

Introductory Physics: Calculus-Based**Name:**

Homework 03_02: Geometrical Optics: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Multiple Choice Questions:

1. Snell's law or the law of refraction is:

a) $n_1 \sin \theta_1 = n_2 \sin \theta_2$. b) $\theta_{\text{incident}} = \theta_{\text{reflected}}$. c) $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. d) $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$.
e) $\theta_{\text{reflected}} = \sin \theta_{\text{incident}}$.

Full Answer Problems:

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.997\,924\,58 \times 10^8 \text{ m/s} \approx 2.998 \times 10^8 \text{ m/s} \approx 3 \times 10^8 \text{ m/s} \approx 1 \text{ ly/yr} \approx 1 \text{ ft/ns}$$

exact by definition

$$e = 1.602\,176\,634 \times 10^{-19} \text{ C} \quad \text{exact by definition}$$

$$G = 6.674\,30(15) \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad (\text{circa } 2025)$$

$$g = 9.8 \text{ m/s}^2 \quad \text{fiducial value}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987\,551\,785\,972(14) \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10} \text{ N m}^2/\text{C}^2$$

$$k_{\text{Boltzmann}} = 1.380\,649 \times 10^{-23} \text{ J/K} = 0.8617\,333\,262 \times 10^{-4} \text{ eV/K}$$

$$\approx 10^{-4} \text{ eV/K} \quad \text{exact by definition in both forms}$$

$$m_e = 9.109\,383\,7139(28) \times 10^{-31} \text{ kg} = 0.510\,998\,950\,69(16) \text{ MeV}$$

$$m_p = 1.672\,621\,923\,95(52) \times 10^{-27} \text{ kg} = 938.272\,089\,32(29) \text{ MeV}$$

$$\epsilon_0 = 8.854\,187\,8188(14) \times 10^{-12} \text{ C}^2/(\text{N m}^2) \approx 10^{-11} \text{ C}^2/(\text{N m}^2) \quad \text{vacuum permittivity}$$

$$\mu_0 = [1.256\,637\,061\,27(20)] \times 10^{-6} \text{ N/A}^2 \approx 4\pi \times 10^{-7} \text{ N/A}^2 \quad \text{vacuum permeability}$$

2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \quad A_{\text{cir}} = \pi r^2 \quad A_{\text{sph}} = 4\pi r^2 \quad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \quad d\Omega = \sin\theta \, d\theta \, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \quad \frac{y}{r} = \sin\theta \quad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \quad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \quad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \quad \cos(\theta + 180^\circ) = -\cos(\theta) \quad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \quad \cos(\theta + 90^\circ) = -\sin(\theta) \quad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \quad \cos(180^\circ - \theta) = -\cos(\theta) \quad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \quad \cos(90^\circ - \theta) = \sin(\theta) \quad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \quad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \quad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}[\cos(a-b) - \cos(a+b)] \quad \cos(a)\cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$$

$$\sin^2 \theta = \frac{1}{2}[1 - \cos(2\theta)] \quad \cos^2 \theta = \frac{1}{2}[1 + \cos(2\theta)] \quad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \quad \frac{1}{1-x} \approx 1+x : (x \ll 1)$$

$$\sin \theta \approx \theta \quad \tan \theta \approx \theta \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2 \quad \text{all for } \theta \ll 1$$

5 Quadratic Formula

$$\text{If } 0 = ax^2 + bx + c, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2} \left[b + \text{sgn}(b) \sqrt{b^2 - 4ac} \right] \quad x_1 = \frac{q}{a} \quad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \quad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\frac{a_y}{a_x}\right) + \pi \text{H}_{\text{Heaviside}}(-a_x) \text{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab \cos \theta = a_x b_x + a_y b_y + a_z b_z \quad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab \sin(\theta) \hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \quad \frac{d(x^0)}{dx} = 0 \quad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

$$\begin{aligned} \text{Taylor's series} \quad f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \dots \end{aligned}$$

$$\int_a^b f(x) dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \quad \int \frac{1}{x} dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \quad v = \frac{dx}{dt} \quad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $g = 9.8 \text{ m/s}^2$

$$x_{\text{rel}} = x_2 - x_1 \quad v_{\text{rel}} = v_2 - v_1 \quad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \quad v' = v - v_{\text{frame}} \quad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \quad \vec{v} = \frac{d\vec{r}}{dt} \quad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \quad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \quad v_{x,0} = v_0 \cos \theta \quad v_{y,0} = v_0 \sin \theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos \theta} \quad y = y_0 + x \tan \theta - \frac{x^2 g}{2v_0^2 \cos^2 \theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin \theta \cos \theta}{g} \quad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2 \theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \quad \theta_{\text{for max}} = \frac{\pi}{4} \quad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \quad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \vec{v} = \vec{v}_2 - \vec{v}_1 \quad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \quad v = r\omega \quad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \quad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \quad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \quad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r}) \vec{r} dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{21} = -\vec{F}_{12} \quad F_g = mg \quad g = 9.8 \text{ m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \quad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \quad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \quad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \quad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \quad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag, lin}} = bv \quad v_{\text{T}} = \frac{mg}{b} \quad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \quad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag, quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \quad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \quad W = \int \vec{F} \cdot d\vec{s} \quad K = \frac{1}{2}mv^2 \quad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \quad P = \frac{dW}{dt} \quad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \quad \vec{F} = -\nabla U \quad U = \frac{1}{2}kx^2 \quad U = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \quad \Delta K_{\text{cm}} = W_{\text{net,external}} \quad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \quad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t) dt \quad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \quad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \quad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \quad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \quad v_{\text{rel}'} = -v_{\text{rel}} \quad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\dots \quad \frac{1}{2\pi} = 0.15915494\dots$$

$$\frac{180^\circ}{\pi} = 57.295779\dots \text{ deg/rad} \approx 60 \text{ rad/deg} \quad \frac{\pi}{180^\circ} = 0.017453292\dots \text{ rad/deg} \approx \frac{1}{60} \text{ rad/deg}$$

$$\theta = \frac{s}{r} \quad \omega = \frac{d\theta}{dt} = \frac{v}{r} \quad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \quad f = \frac{\omega}{2\pi} \quad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \quad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \quad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \quad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \quad \vec{\tau} = \vec{r} \times \vec{F} \quad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \quad \tau_z = RF_{xy} \sin \gamma_\tau \quad L_z = I\omega \quad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \quad I = \int R^2 \rho dV \quad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \quad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \quad I_{\text{cyl}} = \frac{1}{2}MR^2 \quad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \quad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \quad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \quad dW = \tau_z d\theta \quad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} d\theta \quad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}} d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \quad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \quad \vec{\tau}_{\text{ext,net}} = 0 \quad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \quad 0 = F_{\text{net } y} = \sum F_y \quad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \quad \vec{g} = -\frac{GM}{r^2}\hat{r} \quad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \quad V = -\frac{GM}{r} \quad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \quad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \quad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0 \text{ km} \quad R_{\text{Earth,equatorial}} = 6378.1370 \text{ km} \quad M_{\text{Earth}} = 5.9736 \times 10^{24} \text{ kg}$$

$$1 \text{ AU} = 1.495978707 \times 10^{11} \text{ m} \quad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11} \text{ m} = 1.0000001124 \text{ AU} \approx 1.5 \times 10^{11} \text{ m} \approx 1 \text{ AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^8 \approx 109 \times R_{\text{Earth,equatorial}} \quad M_{\text{Sun}} = 1.9891 \times 10^{30} \text{ kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \quad p = \frac{F}{A} \quad p = p_0 + \rho g d_{\text{depth}}$$

$$\text{Pascal's principle} \quad p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \quad \Delta p = \Delta p_{\text{ext}}$$

$$\text{Archimedes principle} \quad F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$$

$$\text{equation of continuity for ideal fluid} \quad R_V = Av = \text{Constant}$$

$$\text{Bernoulli's equation} \quad p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$$

22 Oscillation

$$P = f^{-1} \quad \omega = 2\pi f \quad F = -kx \quad U = \frac{1}{2}kx^2 \quad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \quad P = 2\pi\sqrt{\frac{m}{k}} \quad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \quad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2y}{dt^2} \quad v = \sqrt{\frac{F_T}{\mu}} \quad y = f(x - vt)$$

$$y = y_{\text{max}} \sin[k(x - vt)] = y_{\text{max}} \sin(kx - \omega t) \quad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \quad k = \frac{2\pi}{\lambda} \quad v = f\lambda = \frac{\omega}{k} \quad P \propto y_{\text{max}}^2$$

$$y = 2y_{\text{max}} \sin(kx) \cos(\omega t) \quad n = \frac{L}{\lambda/2} \quad L = n\frac{\lambda}{2} \quad \lambda = \frac{2L}{n} \quad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \quad n\lambda = d\sin(\theta) \quad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \quad \beta = (10 \text{ dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} : n = 1, 3, 5, \dots \quad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \quad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T dS - p dV$$

$$T_K = T_C + 273.15 \text{ K} \quad T_F = 1.8 \times T_C + 32^\circ\text{F}$$

$$Q = mC\Delta T \quad Q = mL$$

$$PV = NkT \quad P = \frac{2}{3} \frac{N}{V} K_{\text{avg}} = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m v_{\text{RMS}}^2 \right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51 \dots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \quad 1 < \gamma \leq \frac{5}{3} \quad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P / \partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \quad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \quad \eta_{\text{heating, Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \quad \eta_{\text{cooling, Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

25 Electrostatics

$$\vec{F}_{1 \text{ on } 2} = \frac{kq_1q_2}{r_{1,2}^2} \hat{r}_{1,2} \quad \vec{F} = q\vec{E} \quad \vec{F} = q \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad \vec{E}(\vec{r}) = \frac{kq}{r^2} \hat{r} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \quad k = \frac{1}{4\pi\epsilon_0}$$

$$\vec{E}(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i) \quad \vec{E}(\vec{r}) = \int_{\text{all space}} \frac{k\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') d\vec{r}' \quad \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\vec{E}(\vec{r}) = \sum_i \frac{kq_i}{r^2} \left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r} \cos\theta_i + \dots \right)$$

$$q = \sum_i q_i \quad \vec{p} = \sum_i \vec{r}_i q_i \quad \vec{p} = 2q\vec{a} \quad \vec{E}(\vec{r}) = \frac{kq}{r^2} \hat{r} + k \left[\frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{r^3} \right]$$

$$\vec{E}(\vec{r}) = k \left[\frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{r^3} \right] = \frac{kp}{r^3} (2 \cos\theta \hat{r} + \sin\theta \hat{\theta})$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n} \quad \vec{\tau} = \vec{p} \times \vec{E} \quad U = -\vec{p} \cdot \vec{E}$$

$$\Delta U = q\Delta V \quad \Delta V = - \int_{\text{i}}^{\text{f}} \vec{E} \cdot d\vec{s} \quad dV = -\vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\nabla V \quad \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \quad V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|} \quad V(\vec{r}) = \int_{\text{all space}} \frac{k\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \quad V(\vec{r}) = \frac{k\vec{p} \cdot \hat{r}}{r^2}$$

$$U = \frac{1}{2} \sum_{i,j,i \neq j} \frac{kq_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2} \sum_i q_i V_i$$

$$U = \frac{1}{2} \int \int \frac{k\rho(\vec{r})\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\mathcal{V} d\mathcal{V}' = \frac{1}{2} \int \rho(\vec{r}) V(\vec{r}) d\mathcal{V}$$

$$C = \frac{Q}{V} \quad C = \frac{\varepsilon_0 A}{d} \quad C_{\text{parallel}} = \sum_i C_i \quad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \quad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \quad \oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

26 Current and Circuits

$$I = \frac{dq}{dt} \quad I = \int \vec{J} \cdot d\vec{A} \quad J = \frac{I}{A} \quad \vec{J} = nq\vec{v}_{\text{drift}} \quad \vec{J} = \sigma \vec{E} \quad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \quad \rho = \frac{1}{\sigma} \quad V = IR \quad R = \rho \frac{L}{A} \quad P = VI \quad P = VI = I^2 R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \quad 0 = \sum_i^{\text{loop}} \Delta V_i \quad R_{\text{series}} = \sum_i R_i \quad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \quad I = \frac{\Delta V_{\text{initial}}}{R} e^{-t/\tau} \quad \Delta V = \Delta V_{\text{final}} (1 - e^{-t/\tau})$$

27 Magnetic Fields and Forces

$$\mu_0 = [1.256 \, 637 \, 061 \, 27(20)] \times 10^{-6} \text{ N/A}^2 \approx 4\pi \times 10^{-7} \text{ N/A}^2 \quad u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{\mu_0} B^2$$

$$1 \text{ N/A}^2 = 1 \text{ T} \cdot \text{m/A} \quad 1 \text{ tesla (T)} = 10^4 \text{ gauss (G)}$$

$$\vec{F} = q\vec{v} \times \vec{B} \quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad d\vec{F} = I d\vec{s} \times \vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \quad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \quad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \quad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \quad \tau = \vec{\mu} \times \vec{B} \quad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2} \quad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad B_{\text{arc}} = \frac{\mu_0 I \theta}{4\pi r} \quad \vec{F}_{1 \text{ on } 2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r} \hat{r}_{2 \text{ to } 1}$$

$$B_{\text{solenoid}} = \mu_0 n I \quad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \quad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z} \quad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \quad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \quad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \quad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \quad \mathcal{E} = -L \frac{dI}{dt} \quad U_L = \frac{1}{2} L I^2 \quad u_B = \frac{1}{2\mu_0} B^2$$

$$L_{\text{series}} = \sum_i L_i \quad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \quad M = M_{12} = M_{21} \quad M_{\text{con-axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \quad I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) = I_{\infty} (1 - e^{-t/\tau_L}) \quad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad q = A \cos(\omega t) + B \sin(\omega t) \quad q = q_0 \cos(\omega t) \quad U = \frac{q_0^2}{2C}$$

$$\alpha_{\pm} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0 \cos(\omega t) + \frac{q_0}{\tau \omega} \sin(\omega t) \right] e^{-t/\tau} \quad \tau = \frac{2L}{R}$$

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0 \cos(\omega t) \quad I = I_0 \cos(\omega t - \phi) \quad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \quad \phi = \tan^{-1} \left[\frac{\omega L - 1/(\omega C)}{R} \right] \quad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad P_{\text{avg}} = \frac{1}{2} I_0 V_0 \cos \phi = I_{\text{rms}} V_{\text{rms}} \cos \phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \quad I_{\text{p}} N_{\text{p}} = I_{\text{s}} N_{\text{s}} \quad I_{\text{p}} \mathcal{E}_{\text{p}} = I_{\text{s}} \mathcal{E}_{\text{s}} \quad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} \quad \oint \vec{B} \cdot d\vec{A} = 0 \quad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \quad f(x, t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \quad c = f\lambda = \frac{\omega}{|k|} \quad \lambda = \frac{2\pi}{|k|} \quad k = \frac{2\pi}{\lambda} \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - \omega t) \quad \vec{B} = \vec{B}_0 \sin(kx - \omega t) \quad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \quad I = S_{\text{avg}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{cB_{\text{max}}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\epsilon E_{\text{max}}^2}{2} = \frac{B_{\text{max}}^2}{2\mu_0} \quad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685 \mu\text{m-K}}{T} \approx \frac{3000 \mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2 \theta \quad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$