Introductory Physics: Calculus-Based **Name:**

Homework 03_11: Particle Physics: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Multiple Choice Questions:

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045 qmult 00100 1 4 4 easy deducto-memory: particle physics defined

1. "Let's play *Jeopardy*! For $100, the answer is: The branch of physics which deals with elementary particles and composite particles up to the scale of protons and neutrons whose theoretical understanding is currently (and probably forever now) based on quantum field theory (QFT) and whose current best theory (but probably not forever the best theory) is called the Standard Model."

   What is ____________, Alex?

   a) atomic and molecular physics b) condensed matter physics c) nuclear physics
   d) particle physics e) quantum mechanics

   SUGGESTED ANSWER: (d)

   Wrong answers:
   a) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2026aug12

Full Answer Problems:

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.997\,924\,58 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns}$$

exact by definition

$$e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition}$$

$$G = 6.674\,30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987\,551\,785\,972(14) \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}$$

$$k_{\text{Boltzmann}} = 1.380\,649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\,333\,262 \times 10^{-4}\,\mathrm{eV/K}$$

$$\approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms}$$

$$m_e = 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\,998\,950\,69(16)\,\mathrm{MeV}$$

$$m_p = 1.672\,621\,923\,95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\,089\,32(29)\,\mathrm{MeV}$$

$$\varepsilon_0 = 8.854\,187\,8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity}$$

$$\mu_0 = [1.256\,637\,061\,27(20)] \times 10^{-6}\,\mathrm{N/A^2} \approx 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}$$

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2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

If $\quad 0 = ax^2 + bx + c \;,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

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6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan\left(a_x, a_y\right) + n\pi = \arctan_{\text{computational}}\left(\dfrac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\mathrm{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

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7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\text{m/s}^2$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\,\text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\,\text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

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19 **Static Equilibrium**

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

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20 **Gravity**

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707 \times 10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^8 \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

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21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

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22 **Oscillation**

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\text{max}}\sin[k(x - vt)] = y_{\text{max}}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\text{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\text{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\text{K}} = T_{\text{C}} + 273.15\,\text{K} \qquad T_{\text{F}} = 1.8 \times T_{\text{C}} + 32^\circ\text{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

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25 **Electrostatics**

$$\vec{F}_{1\text{ on }2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_{\text{i}}^{\text{f}} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\, d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_iV_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\, d\mathcal{V}\, d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\, d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa\vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

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26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J}\cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma\vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R}e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}}\left(1 - e^{-t/\tau}\right)$$

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27 Magnetic Fields and Forces

$$\mu_0 = [1.256\ 637\ 061\ 27(20)] \times 10^{-6}\,\text{N/A}^2 \approx 4\pi\times 10^{-7}\,\text{N/A}^2 \qquad u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2$$

$$1\,\text{N/A}^2 = 1\,\text{T}\cdot\text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad d\vec{F} = I\,d\vec{s}\times\vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s} \times \hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I\theta}{4\pi r} \qquad \vec{F}_{1\text{ on }2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r}\hat{r}_{2\text{ to }1}$$

$$B_{\text{solenoid}} = \mu_0 n I \qquad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt}\int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

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28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2}LI^2 \qquad u_B = \frac{1}{2\mu_0}B^2$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con-axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty\left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0\cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_\pm = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0\cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right]e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

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29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0\cos(\omega t) \qquad I = I_0\cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2} I_0 V_0 \cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \qquad I_{\text{p}} N_{\text{p}} = I_{\text{s}} N_{\text{s}} \qquad I_{\text{p}} \mathcal{E}_{\text{p}} = I_{\text{s}} \mathcal{E}_{\text{s}} \qquad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

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30 **Electromagnetic Radiation (EMR)**

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$