2020jun01 | 3001

Ch 3 Motion along a straight line

1-dimension Kinematics

3.1) ~~Position~~ Position, Displacement,
Velocity, Acceleration
3.2)
3.3)

- Kinematics is the science of Motion Without Reference To Forces
- In Classical Physics it's pretty easy
- In rellativistic physics (which we only touch on) it is trickier, since spacetime has its own 'Force' like propertie and time & length are inertial frame-dependent quantities

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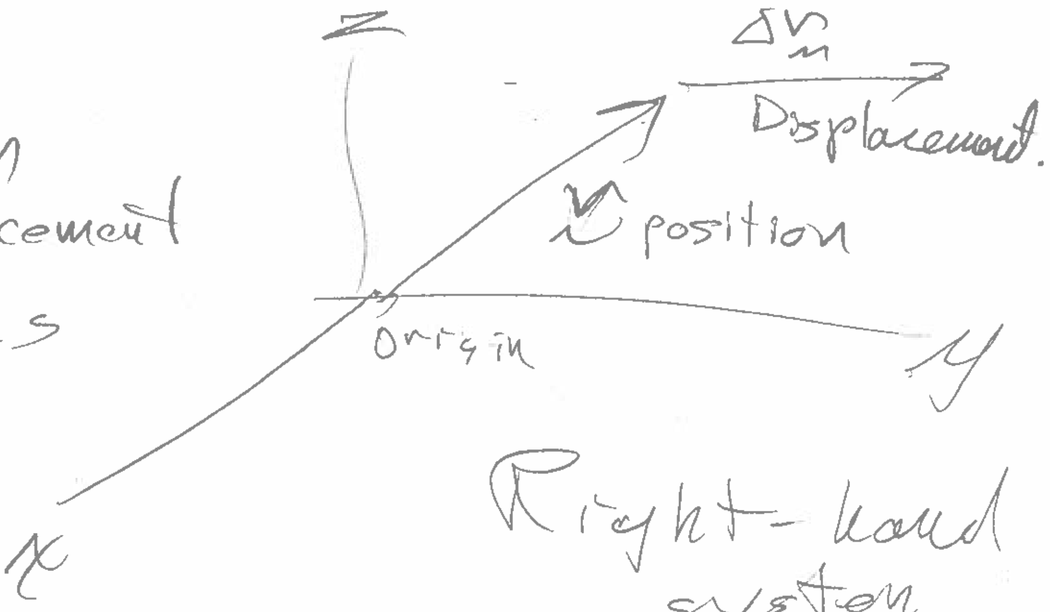
Position + Displacement

synonyms or not depends on what you mean



Right-hand system

Position is displacement from origin

$\underset{\sim}{r}$ squiggle mean vector recall

$\Delta \underset{\sim}{r}$ Greek Capital Delta means change in — usually a finite not infinitesimal (calculus differential) displacement

But no hard rule

$\delta \underset{\sim}{r}$ or $d\underset{\sim}{r}$ is differential displacement, A limiting process is implied.

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Velocity

$$\underset{\sim}{v}_{averg} = \frac{\Delta \underset{\sim}{r}}{\Delta t}$$

$\Delta \underset{\sim}{r}$ vector

in $\underbrace{\Delta t}_{\text{scalar}}$

$$\underbrace{\underset{\sim}{v}}_{\text{instantaneous velocity}} = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{r}}{\Delta t}}_{\text{limiting process}} = \underbrace{\frac{d\underset{\sim}{r}}{dt}}_{\text{derivative of } \underset{\sim}{r} \text{ with respect to time}} = \dot{\underset{\sim}{r}}$$

instantaneous velocity — which we just call velocity

dot over notation
— one bit of Newton's own notation that has stuck around
— most calculus notation is "development" from Leibniz
But dot "•" is only used for differentiation with respect to time

$$\underbrace{v}_{\text{speed is magnitude of velocity}} = |\underset{\sim}{v}| = \sqrt{\underset{\sim}{v}\cdot\underset{\sim}{v}}$$

speed is magnitude of velocity

dot product

dot product

$$\underset{\sim}{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$ unit vectors

$$= (v_x, v_y, v_z)$$ ordered tuple

$$= v_i$$ element representation

3004) [2025 jun 01]

acceleration

$$\underset{\sim}{a}_{\text{Avg}} = \frac{\Delta \underset{\sim}{v}}{\Delta t}$$

$$\underbrace{\underset{\sim}{a}}_{\text{(instantaneous acceleration}} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}}{\Delta t} = \frac{d\underset{\sim}{v}}{dt} = \dot{\underset{\sim}{v}}$$

$$= \frac{d^2 \underset{\sim}{r}}{dt^2} = \underbrace{\ddot{\underset{\sim}{r}}}_{\text{Newton's double dot}}$$

$$\underbrace{a}_{\text{acceleration}} = ||\underset{\sim}{a}| = \sqrt{\underset{\sim}{a} \cdot \underset{\sim}{a}}$$

acceleration
same word
used for
vector and
for its magnitude

$$\underset{\sim}{a} = \overset{\text{ordered triple representation}}{(a_x, a_y, a_z)} = \underbrace{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}_{\text{unit vector representation}}$$

$$= \underbrace{a_i}_{\text{element representation}}$$

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1-d versions

are just x, y, or z components along in some coordinate system

components can be positive or negative unlike magnitudes

Example

x usually used for the horizontal direction

$$v = \frac{dx}{dt}$$

(meaning v_x

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

(meaning a_x)

y usually used for the vertical direction

$$v = \frac{dy}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2}$$

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3.4 Motion with Constant
3.6 Integration Acceleration

The prototype interesting case

Remember your calculus:

constant a

integrate over time to get velocity $v = at + v_0$

Constant of integration
velocity at time zero

integrate over time get position $x = \frac{1}{2}at^2 + v_0 t + x_0$

Constant of integration = position at time zero

$\Delta x = \frac{1}{2}at^2 + v_0 t$

- change in x
- The x_0 parameter is a nuisance in solving problems since you can always just add it, but on the other hand, You want to keep in mind a change in position

The Two equations are algebraically independent (though not independent in a calculus sense)

1) $v = at + v_0$

2) $\Delta x = \frac{1}{2}at^2 + v_0 t$

Question

How many variables are there?

Ans. $4 + 1 = 5$

Question

How many unknowns can you solve for?

Ans. 2

You must know 3 of the variables to solve in general

Two equations give you only two constraints

Any 1-d constant acceleration problem can be solved from these two equations ① and ②

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enerate
ive
-d
nematic
quations

But ~~that~~ is often tedious and so we can 'mix' ~~the~~ equations to solve any 1-d constant acceleration problem with 1 equation.

a) Question
~~Say~~ you two unknowns and one is x, but it's the unwanted one

which equation of ① & ② do you use?

① $v = at + v_0$

b) Question
Say you have ~~two~~ two unknowns and the one you don't want to solve for is v

Which equation do you use?

② $\Delta x = \frac{1}{2}at^2 + vt$

c) Say you have two unknowns and the unwanted one is t

You're stuck t is on both ① & ②

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So you must create ③ by eliminating t

Question ~~Try doing it~~ $\left(\frac{v-v_0}{a}\right) = t$ from

substitute into ② and simplify

$$\Delta x = \frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2 + v_0\left(\frac{v-v_0}{a}\right)$$

$$a\Delta x = \frac{1}{2}(v-v_0)^2 + v_0 v - v_0^2$$

$$= \frac{1}{2}(v^2 + v_0^2 - 2v_0 v) + v_0 v - v_0^2$$

$$= \frac{1}{2}(v^2 - v_0^2)$$

$$v^2 - v_0^2 = 2a\Delta x$$

or $$v = \pm\sqrt{v_0^2 + 2a\Delta x}$$

or $$\Delta x = \frac{v^2 - v_0^2}{2a}$$

But they is a sign ambiguity

"If Δx ~~or~~ a is your unknown to be solved for there is No ambiguity, but ~~if~~ it is v or v_0, there is a sign ambiguity.

Question

Which is right?

They both are righ

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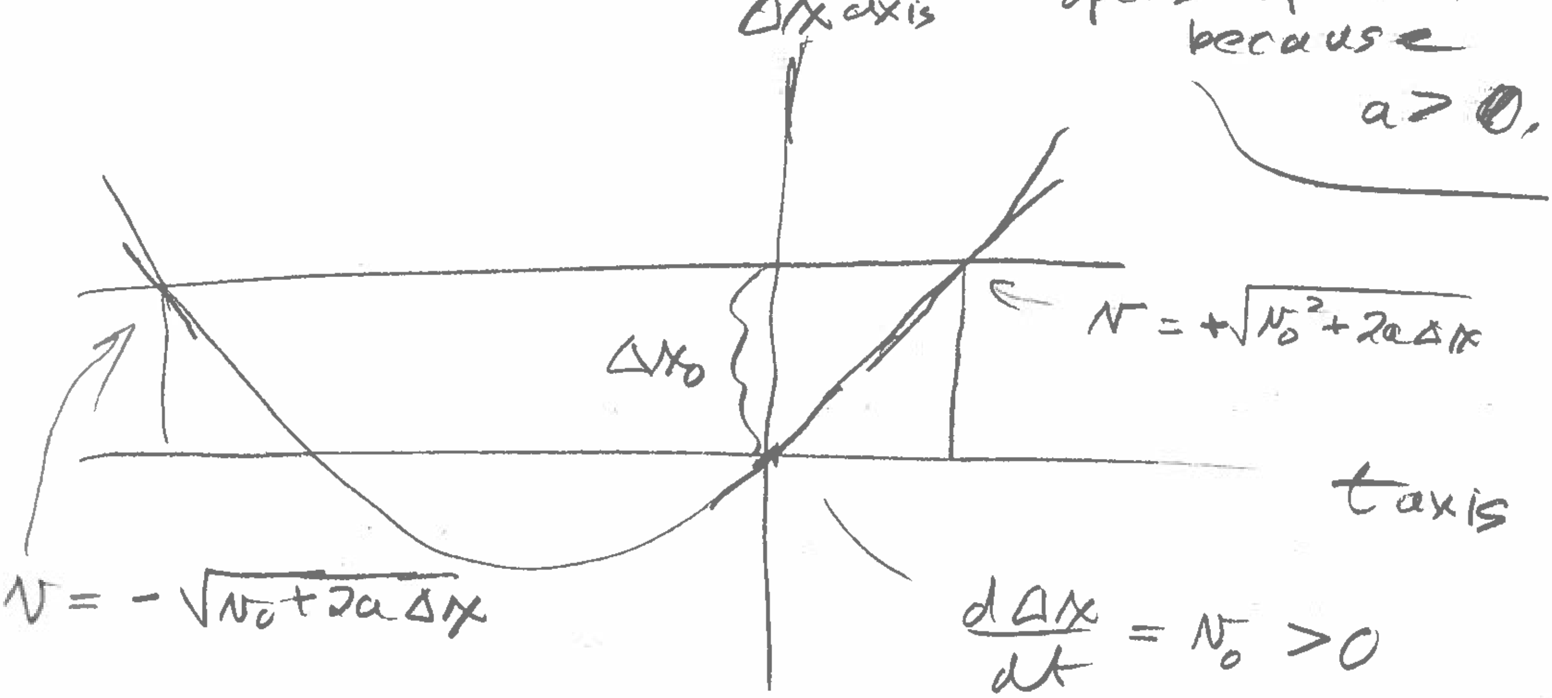
Let's say
v & t where
the unknowns
$\therefore$ you know Δx, a, v_0
and to be definite
Say $a > 0$
and $v_0 > 0$

From eq ① which is a parabola and it opens upward because $a > 0$.



So slope +ve

Both answers right.
You need some other piece
of information in this case know which answer you want
Say $t > 0$, but you don't know what it is
Then $v = +\sqrt{v_0{}^2 + 2a\Delta x}$ and you can solve for it if you want too

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d) Say you have two unknowns and the unwanted one is $\underline{\underline{a}}$

Question Eliminate $\underline{a}$. Go

from ① $a = \frac{v - v_0}{t}$

substitute into ②

$$\Delta x = \frac{1}{2}\left(\frac{v - v_0}{t}\right)t^2 + v_0 t$$

$$\Delta x = \frac{1}{2}(v + v_0)t$$

in $v_{avg} = v + v_0$

e) Say you have two unknowns and the unwanted one is v_0

Question: Generate a kinematic equation without v_0

From ① $v_0 = v - at$

substitute into ②

$$\Delta x = \frac{1}{2}at^2 + (v - at)t$$

⑤ $\Delta x = vt - \frac{1}{2}at^2$

Summarize Five 1-dimensional Kinematic equations of physics

Equation	Unwanted Unknown
① $v = at + v_0$	x
② $\Delta x = \frac{1}{2}at^2 + v_0 t$	v
③ $v^2 - v_0^2 = 2a\Delta x$ timeless equation	t
④ $\Delta x = \frac{1}{2}(v + v_0)t$	a
⑤ $\Delta x = vt - \frac{1}{2}at^2$	v_0

(The timeless eqn actually generalizes to nonconstant acceleration in special cases of conservation of mechanical energy which we do later)

Eq. 5 gets less use than the others because people tend not to write equations that use it

3.5 Free Fall

Near Earth's surface neglecting air drag (which you can do for relatively short fall distances of relatively dense bodies) the acceleration due to gravity of Earth (magnitude) (and centrifugal force)

Fiducial value $g = 9.8 \frac{m}{s^2}$

AKA little g

(as opposed to big G the gravitational constant of Newton's law of Universal gravitation)

Actually g varies with latitude, altitude and geologic terrain and those variations are easily measured with a gravimeter, but below human perception

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Show carnival centrifugal video

Biggest variation is with latitude

9.780 m/s^2 at equator

9.832 m/s^2 at poles

~0.5% just too small for human perception

If it were 10%, everyone would notice

Actually most of variation is centrifugal force

0 at poles, strongest at equator.

It's the force that tries to throw you off carnival centrifuges

Which we discuss more fully later

It is what is called an inertial force

— a force due to structure of spacetime in general relativity jargon

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'ordinary' forces are ultimately due to a field of force

- electromagnetic
- strong & weak nuclear.

gravitation is two-faced

- in classical physics we treat as a field of force, but in general relativity it's an inertial force

We'll discuss both kinds of forces more later

But as a preview

Newton's 2nd law $F = ma$ 1-d form.

Near Earth's surface Not in general

mass of object

$F_{\text{gravity}} = -mg$

(as a field)

up is +ve so down is negative.

a force law for gravity Not $F = ma$

Here $$g = 9.8 \frac{N}{kg} \longleftarrow \text{Newton metric unit of force}$$

is the force constant.

But $F = ma$

net force is true

and $F_{grav} = -mg$ is true

if the only force is gravity,

then

$$-mg = F_{grav} = F_{net} = ma$$

$$a = -g = -9.8 \, m/s^2$$

[Note this is how the Newton as a unit is derived

$$1 N = 1 kg \cdot m/s^2$$

| F_{grav} = weight = $1 kg \cdot 9.8 \, m/s^2$

~~Now~~ $1 N \cong 0.224809$ pounds

1 kg weighs $\cong$ 2.24 pounds

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1 pound is weight of 0.454 kg

In Canada, you don't buy a pound of margarine, You buy 0.454 kg of margarine

Canada metricated in the 1970s, but it did **NOT** change the standard solid quantities but this is good Canadians know 0.454 kg is a pound

---

So $a = -9.8\ m/s^2$

The cancelation of mass using the gravity law was always a mystery in Newtonian physics

Why is the 'charge' of the gravitational force **exactly** equal to the quantity of resistance to acceleration?

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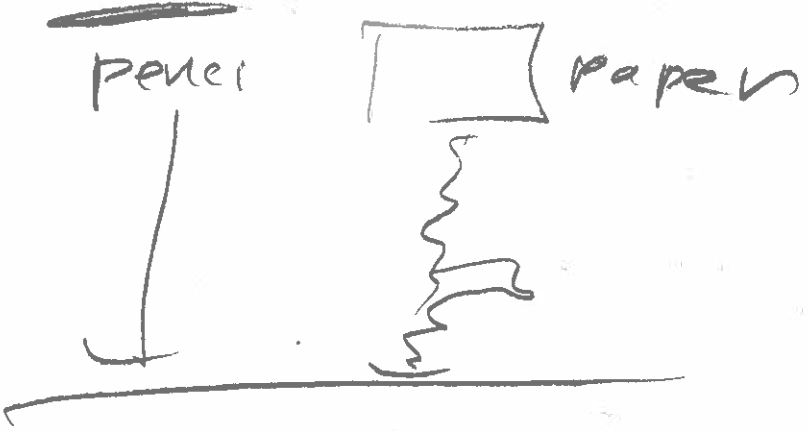
General relativity takes it as an axiom that they must be exactly equal

<u>and</u> axiom has always been confirmed.

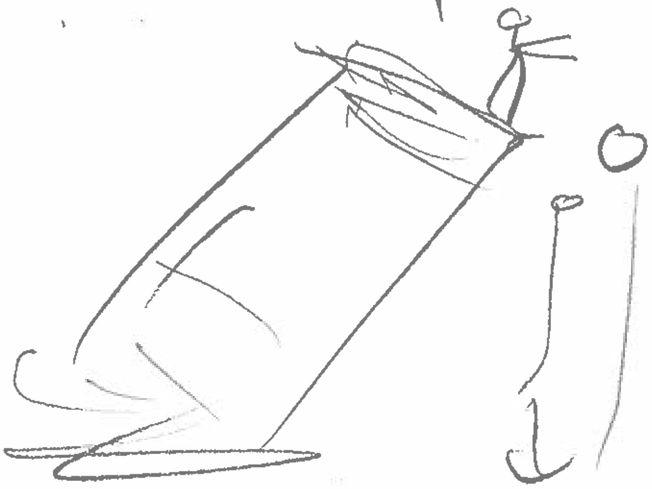
<u>The</u> upshot of $a = -g$ for any object free falling near the Earth's surface is all things fall the same distance in same time

Show Galileo & Moon Videos



If you neglect airdrag.

But Galileo experiment is the ideal limit.



Galileo probably Really did it. But not a careful experiment just a popular demonstration for

And, of course, balls never it exactly at the same time because air drag, variations in release time etc.

But Galileo argued ~~the~~ ideal limit when perturbations have been reduced to negligible object near Earth's have the same acceleration g.

Show videos

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Analysis

$g = 9.8$ m/s

t	$\|v\|$ (m/s)
0	0
1	9.8
2	19.6
3	29.4
4	39.2

Air drag actually grows with speed, and so eventually the acceleration declines and you reach terminal velocity

sky divers $v_{term} \approx 50$ m/s (Wiki)

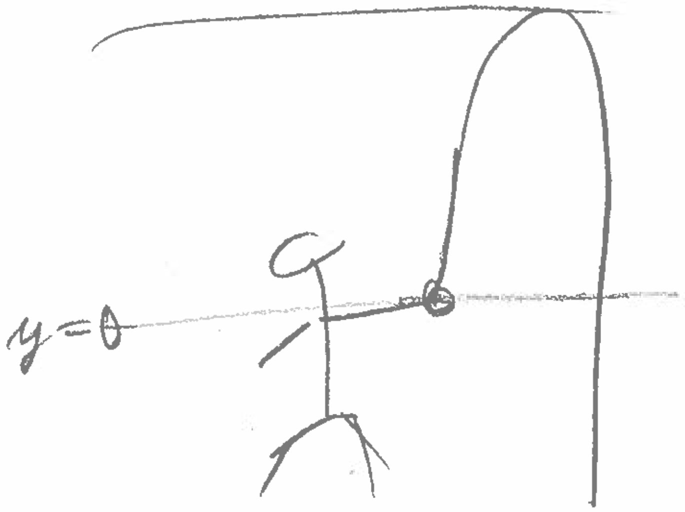
$$= 50 \text{ m/s} * \underbrace{\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)}_{\text{factors of unity}}$$

$$\simeq 200 \text{ km/h}$$

Question for Class

Do conversions to km/h

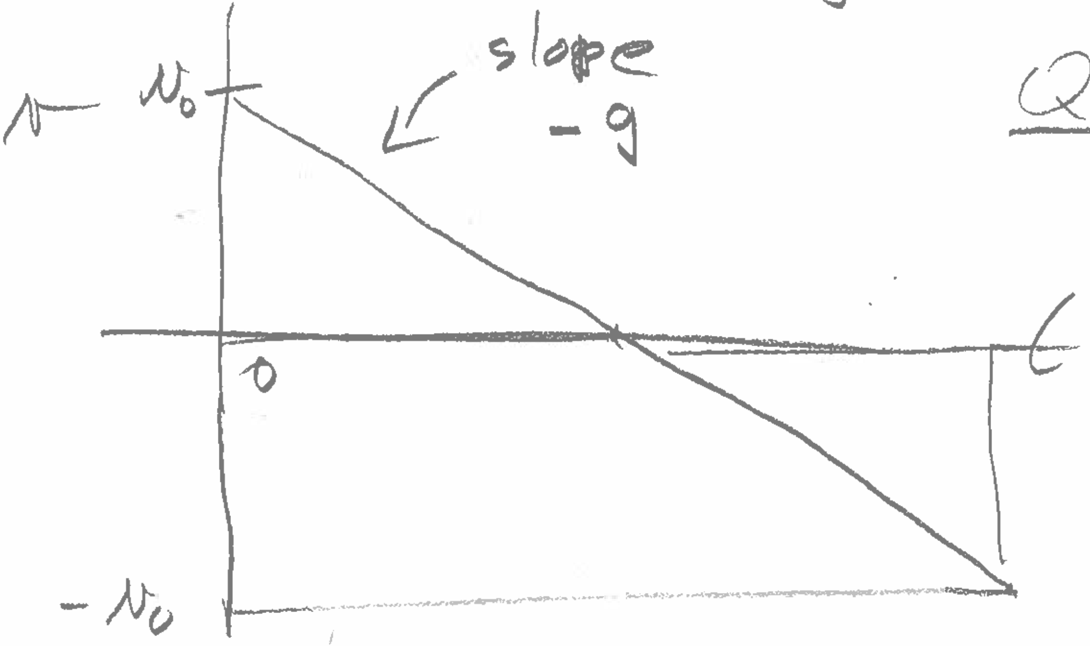
Example



tossing a ball straight up

$v = -gt + v_0$

$$\underbrace{y = -\tfrac{1}{2}gt^2 + v_0 t}_{\text{usually use } y \text{ as the vertical coordinate}}$$



Question

Draw the velocity curve $v(t)$

same speed when returns to tosser

Why? Remember the timeless equation

$$v = \pm\sqrt{v_0^2 + 2(-g)y}$$

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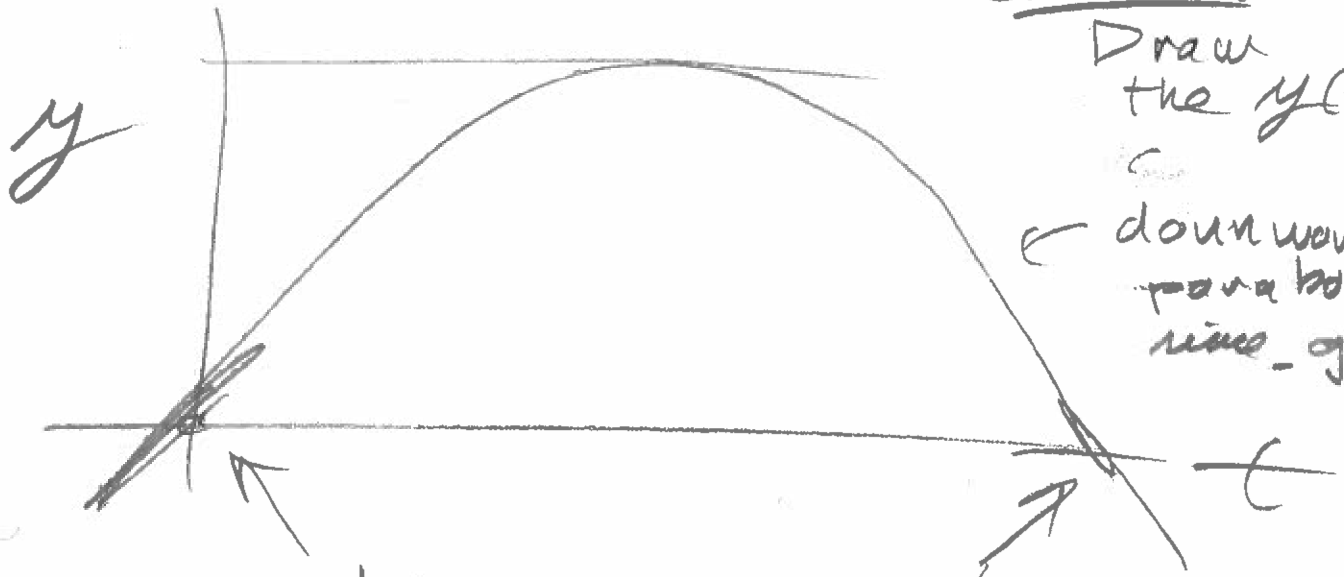
at any height y, the speed going up is the same as the speed ~~going~~ down

Question
Draw the $y(t)$



downward parabola since $-g < 0$

slope $\frac{dy}{dt} = v_0$

slope $\frac{dy}{dt} = -v_0$

Question Determine the maximum height.

$v = 0$ at maximum
— a turning point
(Google AI confirms jargon)

$$0 = v_0^2 + 2(-g)y_{max}$$

$$y_{max} = \frac{v_0^2}{2g}$$

Say $v = 10$ m/s

Question what is y_{max}

$y_{max} \doteq \frac{10^2}{2 \cdot 10} = 5$ m

(you can toss harder than that right)

3.6 Integration

Already done p 3006

End chpt. 3 notes