2025 jun 01 | ~~5~~001

5) Newton's Laws of Motion

5,1, 5,2, 5,3, ~~5~~,5, Conflated and added to

a) As I mentioned in the Chapt. 1 lecture, I believe you must accept elements of Newtonian Physics as a package

force, mass, acceleration, center of mass, inertial frame, inertial forces, time.

They exist in relation to each other.

They can't be defined separately

The elements can't be define in isolation from each other.

A perspective Many people (but some may disagree)

5002) [2025 Jun 01]

But you do have to talk about the elements in some order

b) Inertial frame

All physical laws (including beyond classical physics) are referenced to inertial frames, except general relativity which tells us what are inertial frames

Some may disagree, but I think the disagreement is a matter of perspective

Elementary Inertial frames are free-fall frames in a uniform gravitational field, Not rotating with respect to the bulk mass of observable universe (which often goes without saying)

(2025 JUN 02) [500 3

A local frame NOT accelerating with respect to an inertial frame is also an inertial frame

Now at first thought, You might say this a pretty useless concept since we use Newtonian physics all the time wherever we are on Earth and we are NOT in free-fall

But the center-of-mass of the Earth is under the gravitational field of the whole universe as far as we can tell, but local ~~planetary~~ motions are due local gravitation of Sun + Moon + Planets

~~5004~~

2025 Jun 09

Show Image 1

The observer in an orbiting spacecraft is a uniform gravitational field

so they are in an elementary inertial frame

So is the observer in the falling elevator

For the Earth, neglecting air drag

the Center of Mass (CM) is in Free-fall

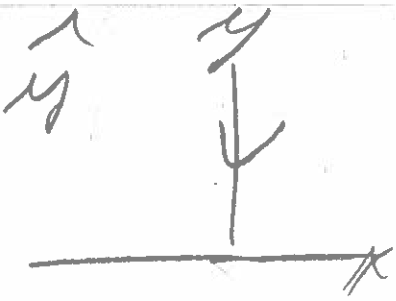
— imagine the Earth was NOT rotating (relative to observable universe)

and the external grav due to Sun & Moon was uniform over the Earth (which is NOT true)

Then any point on the surface of the Earth would be unaccelerated relative CM of Earth →

and would be
an elementary inertial
frame
with Earth gravity
just being a force
in that frame

$$\underbrace{\text{Near Earth's surface}}\; \underset{\sim}{F} = -mg\,\hat{y}$$



Now the
Earth is rotating
(relative to the
observable universe)
and Sun & Moon grav. fields
are NOT uniform over
Earth
But on the size scale of
humans & most of artifacts
those are small perturbations
that can be neglected.
and we can usually treat
any point on surface of Earth
as defining an inertial frame

5006

(2025 jan 01)

On larger scales you need to invoke inertial forces & tidal forces to convert non inertial frames into inertial frames

- force due to structure of spacetime in general relativity understanding

So, in fact, there is an inertial ~~frame~~ wherever you want.

— easy in principle to define that inertial frame but sometimes hard in practice

Images 64 Tidal force

— differential force of graviting on a large body

But it's actually partially an inertial force too

— tends ~~to~~ stretch free-falling bodies

— an being orbit is being in free fall. They give rise to Earth tides

2025 jun 01

5007

Images 5

Centrifugal force (center fleeing force)

(which is NOT the same as centripetal force — though they are closely related)

An inertial force

gives equatorial bulges to Earth + other rotating astro-bodies

We don't notice the centrifugal force much on the small scale, but bulk Earth notices it

It is the cause of effective gravity of earth to vary

from 9.832 m/s² Poles

9.780 m/s² at equator

0.5% variation just below human perception

5008 | 2025 jun 01

Images 6 & 7,8 Coriolis Force

an inertial force
trickier than
centrifugal
- cause by motion
in a rotating
frame
— cause anticyclone,
cyclone
weather
— observable on small scale
by foucault's pendulum.

Whole hierarchy of inertial frames
Images 9, 10
and it tops out with
expanding universe
Why did Celestial Mechanics
from Newton to Einstein
always get right answers without
our modern understanding of inertial frames.
They just treated free-fall frames as non-inertial
frames converted to inertial frames by inertial
forces rather than as elementary inertial frames and
always got the right answers.

2025 jun 01

5009

c) Forces cause

Accelerations relative ~~to~~ inertial frame, if unbalanced

& Also maintain structures like holding up this building when in balance.

They can be inertial forces arise from nature of spacetime

Or 'ordinary' forces cause by fields of force → sometimes obviously between bodies & sometimes Not

— Contact forces are those that at macroscopic scale happen just between matter in contact

5010

2025 jun 01

gravity is a special case
because classically we treat it as a long range force
but general relativity tells its actually an inertial force
because gravity gives the structure of spacetime

d) Accelerations
rate of change of velocity

Since You can define an inertial frame wherever you want, you can say all accelerations are caused by forces ~~ordinary~~ ~~or~~ inertial.
But if you choose to regard a frame as non-ertial there are

Accelerations Not caused by forces

But you need to define to understand both velocity and acceleration.

e) Newton's 2nd Law
For a point mass

An ideal classical point particle

they don't exist, but they are the ideal limit of actual mackosopic bodies

$$\underbrace{\vec{F}_{net}}_{\text{net force on ~~the~~ body}} = \underbrace{m}_{\text{mass of ~~the~~ body}} \underbrace{\vec{a}}_{\text{acceleration}}$$

net force on ~~the~~ body

Forces we regard as inertial frame invariant

mass of ~~the~~ body. It's resistance to acceleration

acceleration

All local inertial frames Not accelerating with respect to each give the same acceleration

5012

$$\underset{\sim}{r}' = \underset{\sim}{r} + \underbrace{\underset{\sim}{v}_{\text{frame}}}_{\text{constant}} t$$

$$\therefore \underset{\sim}{v}' = \underset{\sim}{v} + \underset{\sim}{v}_{\text{frame}}$$

$$\underset{\sim}{a}' = \underset{\sim}{a}$$

<u>Can</u> local frames be accelerated with respect to each other?

Yes, but they differ in their inertial forces

But either way

$$\underset{\sim}{F}_{\text{net}} = m \underset{\sim}{a}$$

is inertial frame invariant

it's Valid with respect to all inertial frames whether invoking inertial forces or NOT

We will clarify ~~p~~point soon.

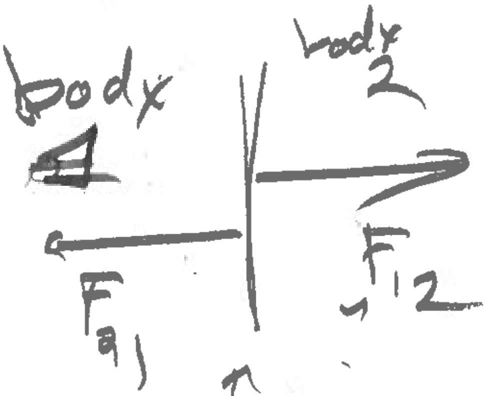
L501B

f) Newton's 3rd law

~~For~~ every force there is an equal & opposite force

ordinary forces, not inertial forces except gravity (considered as an inertial force)

Key point they don't have to be on the same body & so they don't have to cancel for acceleration purposes.



contact force

Otherwise inertial forces do NOT obey the 3rd Law But these lead special problems that I can see.

Long range force like gravity or Coulomb's law

2025 JUN 01

<u>Fine points</u>

(1) Newton's 3^{rd} is classical law — it assumes the ~~speed of~~ signaling of a force is instantaneous.

<u>But</u> the classical limit that is true.

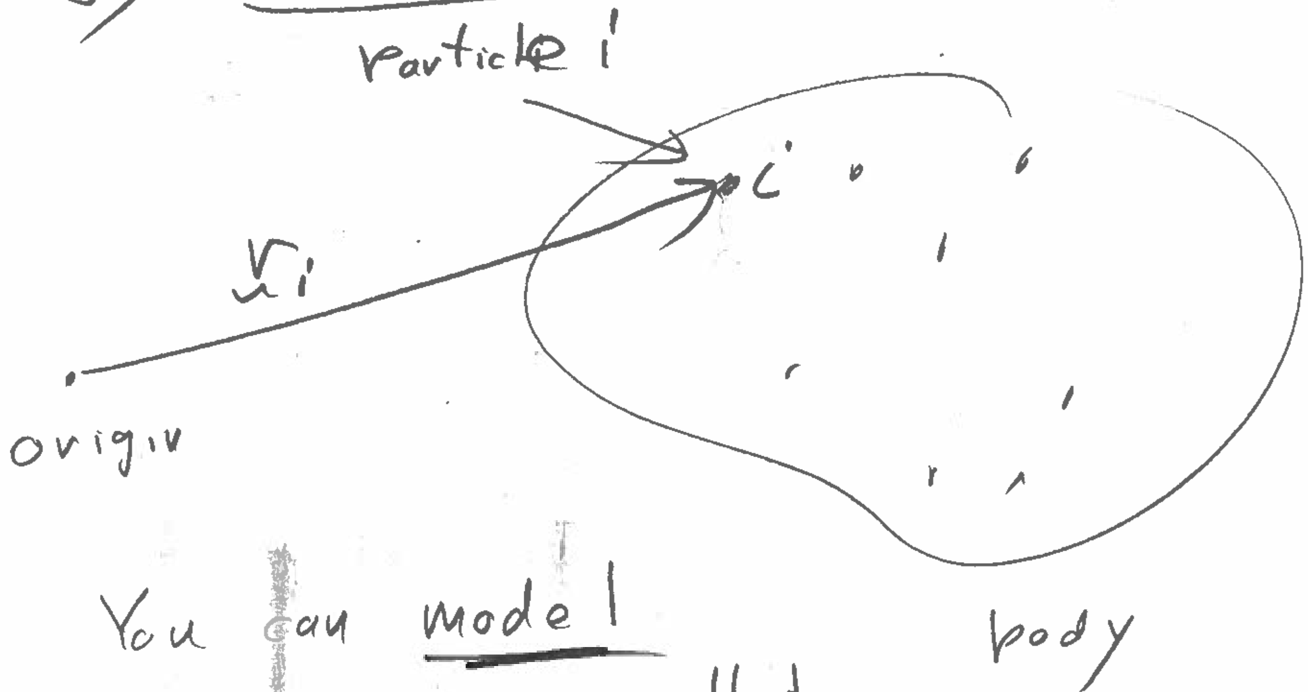
(2) Even classically Newton's 3^{rd} law is NOT strictly true <u>as stated</u>

Magnetic forces can give rise to tricky exceptions

But there is a generalized version that is true

~~But~~ we aren't going to worry about that at our level.

9) Center of Mass



You can model a body as a collection of classical point masses

↳ the ideal limit of actual small bits above the quantum mechanical level

mass of partical i is m_i

$m = \sum_i m_i$ total mass of body

The center of mass position is defined by

$$m\underline{r} = \sum_i m_i \underline{r}_i$$

501~~6~~

2025 jun 04

ov

$$\underline{r} = \frac{\sum_i m_i \underline{r}_i}{M}$$

Mass weighted average position

If you go to the continuum limit

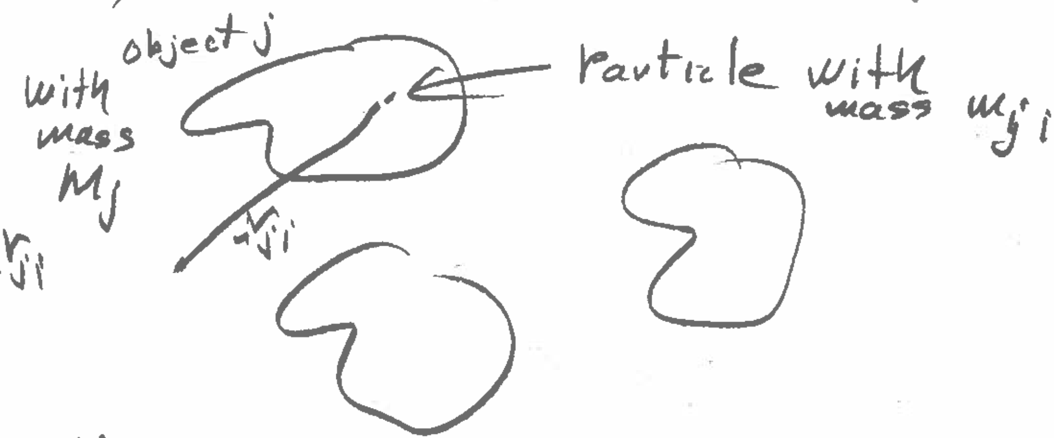
$$\underline{r} = \frac{1}{M} \sum_i \underline{r}_i \, \rho_i \Delta V_i$$

ρ_i average of small volume ΔV_i

If you have a nice regular shape, the integral can even be done

$$\underline{r} = \frac{1}{M} \int \underline{r}' \rho(\underline{r}') \, dV$$

What if you have several objects



$$M_j \underline{r}_j = \sum_i m_{ji} \underline{r}_{ji}$$

$$M \underline{r} = \sum_j M_j \underline{r}_j = \sum_{ji} m_{ji} \underline{r}_{ji}$$

So the center of mass of a set of objects equal the center of mass of set of total point particles

2025 jun 01

5017

Images ~~18, 10,~~ 115T3

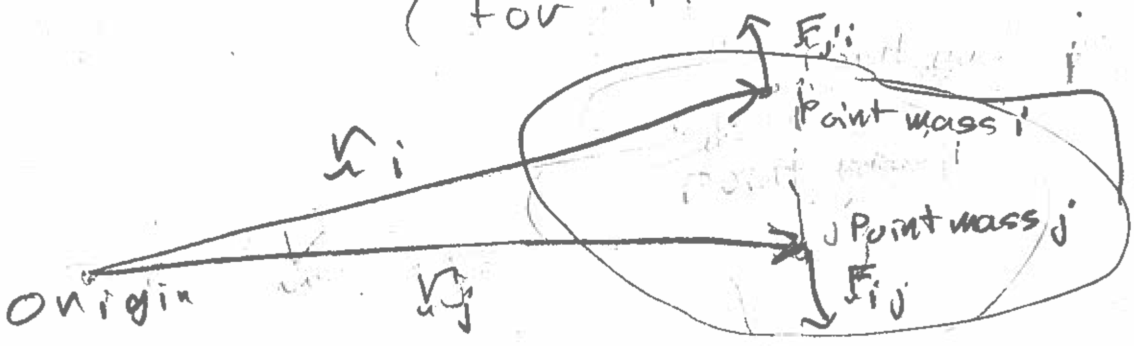
Centers of Mass of symmetric object is easy to find.

— Just geometric center by symmetry

Hanging object at rest
center of mass must below pivot point
— stable equilibrium
perturbation damp out

Balancing equilibrium, ~~center~~ mass above pivot point
— unstable equilibrium

h) Newton's 2nd Law (for finite objects)



$F_{i\,net} = m_i a_i$

Sum over all i

$F_{i\,net} = F_{i\,ext} + \sum_j F_{ji}$

Force of j on i
Sum of all of these

5018

$$\underset{\sim}{F}_{net} = \sum_i F_{net\,i} = \sum_i m_i \underset{\sim}{a}_i$$

on on body

$$= \sum_i F_{i\,ext} + \underbrace{\sum_{\substack{i\,j \\ i \neq j}} \underset{\sim}{F}_{ji}}_{\text{By Newton's 3rd law these forces cancel pairwise}} = \sum_i m_i \underset{\sim}{a}_i$$

$$\underset{\sim}{F}_{net} = \sum_i m_i \underset{\sim}{a}_i$$

$$\underbrace{\underset{\sim}{F}_{net} = M \underset{\sim}{a}_{cm}}$$

This is the way Newton's Law is usually stated

So the net Force on body (which is necessarily the ~~net~~ external force

To be more mathematical

$$\sum_{\substack{i\,j \\ i \neq j}} \underset{\sim}{F}_{ji} = \sum_{\substack{j\,i \\ j \neq i}} \underset{\sim}{F}_{ij}$$

just interchanging labels

$$= \sum_{ji} (-F_{ji})$$

by 3rd law

$$= -\sum_{ji} F_{ji}$$

These two are the same, but ~~this~~ can only be if they are zero.

5019

Key points

i) $\underset{\sim}{F}_{net} = 0$

~~then~~ $\underset{\sim}{a}_{cm} = 0$

and ~~the~~ center of mass moves in straight line

uniform motion

This is Newton's 1^{st} law

One perspective is that, it is a special case of the 2^{nd} law

and so there really are only Two laws of Motion.

I believe this the good perspective

since you really need some way of measuring time to say there is uniform motion and that requires 2^{nd} law and mass to verify means of counting time flow.

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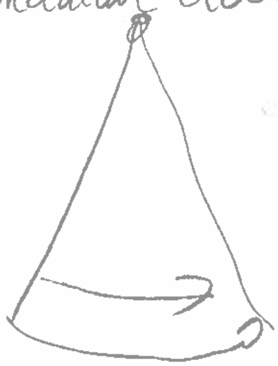
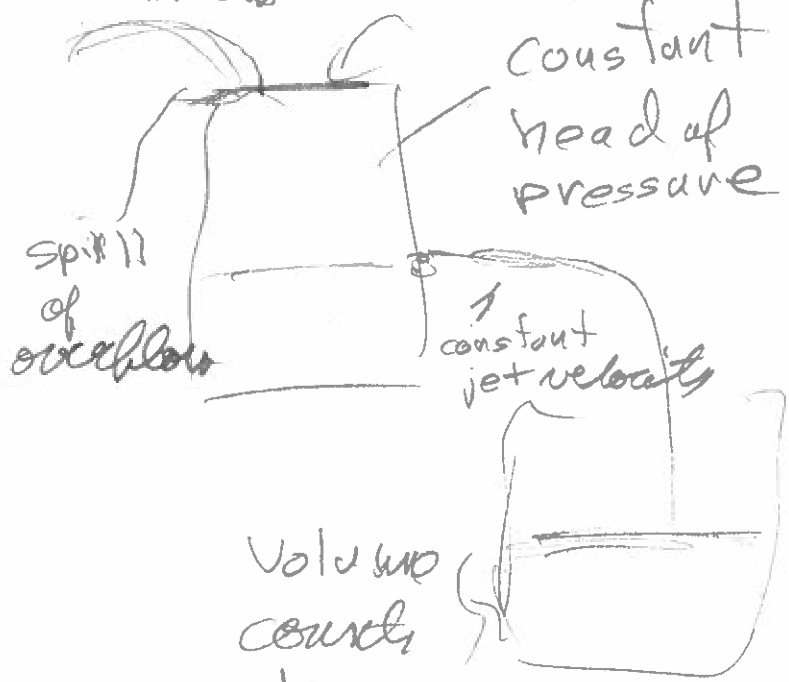
e.g.

pendulum clock



- oscillator like a pendulum
- we'll analyze later, but the periods of oscillation are ideally constant

Ideal water clock



But you need Newtonian physics to tell you jet has constant flow speed.

But there is a perspective that the 1st law is a necessary independent law

Google AI answers Both ways depending on how

a) is Newton's 1st law a special case of 2nd law? Yes

b) is Newton's 1st law redundant? No.

ii) $F_{net} = m\, a_{cm}$

Just tells you about the CM motion

Internal motions have to be analyzed on their own

— Also using Newtonian physics applied to parts,

Image 15 Fosbury flop

iii) ~~Changing Mass Systems~~ (OMIT in lecture)

(e.g., Rockets) need a generalization of Newton's 2nd Law

Wiki: 2nd law sort of confuses

momentum flow inertial frame

$$\underset{\sim}{F}_{net} + \left(-\frac{dM}{dt}\right)(\underset{\sim}{v}_{eject} - \underset{\sim}{v}) = \frac{d\underset{\sim}{p}}{dt}$$

$\underset{\sim}{p} = m\ldots$ is momentum

ejected

Rocket

red

$d\underset{\sim}{p}_{momentum\ of\ dm_{ej}} = dm_{ej}(\underset{\sim}{v} - \underset{\sim}{v}_{ej})$ in outside frame

but $dm_{Rocket} = -dm$

$$\left(\frac{d\underset{\sim}{p}}{dt}\right)_{loss} = \left(\frac{dM}{dt}\right)(\underset{\sim}{v} - \underset{\sim}{v}_{ej})$$

velocity

$$\frac{d\underset{\sim}{p}}{dt} = m\frac{d\underset{\sim}{v}}{dt} + \underset{\sim}{v}\frac{dm}{dt} = m\underset{\sim}{a} + \underset{\sim}{v}\frac{dm}{dt}$$

e.g., rocket

5022 (2023Jun01)

iii) A generalization of Newton's 2nd Law is needed for mass changing objects

mit in class, include vecopitut- tion

$$\underbrace{\underset{\sim}{F}_{net}}_{\text{Net external force}} + \underbrace{\frac{dm}{dt}}_{\substack{\text{Rate of mass}\\ \text{inflow to system}\\ \frac{dm}{dt}<0 \text{ for outflow}}} \underbrace{\underset{\sim}{v}_{Inflow}}_{\substack{\text{velocity relative}\\ \text{to outside}\\ \text{inertial frame}}} = \underbrace{\frac{d\underset{\sim}{p}}{dt}}_{\substack{\text{Rate of change}\\ \text{of momentum}\\ \text{of object.}}} = m\frac{d\underset{\sim}{v}}{dt} + \underset{\sim}{v}\frac{dm}{dt}$$

$$= m\underset{\sim}{a} + \underset{\sim}{v}\frac{dm}{dt}$$

pt. 6 ... ty ... nk ... restion) complicated class

Example Rocket

The Rocket ejects burnt f at speed v_{ej} opposite direction of motion.

Assume motion along x-axis and $\underset{\sim}{F}_{net} = 0$.

$\frac{dm}{dt} < 0$ for mass loss

$$v_{inflow} = \underbrace{v}_{\text{velocity of rocket}} - v_{ej}$$

The outflow velocity is reduced from v by v_{ej}.

$$\therefore \frac{dm}{dt}(v - v_{ej}) = ma + v\frac{dm}{dt}$$

$$ma = -v_{ej}\frac{dm}{dt}$$

$$m\frac{dv}{dt} = -v_{ej}\frac{dm}{dt}$$

$$m\,dv = -v_{ej}\,dm$$

$$dv = -v_{ej}\frac{dm}{m}$$

$$v - v_0 = -v_{ej}\ln\left(\frac{m}{m_0}\right)$$

$$v - v_0 = v_{ej}\ln\left(\frac{m_0}{m}\right) = v_{ej}\ln\left[\frac{m_0}{m(t)}\right]$$

Tsiolkovsky Rocket Equation (Wik)

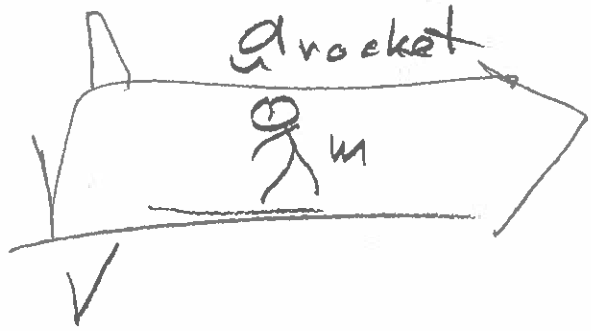
$m_0 > m$ because of mass loss

$\therefore v > v_0$

You have to specify $m(t)$ to find $v(t)$

iv) Inertial Forces

$$\underset{\sim}{F}_{net} = m\underset{\sim}{a}$$



But say you are in an accelerating rocket relative outside inertial frame

but you want to treat the inside as an inertial frame ∠Not derivation

Separate $\underset{\sim}{a} = \underset{\sim}{a}' + \underset{\sim}{a}_{rocket}$

$$\underset{\sim}{F}_{net} = m\underset{\sim}{a}' + m\underset{\sim}{a}_{roc}$$

$$F'_{net} = \underset{\sim}{F}_{net} - m\underset{\sim}{a}_{roc} = m\underset{\sim}{a}'$$

$$F'_{net} = m\underset{\sim}{a}' \quad \text{recovered}$$

$$\underset{\sim}{F}_{fictual} = -m\underset{\sim}{a}_{roc}$$

it pulls ~~on~~ you atom by atom like gravity backward.

All inertial forces are mass proportional like gravity

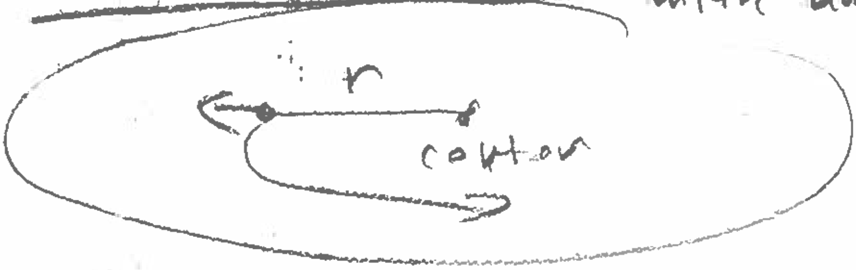
5024

toward the rear of the ~~rocket~~ rocket

and like gravity it is proportional to your mass.

These facts are what led Einstein to conclude gravity is an inertial force is developing general relativity

<u>Rotating frame</u> with angular velocity ω



$$\underset{\sim}{F}_{net} = m\underset{\sim}{a} = m\underset{\sim}{a}' + m\underset{\sim}{a}_{frame}$$

$$= m\underset{\sim}{g}' - \underbrace{m\omega^2 r\hat{r}}_{\text{centripetal acceleration formula But it's NOT the centripetal force}}$$

$$\underset{\sim}{F}'_{net} = \underset{\sim}{F}_{net} + \underbrace{m\omega^2 r\hat{r}}_{\text{outward pointing centrifugal force}} = m a'$$

the centrifugal force
pushes you outward
atom by atom.

To stay in place in
~~the~~ rotating Frame
there must be
some ordinary force
to act as the
centripetal force

Show
image Video
"Once the
door closes,
there is
no escape!"

say you are at rest
in carnival centrifuge
You need ~~the~~
wall to supply
a contact force
on your back.

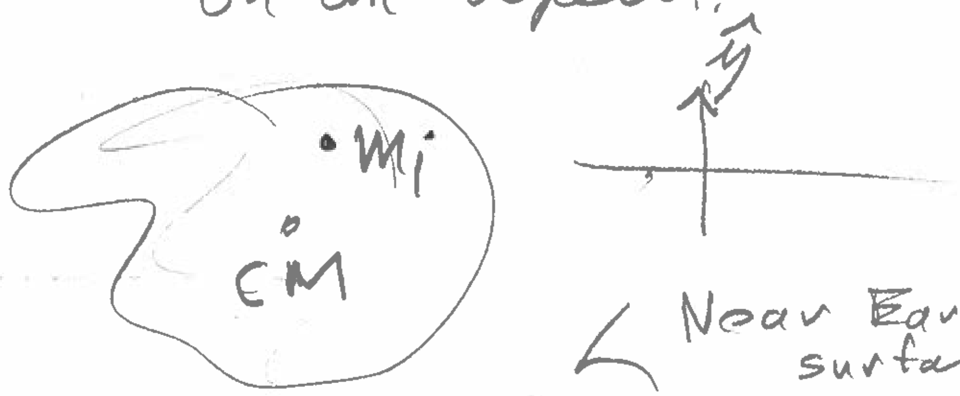
What if you are moving in ~~the~~
rotating frame. Then
there is still ~~the~~ the Coriolis Force, but that
is a velocity-dependent force and is beyond our scope to analyze.

5026 (2025jun01)

5.4 Mass & Weight

Weight is force of gravity on an object.

total Mass $m = \sum_i m_i$



Near Earth's surface

$$\underline{F}_{grav} = \sum_i m_i \underbrace{g(-\hat{y})}_{\text{independent of position}}$$

$$= \left(\sum_i m_i\right) g(-\hat{y})$$

$$= mg(-\hat{y})$$

or for just magnitude $W = F_{grav} = mg$

[2025 jun 01]

5027

What is mass?

Omit 5027-5033 in class

To reiterate from pp 1001-1006 with some expansion

a) Quantity of resistance to acceleration (relative to an inertial frame — goes without saying usually)

b) "Charge" of gravitational force

equality in Newtonian Physics is an unexplained coincidence.

In general relativity (our modern understanding it's fundamental. Gravity is mass proportional, so it's an inertial force

B028 (2025 jun 01)

A brief ~~into~~ ~~to~~ the world's most famous physics equation $E=mc^2$

What does it mean?

Two things,

i) All energy has mass
a resistance to acceleration and a gravitational effect

Note sometimes we talk of negative energies

But they are in fact always contributions to total energy at any point and the total is always positive

So as we know there is no negative mass in a total sense.

2025 Jun | 5029

ii) There is such a
thing a rest mass (energy)
an intrinsic
energy of a fundamental
particle that
can be a rest
in an inertial frame
e.g., electrons, protons,
neutrons

We can them
massive particles

and they are fundamentally
identical in a way no
macroscopica particle
can be

Why do we believe this:
All QM relies ~~on~~ it and
QM is the best verified
of all physical theories

All energy can ~~able~~ converted
into all other energy
forms but <u>not</u>
necessarily easily

5030

2025 jun 01

There is no practical way of converting large amounts of rest mass energy into other forms

But macroscopic amounts of antimatter don't exist in our universe

$\frac{1}{2}$ kg matter → ← $\frac{1}{2}$ kg antimatter

ram them together

and $E = mc^2 = 1\,\text{kg} \cdot (3 \times 10^8 \frac{m}{s})^2$

$\hat{=} 10^{17}\,\text{J} \times \left(\frac{1\,\text{Mt TNT}}{4 \times 10^{15}\,\text{J}}\right)$ wiki

$\hat{=} 25\,\text{Mt TNT}$

Equivalent to large Hydrogen bombs.

photons are massless

— but they do have mass because they have energy

But they have NO rest mass

and they can't be at rest.

Special relavity dictates they have an invariant speed $c = 2.99792458 \times 10^8$ m/s

relative to all inertial frame (which are all frames really)

Why wasn't E = mc2 noticed before special relativity (in which it is actually a very easy to prove result)?

In the 19th century, energy and mass were considered separate conserved things.

The changes in mass in all ordinary reactions below measurement.

5032)

For ~~example~~

heat a sample

You add heat energy

~~heat~~ change is far too small for ordinary measurements

Only with radioactivity an nuclear reactions were energy changes big enough to create noticeable mass changes

Nuclear reactions are of order 10^6 times more energetic than chemical reactions.

Ever since 1900 people have been mesmerized by that factor of 10^6

(2024 jun 06) | 5033

So much energy
from ~~so~~ so little fuel,
It's hard to access, but
we do in nuclear reactors
& nuclear bombs

For example
H nuclear burning in the Sun
— it a multistep process
but the net reaction is

$$\underbrace{4p^+}_{\text{photons}} + \underbrace{2e^-}_{\text{2 electrons}} \rightarrow \underbrace{{}^4He}_{\substack{\text{He 4 nucleus} \\ 2p^+, 2n}} + \underbrace{2\nu_e}_{\text{neutrino}} + \underbrace{\text{energy}}_{26.73\ \text{MeV}}$$

— an exothermic reaction

comparable chemical reaction are of order eV

Overall 0.7 % decrease in rest
from products to results

But "rest mass" here
includes nuclear binding
energy not just intrinsic rest.

Actually intrinsic rest mass energy
increases since 4 protons have less
than 2 protons, 2 neutrons

5034

2020 jan 01

5.6 Common Forces

Actually just 3

i) Gravity near Earth's surface

$$\vec{F}_{grav} = mg(-\hat{y})$$

The only one here for which we have force law formally

ii) Normal Force

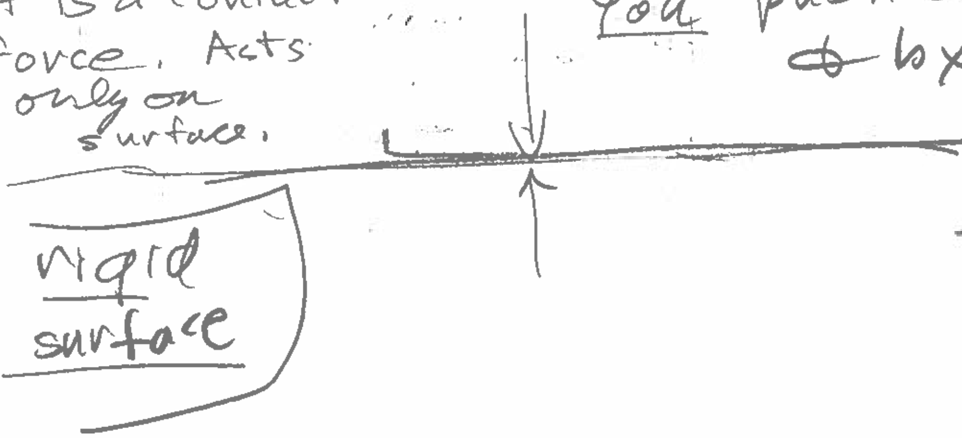
perpendicular to surface

The ideal force of ideal rigid surface

perpendicular

Normally

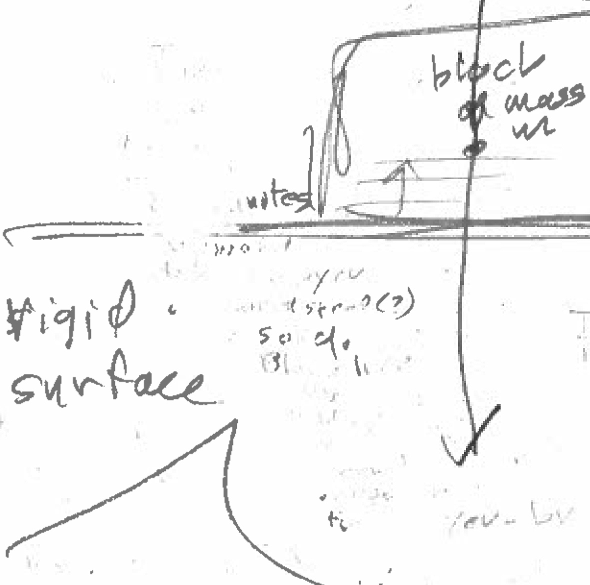
It is a contact force. Acts only on surface.



You push on it by 3rd law it pushes back with equal and opposite force.

2025jan01

L5035



Solve for F_{Normal}

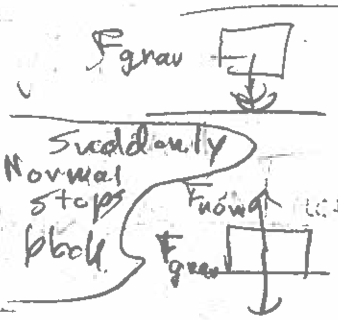
By 3rd Law

$F_{Normal} = mg$

By 2nd law on block

$ma = 0 = F_g - F_N$

$F_{normal} = mg$



In free-fall no internal pressure in block.

Hits table and at speed of sound (or so), the pressure force communicates stop through Block layer-by-layer pressure holds up block. But those are internal and/do not change center of mass motion

Free-body-diagram

You draw on it the forces exerted on on a body by external forces

NOT forces the body exerts.

If you draw them, You get into a complicated mess of cancelling forces.

Note gravity is a body force
→ it acts on body atom by atom

The Normal force is a contact force
only acts at the surface of contact.

5036

2025 jun 01

Emphasis the Normal Force is an ideal force with no intrinsic force law.

We only know its value using 2nd law or 3rd law

Real rigid bodies compress and from the amount of compression you can calculate the intrinsic resistance force.

like little springs

We touch on elasticity later.

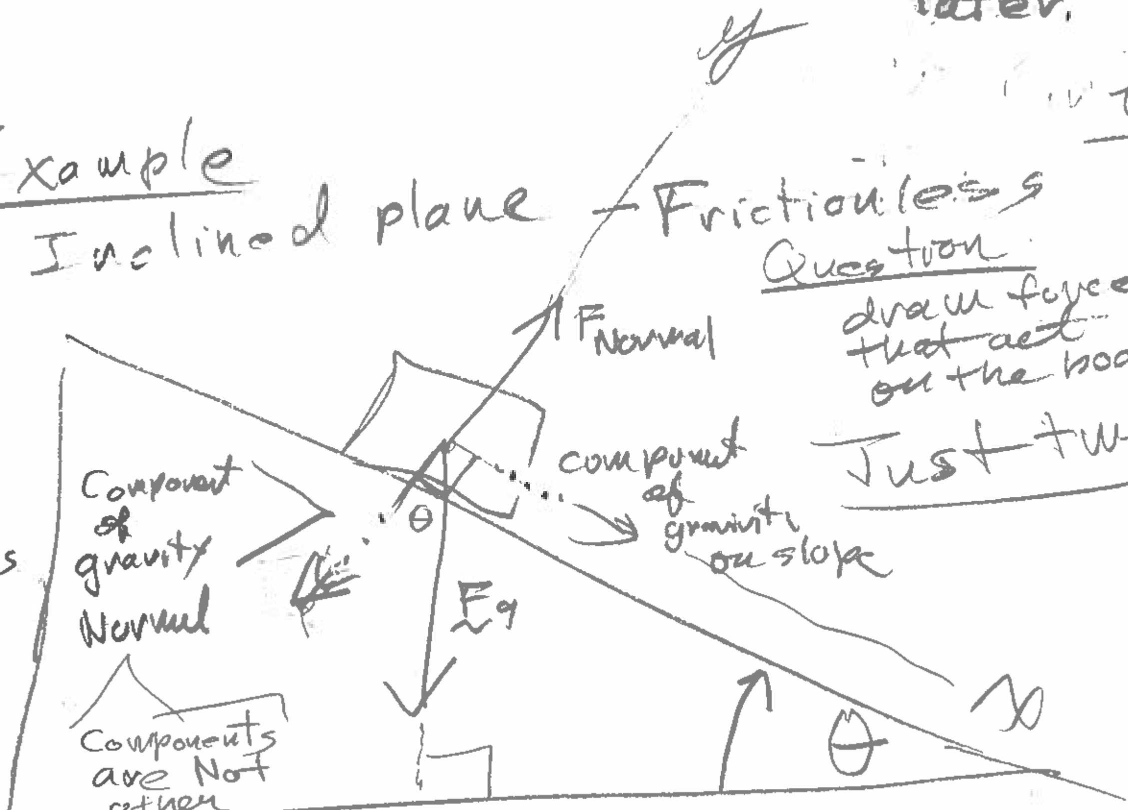
d) Example

Inclined plane - Frictionless

Question: draw forces ~~that act~~ on the body

Just two

The inclined plane is rigid and provides a constraint force.



Components are Not other forces.

There are only 2 forces on Block

2025 jun 01

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Question What is F_{Normal}

$F_{normal} = mg\cos\theta\ \hat{y}$

it must cancel gravity

Question What is the net force in the x-direction

$F_x = F_g \sin\theta$

$= mg\sin\theta$

Question What is the acceleration down the incline?

$F_{x\,net} = ma_x = mg\sin\theta$

$\therefore\ a_x = g\sin\theta$

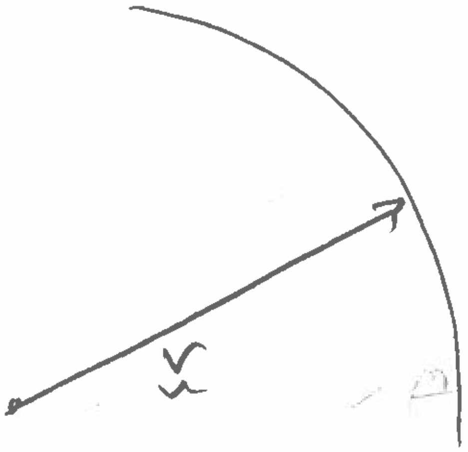
independent of mass

a consequence of gravity be a mass dependent force.

5038

b) Example Road banking

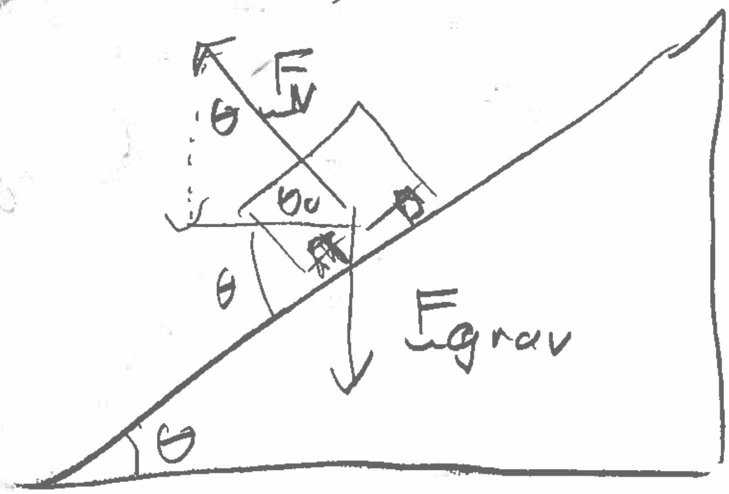
Top view



car velocity v

Side View

Free body diagram



Rigid Bank is a constraint force

Given the car is making the turn with no friction force acting.

True for all contents as well.

$$\therefore \quad \underset{\sim}{F}_{centripetal} = \frac{mv^2}{r}(-\hat{r})$$

$$\underset{\sim}{F}_{net} = \underset{\sim}{F}_N + \underset{\sim}{F}_{grav}$$

$a_{vertical} = 0$, $F_N \cos\theta = mg$ to cancel gravity

$a_{horizontal} = \frac{mv^2}{r}$, $F_N \sin\theta = \frac{mv^2}{r}$ to supply centripetal Force

Solve for θ

$$\tan\theta = \frac{v^2}{gr}$$

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$$\theta = \tan^{-1}\left[\frac{v^2}{gr}\right]$$

dimensional analysis

$$\left[\frac{v^2}{gr}\right] = \frac{L^2/T^2}{(L/T^2)L} = 1,$$

and so dimensions are Ok.

$v = 10\ m/s$

$r = 100\ m$

$g = 9.8\ m/s^2 \doteq 10$

$$\theta = \tan^{-1}\left[\frac{100}{10^3}\right]$$

$$\theta = \tan^{-1}(0.1) \doteq 0.1 \text{ radians} \doteq 6^\circ$$

* ~~60°/rad~~

using small angle approximation

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2}{15}\theta^5 + \ldots$$

$\doteq \theta$ when

$$\frac{\theta^3/3}{\theta} = \frac{\theta^2}{3} << 1$$

and $\theta = 0.1$ means

$$\frac{\theta^2}{3} \doteq 0.003 << 1$$

What of F_{Normal}?

We have no intrinsic formula, but the 2^{nd} law tells us

$0 = F_{Norm} - mg\cos\theta$

$$F_{Norm} = \frac{mg}{\cos\theta} \quad \text{since } a_y = 0$$

$$= m\left(\frac{10}{1 - \frac{1}{2}\theta^2}\right)$$

Using Taylor's series

$$\doteq m \cdot 10\ \frac{N}{kg}$$

Car mass $= 10^3 kg$, $F_N = 10^4 N$

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So a car moving at $v = 10\,m/s$

will make the turn without

friction. { Also true for contents, Passengers Just sink in there seats. Little stress force on structure of car, Just compression.

Faster, it will tend to slide up.

Slower, it will tend to slide down

Now relying on friction is bad.

Hard on tires & road

It can go to zero

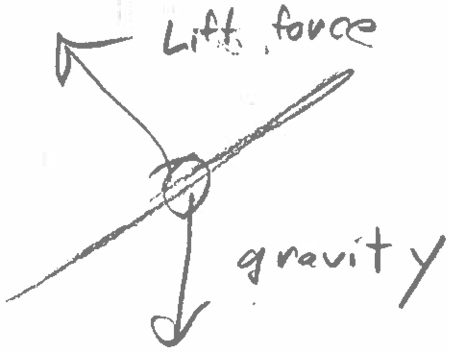
if the road is icy.

The conclusion is always

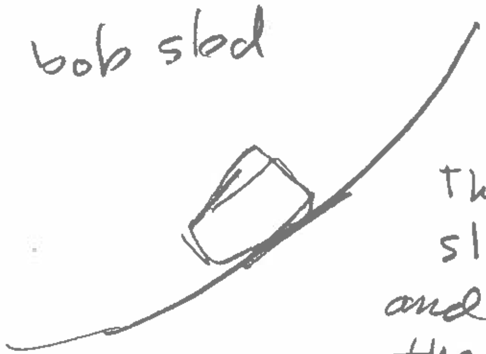
drive at the ramp speed.

It's there for a reason.

Analogus systems



bob sled



The sled
slides up
and down until
the required
centripetal force doesn't need friction

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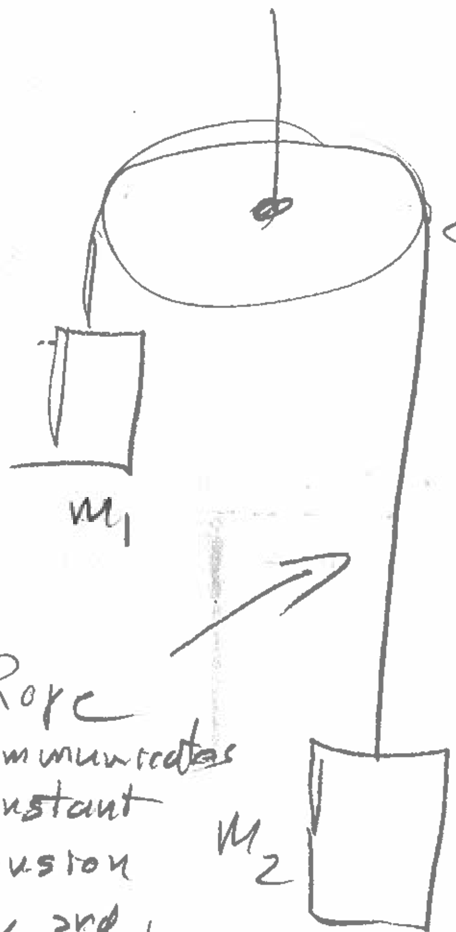
Example b) Atwood's Machine

invented 1784 (wik) to verify laws of Motions thousands year after pulleys had been invented for practical uses



Pulley with no-slip condition ~~on~~ on rope (realistic)

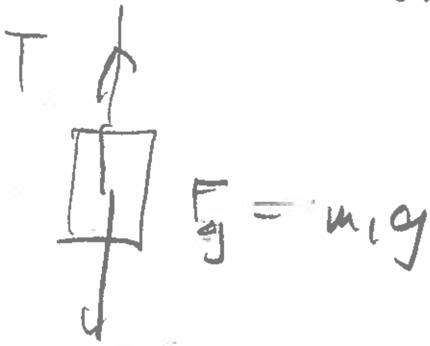
and pulley having no rotational inertia (idealized) and the wheel turns without friction

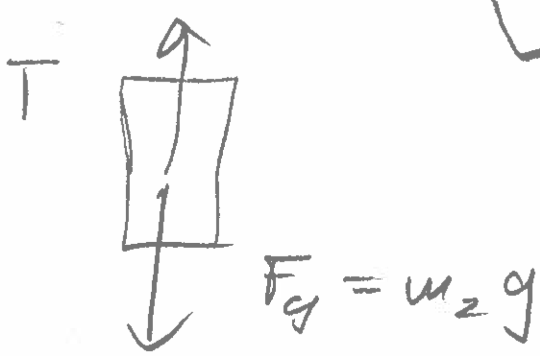
Rope communicates constant tension by 3rd law along it's length

It's massless

Question

Draw Free body diagram for both masses.





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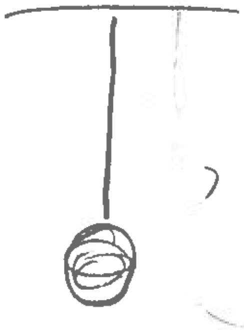
iii)

Tension Force of ideal rope



It cannot resist compression or shearing

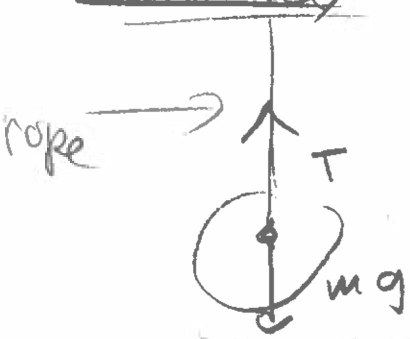
- No mass
- can only exert a tension force and Tension force cannot vary

You can push on on a rope, But it doesn't resist.

You can't push with a rope.

The ideal rope just exerts a pulling force.

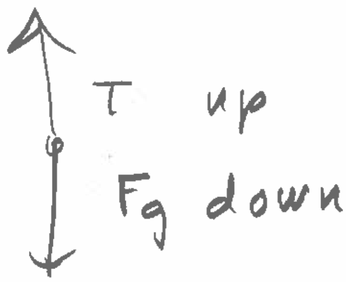
Example (a)



10 kg object hanging

Question

Draw the free body diagram.



No intrinsic force law for ideal rope.

What is T by 2nd Law

$F = ma$

but a

$0 = T - mg$

$T = mg$

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Solve for acceleration of m_1 and m_2?

Write down F=ma for both masses

Assume a From 1 to 2 is ~~the~~ +ve direction

Eliminate T.

① $m_1 a_1 = T - m_1 g$

② $m_2 a_2 = m_2 g - T$

Apply constraint $a_1 = a_2$

Exploit symmetry

①+②

$(m_1 + m_2) a = (m_2 - m_1) g$

$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$

You can make a as small as you like making it much easier to study than $a = g$

Solve for T eliminating a

①/m_1 − ②/m_2

$0 = \frac{T}{m_1} - g - \left(g + \frac{T}{m_2}\right)$

$= T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - 2g$

$T = \frac{2g}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = \frac{2 m_1 m_2 g}{m_1 + m_2}$

$T = \frac{2g}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$

$= \frac{2 m_1 m_2 g}{m_1 + m_2}$

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Remember we have no intrinsic force law for ideal tension ~~and~~ must rely on 2nd or 3rd law to determine it

Question

$$T(m_1 = m_2 = m) \overset{?}{=} mg \quad \text{reasonable}$$

$$T(m_2 \to \infty) = 2m_1 g \quad \text{not easily anticipated.}$$

5.7) Free body diagrams

We have already covered them.