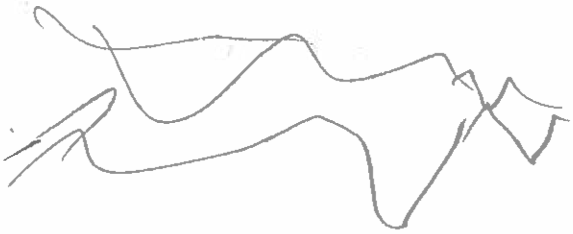
But if $F_{applied} \geq \mu_s F_{Normal}$

the Friction forces break and the block slides

6026 | 2025jun01

Why is F_{Normal} the determining factor and not contact area?

Microscopically contact area is key



← Microscopic contact area is much smaller than macroscopic contact area

It turns out

$F_{friction} \underset{\sim}{\propto} A_{contact\ Microsopic}$

$\underset{\sim}{\propto} F_{Normal}$ which is pressing the surface together.

Kinetic Friction

Magnitude $F_{friction} = \mu_K F_{Normal}$

Vector $\vec{F}_{friction}$ points opposite direction of motion.

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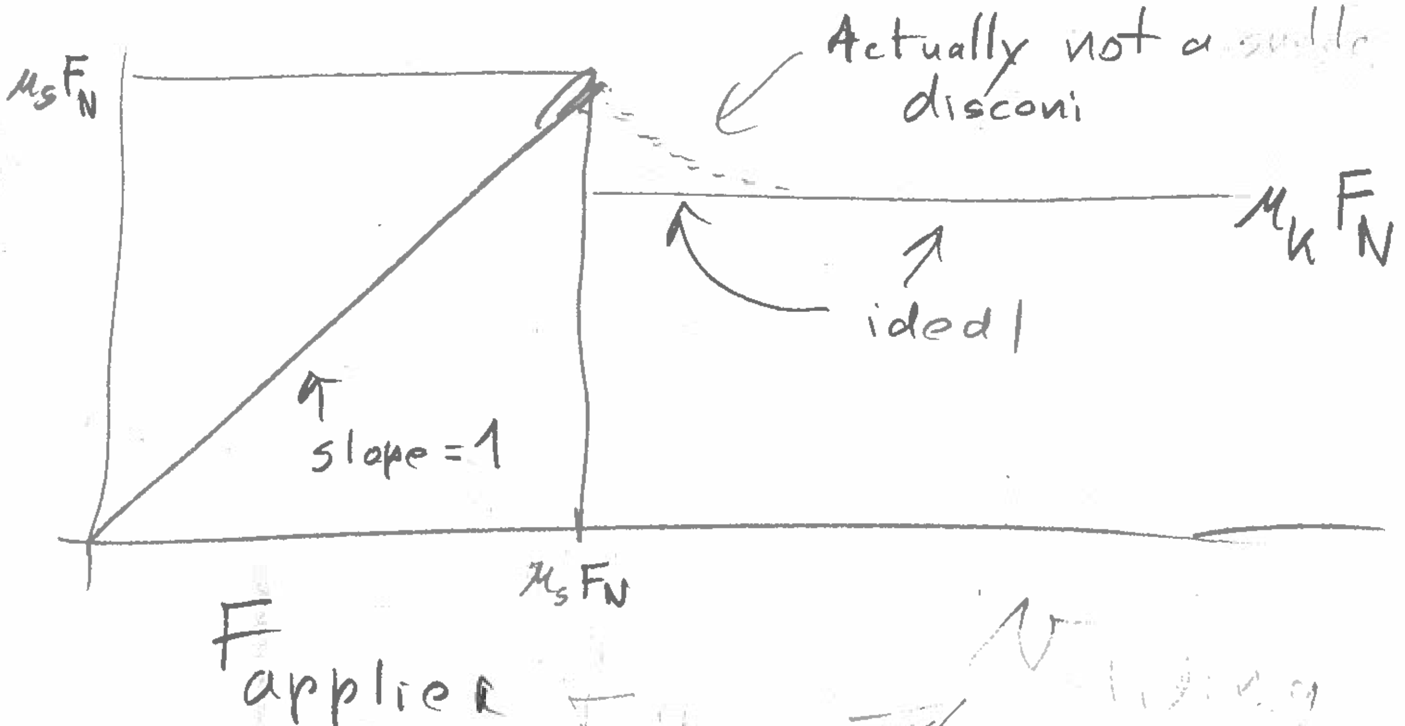
Plot of Friction



$\mu_k < \mu_s$ usually

Reason: once sliding starts some chemical bonding can't occur. The bonding decreases for a while as $F_{applied}$ increases, but reaches a floor

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Of course

μ_s and μ_k

are dependent on

both materials

at the interface

and there are approximate

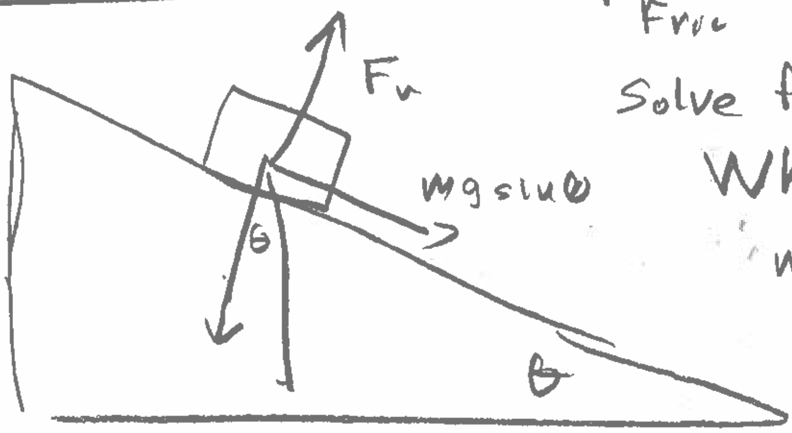
though they work

very well very often

Exactly true maybe in

very extreme limit?

Example



$F_{Fric} = mg\sin\theta \le \mu_s mg\cos\theta$

Solve for sliding angle?

When an equality

$mg\sin\theta = \mu_s mg\cos\theta$

$\tan\theta = \mu_s$

$\theta_{slide} = \tan^{-1}(\mu_s)$

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For example for Wood on Wood

$\mu_s = 0.5$

$\theta_{slide} = \tan^{-1}(\mu_s) = \tan^{-1}(0.5)$

$\cong 0.5$ radians applying

$= 0.5 \text{ rad} \left(\frac{60^\circ}{1 \text{ rad}}\right)$

$= 30^\circ$

small angle approximation

$\tan\theta \cong \theta$

Above θ_{slide}

$ma = mg\sin\theta - \mu_k mg\cos\theta$

taking downhill as positive

$$a = \begin{cases} g(\sin\theta - \mu_k\cos\theta) \\ g\cos\theta_{slide}(\mu_s - \mu_k) \\ g \quad \text{for } \theta = 90^\circ \end{cases}$$

An ideal knife edge case $\theta = \theta_{slide}$

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Appendix A

Recapitulation ~~of~~
the generalized
Newton's 2nd law
and the Rocket problem

III) A generalization of Newton's 2nd Law - is needed for mass changing objects

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$$\underset{\text{Net external force}}{\underbrace{\underset{\sim}{F}_{net}}} + \underset{\substack{\text{Rate of mass inflow to system}\\ \frac{dm}{dt}<0 \text{ for outflow}}}{\underbrace{\frac{dm}{dt}}} \underset{\text{velocity relative to outside inertial frame}}{\underbrace{\underset{\sim}{v}_{inflow}}} = \underset{\text{Rate of change of momentum of object.}}{\underbrace{\frac{dp}{dt} = m\frac{d\underset{\sim}{v}}{dt} + \underset{\sim}{v}\frac{dm}{dt} = m\underset{\sim}{a} + \underset{\sim}{v}\frac{dm}{dt}}}$$

Example Rocket

The Rocket ejects burnt f at speed v_{ej} opposite direction of motion.

Assume motion along x-axis and $\underset{\sim}{F}_{net} = 0$.

$\frac{dm}{dt} < 0$ for mass loss

$$v_{inflow} = \underset{\text{velocity of rocket}}{\underbrace{v}} - v_{ej}$$

The outflow velocity is reduced from v by v_{ej}.

$$\therefore \quad \frac{dm}{dt}(v - v_{ej}) = ma + \underset{\sim}{v}\frac{dm}{dt}$$

$$ma = -v_{eg}\frac{dm}{dt}$$

$$m\frac{dv}{dt} = -v_{ej}\frac{dm}{dt}$$

$$m\,dv = -v_{ej}\,dm$$

$$dv = -v_{ej}\frac{dm}{m}$$

$$v - v_0 = -v_{ej}\ln\left(\frac{m}{m_0}\right)$$

$$v - v_0 = v_{ej}\ln\left(\frac{m_0}{m}\right) = v_{ej}\ln\left[\frac{m}{m_0}\right.$$

$m_0 > m$ because of mass loss $\therefore v > v_0$

You have to specify $m(t)$ to find $v(t)$

Tsiolkovsky Rocket Equation (Wiki)