Introductory Physics: Calculus-Based **Name:**

Homework 02_05: Electric Charge and Electric Fields: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Multiple Choice Questions:

---

022 qmult 00100 1 4 3 easy deducto-memory: clasical electromagnetism and Maxwell's equations

1. "Let's play *Jeopardy*! For \$100, the answer is: The equations that summarize most of classical electromagnetism:

$$1)\quad \nabla\cdot\vec E=\frac{\rho}{\varepsilon_0}\qquad 2)\quad \nabla\cdot\vec B=0\qquad 3)\quad \nabla\times\vec E=-\frac{\partial\vec B}{\partial t}\qquad 4)\quad \nabla\times\vec B=\mu_0\vec J+\mu_0\varepsilon_0\frac{\partial\vec E}{\partial t}$$

which are the vacuum form. To complete summary, one also needs the Lorentz force formula

$$\vec F=q\left(\vec E+\vec v\times\vec B\right)\ .$$

What are ____________, Alex?

a) Ehrenfest's equations b) Friedmann's equations c) Maxwell's equations
d) Maxwell's relations e) the Schrödinger and Dirac equations

SUGGESTED ANSWER: (c)

Wrong answers:
b) As Lurch would say AAAARGH.
d) Yes, they exist and are not the same as Maxwell's equations.

Redaction: Jeffery, 2008jan01

---

022 qmult 00150 1 1 1 easy memory: electric charge definition, incomplete
Extra keywords: physci

2. Electric charge is:

a) a fundamental property of matter: it comes in invariant quantized amounts of size $\pm e$.
b) a derived property of matter: it comes in somewhat variable quantized amounts of size $\pm e$ more or less.
c) a fundamental property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
d) a derived property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
e) amber.

SUGGESTED ANSWER: (a)

Wrong answers:
e) Forever Amber. Where it all began since the Greek word for amber mutated into electricity and electron.

Redaction: Jeffery, 2001jan01

---

022 qmult 00160 1 1 5 easy memory: Ben Franklin named charges
Extra keywords: physci KB-137

3. Who named positive and negative electric charge?

a) George Washington (1732–1799). b) John Adams (1735–1826).
c) Benedict Arnold (1741–1801). d) Aaron Burr (1756–1836).
e) Benjamin Franklin (1706–1790).

SUGGESTED ANSWER: (e) KB-137 and WP-513 confirm it was old Ben.

Wrong answers:
c) Oh, c'mon.

Redaction: Jeffery, 2001jan01

---

022 qmult 00170 1 1 1 easy memory: elementary charge 1
Extra keywords: physci KB-169-1

4. The absolute value of the smallest nonzero physical amount of electric charge (not considering quarks) is:

a) e or $1.602\ldots \times 10^{-19}$ C. b) 1 C. c) indefinitely small. d) indefinitely large.
e) indeterminate.

SUGGESTED ANSWER: (a)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

---

022 qmult 00200 1 4 5 easy deducto-memory: ordinary matter neutral
Extra keywords: physci

5. "Let's play *Jeopardy*! For $100, the answer is: It is usually approximately electrically neutral."

What is ____________, Alex?

a) an electron b) a proton c) highly charged matter
d) highly negatively charged matter e) ordinary, terrestrial matter above the atomic scale

SUGGESTED ANSWER: (e)

Wrong answers:
a) C'mon.

Redaction: Jeffery, 2001jan01

---

022 qmult 00210 1 4 5 easy memory: no microscopic charge neutralization
Extra keywords: physci

6. Why don't positive and negative charge largely neutralize each other at the microscopic level just like they mostly do at the macroscopic level? The short answer is it is forbidden in most circumstances by ____________.

a) classical physics b) James Clerk Maxwell c) Michael Faraday d) Ben Franklin
e) quantum mechanics

SUGGESTED ANSWER: (e): Quantum mechanics is formulated in terms of differential equations and a lot of other rules that have been found to work. When you have elementary positive and negative charges in proximity and they don't have the energy to escape each other and they don't annihilate each other (or undergo some other metamorphosis), they can form stable, charge-separated structures according to these quantum mechanical rules: e.g., atoms.

Wrong answers:
a) No it doesn't and the stability of atoms began to puzzle people at the end of the 19th century.

Redaction: Jeffery, 2001jan01

---

022 qmult 00220 1 1 3 easy memory: positively charged matter
Extra keywords: physci KB-169-9

7. Ordinary matter is positively charged whenever protons:

a) outnumber neutrons. b) outvote electrons. c) outnumber electrons.
d) are fewer than electrons. e) are absent.

SUGGESTED ANSWER: (c)

Wrong answers:
d) Exactly wrong.
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

---

022 qmult 00310 1 1 2 easy memory: electrons not permanently bound
Extra keywords: physci KB-169-7

8. Electrons in an atom are:

a) permanently bound to the atom. b) not permanently bound to the atom.
c) confined to the nucleus. d) more massive than the nuclues. e) green.

SUGGESTED ANSWER: (b)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

---

022 qmult 00320 1 1 5 easy memory: electron or proton removal
Extra keywords: physci KB-171-3

9. Which of electrons, protons, neutrons are easier to remove from an atom?

a) Protons. b) They are all irremovable. c) They are equally easily removable.
d) Neutrons. e) Electrons.

SUGGESTED ANSWER: (e)

Wrong answers:
b) Niaoow.

Redaction: Jeffery, 2001jan01

---

022 qmult 00330 2 3 3 moderate math: charge on 1 g of protons
Extra keywords: physci KB-173-1

10. Given the mass of a proton is approximately 1.67×10^{-24} g and the elementary charge is approximately 1.602×10^{-19} C, what approximately is the total charge of 1 g of protons?

a) 1.67×10^{-24} C. b) 1.602×10^{-19} C. c) 10^5 C. d) 2.5×10^{-43} C. e) zero.

SUGGESTED ANSWER: (c) Behold:

$$Q = Ne = \frac{M}{m_{\rm proton}} e = \frac{1}{1.67 \times 10^{-24}} \times 1.602 \times 10^{-19} \approx 10^5 \,\text{C} \;,$$

where N is the number of protons, $M = 1$ g, and $m_{\rm proton}$ is the mass of a proton.

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

---

022 qmult 00400 1 1 4 easy memory: conduction classes
Extra keywords: physci

11. There are four main charge conduction categories for materials:

a) insulator, conductor, demiconductor, orchestra conductor.
b) insulator, conductor, semiconductor, infraconductor.
c) insulator, conductor, semiconductor, Superman-conductor.
d) insulator, conductor, semiconductor, superconductor.

e) insulator, conductor, semiconductor, demisemiconductor.

SUGGESTED ANSWER: (d) We usually think only of metals as conductors. But ionic fluids and ionized gases can also conduct. Ionic fluids can be pure fluids, but the more strongly conducting ones have solutes in them.

Wrong answers:
a) Orchestra conductor?
c) The last one on trains is a very difficult man get away with fooling.

Redaction: Jeffery, 2001jan01

---

022 qmult 00420 1 4 1 easy deducto-memory: insulators defined
Extra keywords: physci

12. "Let's play *Jeopardy*! For $100, the answer is: These materials do **NOT** allow an electric current to flow through them easily."

What are ____________, Alex?

a) insulators b) conductors c) vacuum states d) solids e) crystals

SUGGESTED ANSWER: (a)

Wrong answers:
b) Exactly wrong

Redaction: Jeffery, 2001jan01

---

022 qmult 00510 1 1 4 easy memory: electromagnetism
Extra keywords: physci

13. The electricity and magnetism are now understood to be a united set of phenomena which we call:

a) magnoelectrism. b) magmaelectrism. c) electromagi-ism. d) electromagnetism.
e) electromagnanimousism.

SUGGESTED ANSWER: (d)

Wrong answers:
a) Makes sense anyway. But from a heuristic point of view one always starts with electric effects.

Redaction: Jeffery, 2001jan01

---

022 qmult 00520 1 4 1 easy deducto-memory: contact forces

14. What are contact forces?

a) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
b) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
c) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
d) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
e) The forces holding contact lenses in place. They're not for the squeamish.

SUGGESTED ANSWER: (a)

Wrong answers:
e) Those are contact forces and they arn't for the squeamish, but it's not a best answer.

Redaction: Jeffery, 2001jan01

---

022 qmult 00610 1 1 1 easy memory: Coulomb's law vector form

15. The formula

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, is ________ law.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

SUGGESTED ANSWER: (a)

Wrong answers:

e) Gauss's law is equivalent to Coulomb's law: each implies the other.

Redaction: Jeffery, 2001jan01

---

022 qmult 00620 1 1 1 easy memory: Coulomb's law scalar form

Extra keywords: physci

16. The formula

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} \ ,$$

where $k \approx 8.99\ldots \times 10^9\,\mathrm{N\cdot m^2/C^2}$, is ________ law in scalar form.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

SUGGESTED ANSWER: (a)

Wrong answers:

e) Gauss's law is equivalent to Coulomb's law: each implies the other.

Redaction: Jeffery, 2001jan01

---

022 qmult 00622 1 4 5 easy deducto-memory: Coulomb's constant

17. "Let's play *Jeopardy*! For \$100, the answer is:

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987\,551\,785\,972(14)\times10^9\,\mathrm{N\cdot m/C^2} \approx 8.99\times10^9\,\mathrm{N\cdot m/C^2} \approx 9\times10^9\,\mathrm{N\cdot m/C^2} \approx 10^{10}\,\mathrm{N\cdot m/C^2} \ ,$$

where ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, and c is the vacuum light speed. The k constant is an exact rational number because of the way the basic units are defined.

What is ____________, Alex?

a) Boltzmann's constant b) Planck's constant c) Avogadro's number
d) Fred's constant e) Coulomb's constant

SUGGESTED ANSWER: (e)

Wrong answers:

a) Boltzmann's constant also has the symbol k, but it's a different thing.
d) Fred's constant?

Redaction: Jeffery, 2008jan01

---

022 qmult 00630 1 1 5 easy memory: category of Coulomb's law

Extra keywords: physci KB-169-5

18. Coulomb's law is a:

a) societal law. b) law of thermodynamics. c) law of motion. d) conservation law.
e) force law.

SUGGESTED ANSWER: (e)

Wrong answers:

a) Oh, c'mon.

Redaction: Jeffery, 2001jan01

---

022 qmult 00640 1 1 2 easy memory: electric and gravity force inverse-square

Extra keywords: physci KB-140 sort of

19. The electrostatic force and gravity are both ____________ force laws.

a) inverse-linear b) inverse-square c) inverse-cube d) linear e) quadratic

SUGGESTED ANSWER: (b)

Wrong answers:

d) This is the Hooke's law force or the linear restoring force: $F = -kx$.

Redaction: Jeffery, 2001jan01

---

022 qmult 00650 1 1 2 easy memory: net charge and force

Extra keywords: physci KB-172-9 This is a simplified version.

20. For two **POINT** masses to exert a Coulomb force on each other:

a) both must be uncharged. b) both must be charged. c) one must charged and the other uncharged. d) their charges must both be greater than 1 C. e) their charges must both be less than 1 C.

SUGGESTED ANSWER: (b)

Bodies with structure that have no net charge can in many cases exert electric force and have electric net force exerted on them. The most obvious case is that of dipoles which have no net charge, but have positive and negative ends. More complex (i.e., higher) multipole structures can also interact via the electric force.

Wrong answers:

e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

---

022 qmult 00660 3 5 4 hard thinking: neutral finite body attract-repel

Extra keywords: physci-172-9 This is the hard version

21. Can two finite bodies each electrically neutral overall ever attract or repel each other?

a) No.
b) Always.
c) Always and no.
d) **YES**. For example, consider two small, non-conducting balls attached by a non-conducting bar: give one ball a positive charge (uniformly spread) and the other ball a negative charge (uniformly spread) of the same magnitude. The structure is an electric dipole that is overall neutral. Now consider a second identical dipole. Align the two with unlike ends closest and then with like ends closest. The distance between the balls is a fixed distance a in both cases. The force between the two unlike ends is attractive and between the like ends is repulsive. What of the other forces between the balls? We can make the bars as long as we like. The other forces between the balls get smaller and smaller as we make the bars longer and longer. Eventually the other forces become negligible and the closest ball forces dominate.
e) They can **REPEL**, but never **ATTRACT**. That is the valid conclusion of answer (d).

SUGGESTED ANSWER: (d)

The longest-answer-is-right rule triumphs again.

Wrong answers:

a) The right answer proves this is not the case.
b) Not always. Two neutral non-polarized objects would not attract or repel. Such objects are an idealization strictly speaking, but are approximately realized to high accuracy by most macroscopic objects most of the time.
c) A logical impossibility.
e) C'mon, why wouldn't there be attraction.

Redaction: Jeffery, 2001jan01

---

022 qmult 00710 2 3 1 moderate math: Coulomb's law in action
Extra keywords: physci KB-171-29

22. If the distance between charges is changed by a **MULTIPLICATIVE** factor of 1/4, the magnitude of electric force between the charges changes by a **MULTIPLICATIVE** factor of:

    a) 16. b) 4. c) 1. d) 1/4. e) 1/16.

    SUGGESTED ANSWER: (a)

    Wrong answers:
    e) As Lurch would say: "Aaaarh."

    Redaction: Jeffery, 2001jan01

---

022 qmult 00720 3 3 3 hard math: Coulomb and gravitational force
Extra keywords: physci KB-171-5

23. Coulomb's law and the point-mass gravitational formula in scalar form are, respectively:

$$F_{\mathrm{C}} = \frac{kq_1q_2}{r^2} \qquad \text{and} \qquad F_{\mathrm{G}} = \frac{Gm_1m_2}{r^2} ,$$

where $k = 8.98755179 \times 10^9\,\mathrm{N\,m^2/C^2}$, $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25), q stands for charge, m for mass, 1 and 2 for particles 1 and 2, and r is the distance between the particles. The mass of electrons is $9.1093826 \times 10^{-31}$ kg and their electric charge is $1.60217653 \times 10^{-19}$ C. Approximately, what is the ratio of the magnitude of gravitational force to the magnitude of the electrical force between two electrons (i.e., $F_{\mathrm{G}}/F_{\mathrm{C}}$) for any distance r?

a) 1. b) 10. c) 3×10^{-43}. d) 3×10^{43}. e) 10^{-19}.

SUGGESTED ANSWER: (c) The answer can actually be deduced with only a little math.
Behold

$$\frac{F_{\mathrm{G}}}{F_{\mathrm{C}}} = \frac{Gm_1m_2}{kq_1q_2} \approx \frac{7 \times 10^{-11}(10^{-30})^2}{10^{10} \times (1.6 \times 10^{-19})^2} \approx 3 \times 10^{-43} .$$

This is pretty small. Clearly, the electrical force between elementary charges is very dominant over the gravitational force. The gravitational force is negligible, in fact.

Wrong answers:
e) Gauss's law is equivalent to Coulomb's law: each implies the other.

Redaction: Jeffery, 2001jan01

---

022 qmult 00800 1 1 3 easy memory: polarization

24. Charge separation in a object is called:

    a) rasterization. b) pasteurization. c) polarization. d) north polarization.
    e) miniaturization.

    SUGGESTED ANSWER: (c) The object doesn't have to be neutral, but usually we think of neutral objects first when thinking of polarization.

    Wrong answers:
    a) Something computer system nerds go on about.
    d) Only by Santa Claus.

    Redaction: Jeffery, 2001jan01

---

022 qmult 00910 1 1 3 easy memory: two-ball induction charging 1

25. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged rod is brought from infinity to near B, but not near A. The balls are then separated and the rod is put back at infinity. The charges on A and B are, respectively:

    a) positive and positive. b) negative and negative. c) negative and positive.
    d) positive and negative. e) zero and zero.

    SUGGESTED ANSWER: (d)

When the rod is brought up to B, negative charge is attracted to the rod and positive charge is repelled. So B becomes a bit negative and A a bit positive. When the balls are separated, they can no longer exchange charge and so A remains positive and B becomes negative.

If we specify that the conductors are metals (which implicitly we have since we have treated them like metals), then in a microscopic description B acquires an excess of electrons and A has a deficiency. In metals, electrons are the charge carriers and are mobile. The positive charge is the protons in the atomic nuclei and that is essentially immobile in metals. In fluids and gases, there can be positive and negative ions both of which are mobile.

If the rod were removed before the balls were separated, then the attractive force between positive and negative would have quickly neutralized both balls.

Wrong answers:

a) This violates the conservation of charge.

Redaction: Jeffery, 2008jan01

---

022 qmult 00912 1 1 3 easy memory: two-ball induction charging 2

26. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged conducting rod is brought from infinity to near B, but not near A. The rod touches B. Then the rod is then removed and the balls are separated. The charges on A and B are ____________.

a) both positive b) both negative c) negative and positive d) positive and negative e) both zero.

SUGGESTED ANSWER: (a)

The positive charge on the rod is self-repellent. It will try to spread out as far a it can and thus will spread to both balls A and B. So both A and B get positively charged. Removing the rod and separating the balls doesn't change this.

If we specify that the conductors are metals (which implicitly we have since we have treated them like metals), then in a microscopic description the rod has a deficiency of electrons and electrons from A and B flow into the rod. The positive charge is the protons in the atomic nuclei and that is essentially immobile in metals. In fluids and gases, there can be positive and negative ions both of which are mobile.

If the rod were removed before the balls were separated, then the attractive force between positive and negative would have quickly neutralized both balls.

Wrong answers:

a) This violates the conservation of charge.

Redaction: Jeffery, 2008jan01

---

023 qmult 00100 1 4 1 easy deducto-memory: electric field definition

Extra keywords: physci

27. The electric field is a:

a) vector field that emanates from charge (but not only charge) and causes the electric force.
b) scalar field that emanates from charge (but not only charge) and causes the electric force.
c) scalar field that emanates from current (but not only current) and causes the electric force.
d) vector field that emanates from current (but not only current) and causes the magnetic force.
e) scalar field that emanates from current (but not only current) and causes the magnetic **FARCE**.

SUGGESTED ANSWER: (a) This is not a complete definition. Electric fields can be created by time-varying magnetic fields. Charge does not have to be present in an immediate sense.

Wrong answers:

e) All things are wrong.

Redaction: Jeffery, 2001jan01

---

023 qmult 00150 1 1 3 easy memory: electric field line

Extra keywords: physci

28. A curve through space that is tangent to the electric field vector at each point is a/an:

a) streamline. b) stream curve. c) electric field line. d) magnetic field line.
e) straight line always.

SUGGESTED ANSWER: (c)

Wrong answers:
a) Streamlines turn up in fluid dynamics.
e) Electric field lines emanating from a point charge are straight lines. But in general electric field are curved.

Redaction: Jeffery, 2001jan01

---

023 qmult 00200 1 4 5 easy deducto-memory: E-force on charge
29. The force on a point charge q caused by an electric field $\vec{E}$ is given by:

a) $\vec{F} = m\vec{a}$. b) $\vec{F} = \dfrac{\vec{E}}{q}$. c) $\vec{\tau} = \vec{p} \times \vec{E}$. d) $\vec{F} = \dfrac{\left(\vec{E}\right)^2}{q}$. e) $\vec{F} = q\vec{E}$.

SUGGESTED ANSWER: (e)

Wrong answers:
d) Not even a proper vector equation.

Redaction: Jeffery, 2001jan01

---

023 qmult 00202 1 1 3 easy memory: electric force and a charge distribution
30. Say that you have charge distribution of point charges with net charge q in an external electric field $\vec{E}$. Since it is external, $\vec{E}$ does not include contributions from the charge distribution. Also $\vec{E}$ is **NOT** necessarily uniform. The electric force on the distribution due to $\vec{E}$ is

$$\vec{F} = \sum_i q_i \vec{E}_i \ ,$$

where q_i is the charge of one member of the distribution and $\vec{E}_i$ is the external electric at the location of charge q_i. The force $\vec{F}$ is ____________ on the distribution and in the absence of any other net external force dictates ____________ of the distribution.

a) the net electric force; individual charge accelerations
b) not the net electric force; center-of-mass acceleration
c) the net electric force; center-of-mass acceleration
d) not the net electric force; individual charge accelerations
e) half the net electric force; individual charge accelerations

SUGGESTED ANSWER: (c)

The internal electrical forces cancel out pairwise and so the external electric field on the distribution is the net electrical field.

Do all the internal forces cancel out pairwise. By Newton's 3rd law they should and we assume that law is obeyed in many cases. But actually Newton's 3rd law is **NOT** always obeyed between matter particles. This is a fine point that is often skipped over in intro physics classes. Magnetic forces can violate it for swarm of free-flying charges for example (Go-8). But nonetheless once one includes the mass and momentum of the electromagnetic field in the total mass and momentum of the system, it still turns out to be true that $\vec{F}_{\text{net external force}} = ma_{\text{CM}}$—or at least so I believe.

Now what about real point charges? What of their internal forces? Well maybe they don't really exist. But the electron is a point charge as far as we know. So what about the electrons internal electric forces? Are they infinite? There's a special place on the far side of the River Styx for people who ask such questions.

Wrong answers:
e) Half?

Redaction: Jeffery, 2008jan01

---

023 qmult 00210 1 1 4 easy memory: electromagnetic field

31. A self-propagating electromagnetic field is:

a) a static electric field. b) a static magnetic field. c) a grass field. d) light. e) sound.

SUGGESTED ANSWER: (d)

Wrong answers:
c) C'mon.

Redaction: Jeffery, 2001jan01

---

023 qmult 00220 1 1 1 easy memory: point charge electric field

32. The electric field about a point charge or any spherically symmetric charge distribution is given by

$$\vec{E} = \frac{kq}{r^2}\hat{r} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, r is the radius from the center of system, q is the charge inside of a shell of size r, and $\hat{r}$ is a/an:

a) radially outward pointing unit vector. b) a radially inward pointing unit vector. c) an azimuthal angle unit vector. d) a polar angle unit vector. e) an r with tilde on it.

SUGGESTED ANSWER: (a)

Wrong answers:
e) It's a hat, not a tilde. An r with at tilde is $\tilde{r}$.

Redaction: Jeffery, 2001jan01

---

023 qmult 00230 1 4 4 easy deducto-memory: E-field for arbitrary charge distribution

33. "Let's play *Jeopardy*! For \$100, the answer is: The formula for a discrete set of charges is

$$E(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \ ,$$

where the sum is over all charges q_i, $\vec{r}_i$ is the charge location, $\vec{r}$ is the point of evaluation, and $\vec{r}$ is the point of evaluation."

What is the discrete charge electric field formula for ____________, Alex?

a) an overall neutral charge distribution b) a point charge c) a dipole charge distribution. d) an arbitrary charge distribution e) a huge charge distribution

SUGGESTED ANSWER: (d)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

---

023 qmult 00232 1 1 3 easy memory: changing charge signs

34. Say you had a static distribution of charge that gave rise to an electric field $\vec{E}$ and a test charge q (which not counted in the distribution). Say you magically changed all the charge signs in the distribution, but not q sign. The force on the charge is now ____________. Now say you magically changed all the charge signs including changing q to $-q$. The force on the charge is now ____________. Note we do **NOT** redefine the values assigned to the symbols $\vec{E}$ and q.

a) $q\vec{E}$; $q\vec{E}$ b) $-q\vec{E}$; $-q\vec{E}$ c) $-q\vec{E}$; $q\vec{E}$ d) $q\vec{E}$; $-q\vec{E}$ e) $-\vec{E}$; $\vec{E}$

SUGGESTED ANSWER: (c)

Wrong answers:
e) Dimensionally incorrect.

Redaction: Jeffery, 2008jan01

---

023 qmult 00240 1 1 2 easy memory: far field limit

35. The electric field around a localized distribution of charge (with a non-zero net charge) in the far-field limit looks like the electric field of a/an:

a) dipole. b) point charge. c) infinite plane of charge. d) infinite cylinder of charge. e) infinite line of charge.

SUGGESTED ANSWER: (b)

Wrong answers:

a) A dipole field dominates in the far-field limit only if the distribution has zero net charge and a dipolar component. The dipole field falls off like $1/r^3$ and has a complicated angular pattern. A point charge field falls off like $1/r^2$ and the field vectors always point radially inward or outward.

c) Infinite distributions never look like point charges in the far-field because you can never get far enough away to make them approximately a point.

Redaction: Jeffery, 2001jan01

---

023 qmult 00300 1 1 1 easy memory: electric dipole moment definition

36. The quantity

$$\vec{p} = \int \vec{r}\rho(\vec{r}\,)\,dV$$

for a continuous distribution (and where the integral is over the whole distribution) or

$$\vec{p} = \sum_i \vec{r}_i q_i$$

for a discrete distribution (and where the sum is over the whole distribution) is called:

a) an electric dipole moment. b) an electric monopole moment.
c) a magnetic dipole moment. d) a magnetic monopole moment.
e) an electric field moment.

SUGGESTED ANSWER: (a)

Wrong answers:

d) In some theories, these should exist, but none have ever been observed.

e) Woe, WOE.

Redaction: Jeffery, 2008jan01

---

023 qmult 00320 1 4 2 easy deducto-memory: dipole moment electric field

37. "Let's play *Jeopardy*! For \$100, the answer is:

$$E(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] \ ."$$

What is the ____________ moment electric field in the far-field limit, Alex?

a) monopole b) dipole c) quadrupole d) octupole e) hexacontatetrapole

SUGGESTED ANSWER: (b) (e.g. Wikipedia: Dipole)

Some of the orders of a multipole expansion have standard names: 0th order (monopole), 1st order (dipole), 2nd order (quadrupole), 3rd order (octupole), and 6th order (hexacontatetrapole). I don't know about the other orders.

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

---

023 qmult 00330 1 1 3 easy memory: dipole far-field

38. The far-field electric field of an electric dipole falls off as:

    a) $1/r$. b) $1/r^2$. c) $1/r^3$. d) $1/r^4$. e) $1/r^5$.

    SUGGESTED ANSWER: (c) The far-field electric field of an electric dipole falls off as: $1/r^3$.

    Wrong answers:
    a) A line charge.
    b) A point charge or monopole.
    d) A quadrupole.
    e) An octupole.

    Redaction: Jeffery, 2001jan01

---

023 qmult 00340 1 1 1 easy memory: dipole electric field description

39. The pattern of the electric field lines about a dipole viewed in a cross section plane in which the dipole axis lies is sort of:

    a) butterfly-like. b) ladybug-like. c) dragonfly-like. d) aphid-like.
    e) praying-mantis-like.

    SUGGESTED ANSWER: (a)

    Wrong answers:
    d) What's an aphid look like?

    Redaction: Jeffery, 2001jan01

---

023 qmult 00350 1 1 2 easy memory: dipoles in nature

40. The permanent or induced dipoles at the microscopic level are:

    a) uncommon. b) common. c) never found. d) never found on Earth.
    e) never found in space.

    SUGGESTED ANSWER: (b)

    Wrong answers:
    a) There common as, as ... whatever.

    Redaction: Jeffery, 2001jan01

---

023 qmult 00360 1 1 3 easy memory: electric dipole torque

41. The torque on an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$ is:

    a) $U = \vec{p} \cdot \vec{E}$. b) $U = -\vec{p} \cdot \vec{E}$. c) $\tau = \vec{p} \times \vec{E}$. d) $\tau = -\vec{p} \times \vec{E}$. e) $\tau = -\vec{p} \cdot \vec{E}$.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    b) This the potential energy of a dipole in an electric field.
    d) The negative sign means the torque is trying to disalign the dipole from the field direction: the opposite is the case.

    Redaction: Jeffery, 2001jan01

---

023 qmult 00370 1 1 2 easy memory: electric dipole potential energy

42. An electric dipole (with moment $\vec{p}$) in an electric field $\vec{E}$ has a potential energy of alignment:

    a) $U = \vec{p} \cdot \vec{E}$. b) $U = -\vec{p} \cdot \vec{E}$. c) $\tau = \vec{p} \times \vec{E}$. d) $\tau = -\vec{p} \times \vec{E}$. e) $\tau = -\vec{p} \cdot \vec{E}$.

    SUGGESTED ANSWER: (b) The potential energy is lowest for complete alignment. Complete alignment is a point of stable equilibrium: i.e., the torque is zero for alignment and the torque of electric field opposes all perturbations.

    Wrong answers:
    a) The potential energy is lowest for complete alignment: this answer says it is highest for complete alignment. the dipole from the field direction: the opposite is the case.

Redaction: Jeffery, 2001jan01

---

023 qmult 00380 1 1 1 easy memory: electric dipole simple harmonic motion

43. The equation of angular motion for a rigid dipole (of dipole moment $\vec{p}$ and rotational inertia I) in a uniform electric field $\vec{E}$ is

$$I\alpha = I\frac{d^2\theta}{dt^2} = -pE\sin\theta \ ,$$

where θ is the angle of the dipole moment from the electric field direction. This innocent-looking differential equation (DE) has no exact analytic solution—that gosh-darn transcendental function of the solution θ is pretty intractable. But one can make the small angle approximation

$$\sin\theta \approx \theta$$

(with θ in radians of course) which is exact to 2nd order in small θ since the Taylor's expansion for $\sin\theta$ is

$$\sin\theta = \sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{\theta^{2\ell+1}}{(2\ell+1)!}$$

(Ar-264). The angle θ in radians is larger than $\sin(\theta)$ at 10° by 0.51 %, 20° by 2.1 %, 30° by 4.7 %, 45° by 11 %, and 60° by 21 %. So the small angle approximation is pretty good even for not-so-small angles. With the small angle approximation, the DE becomes

$$I\frac{d^2\theta}{dt^2} = -pE\theta$$

which is the simple harmonic oscillator DE. The general solution of this linear 2nd order DE (linear because the solution function and its derivatives appear linearly and 2nd order because the highest derivative is 2n order) is

$$\theta = A\cos(\omega t) + B\sin(\omega t) \ ,$$

where ω here is not the angular velocity (not $d\theta/dt$), but the angular frequency and is given by

$$\omega = \sqrt{\frac{pE}{I}} \ .$$

We see that the dipole oscillation is simple harmonic oscillation in the small angle approximation. The period of the solution is

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}} \ .$$

The period increases as I increases (increasing resistance to angular acceleration) and decreases as pE increases (increasing torque magnitude per angular displacement from equilibrium). The constants A and B are independent of ω and P and set by the initial conditions. Say at time $t = 0$, the dipole is at $\theta = 0$ and has angular velocity $\omega\theta_0$. The solution for this special case is:

a) $\theta_0\sin(\omega t)$. b) $\theta_0\cos(\omega t)$. c) $\omega\theta_0\sin(\omega t)$. d) $(\theta_0/\omega)\sin(\omega t)$. e) ωt.

SUGGESTED ANSWER: (a)

The solution

$$\theta = \theta_0\sin(\omega t)$$

has $\theta = 0$ and time $t = 0$ Its angular velocity is

$$\theta' = \omega\theta_0\cos(\omega t) \ ,$$

and so its angular velocity at time $t = 0$ is $\omega\theta$. Thus, the solution matches the initial conditions, and thus is the solution we desire.

Wrong answers:

e) Oh, c'mon.

Fortran-95 Code

```
      print*
!      pi=acos(-1.d0)
!      raddeg=180.d0/pi
      thetaa(1)=10.d0/raddeg
      thetaa(2)=20.d0/raddeg
      thetaa(3)=30.d0/raddeg
      thetaa(4)=45.d0/raddeg
      thetaa(5)=60.d0/raddeg
      print*,'i,thetaa(i)*raddeg,thetaa(i),sin(thetaa(i)),del'
      do i=1,5
       del=100.d0*(thetaa(i)-sin(thetaa(i)))/sin(thetaa(i))
       print925,'!',i,thetaa(i)*raddeg,thetaa(i),sin(thetaa(i)),del
      end do
  925 format(a1,i5,f10.2,2f10.4,f10.2)
! i,thetaa(i)*raddeg,thetaa(i),sin(thetaa(i)),del
!    1     10.00    0.1745    0.1736      0.51
!    2     20.00    0.3491    0.3420      2.06
!    3     30.00    0.5236    0.5000      4.72
!    4     45.00    0.7854    0.7071     11.07
!    5     60.00    1.0472    0.8660     20.92
```

Redaction: Jeffery, 2008jan01

Full Answer Problems:

---

022 qfull 00100 1 3 0 easy math: elementary charges in a coulomb

44. A typical current is 1 A (A for ampere) or 1 Coulomb per second. Given that the elementary charge is $e = 1.602 \times 10^{-19}$ C, how many elementary charges flow past a point in 1 second if the current is 1 A?

SUGGESTED ANSWER: The charge flow is $I\,dt = q$. In this case 1 A for 1 second gives 1 coulomb of charge. The number of charges to flow past is thus

$$\frac{1\,\mathrm{C}}{e} = \frac{1\,\mathrm{C}}{1.602\ldots \times 10^{-19}\,\mathrm{C}} \approx 0.6 \times 10^{19} \ .$$

Redaction: Jeffery, 2001jan01

---

022 qfull 00110 2 3 0 moderate math: elementary charge disparity

45. A copper penny weighs about 2.5 g (by actual, but not terribly accurate, measurement with a mail weighing scale). It is a pure copper penny, not one of those new-fangled things with zinc cores.

a) Given that copper's atomic mass is 63.5 in atomic mass units (i.e., in units of 1.66×10^{-27} kg), how many copper atoms are there in the penny? **HINT:** If you have 10 kg of eggs and each egg weighs 50 g, then you how many eggs?

b) Given that a copper atom has 29 protons (i.e., atomic number Z=29), how much gross positive charge in Coulombs does the penny have?

c) Imagine (contrafactually as they say in Washington) that the absolute value of the elementary proton charge was 1.000001 times larger than the absolute value of the elementary electron charge (i.e., there was a fractional charge difference of $(\delta e/e) = 10^{-6}$), what would be the force between two pennies a distance 1 meter apart assuming that the pennies would be neutral if there were no fractional charge difference.

d) Is a charge difference between proton and electron of the order suggested in the part (c) question likely? Explain.

SUGGESTED ANSWER:

a) The number of copper atoms is

$$N = \frac{m_{\rm penny}}{A_{\rm cu} m_{\rm amu}} = \frac{2.5 \times 10^{-3}}{63.5 \times 1.66 \times 10^{-27}} = 2.37 \times 10^{22} \ .$$

And by the way about those eggs, one has

$$\frac{10\,{\rm kg}}{50\,{\rm g}} = \frac{10000\,{\rm g}}{50\,{\rm g}} = 200 \qquad \text{eggs.}$$

b) The number of protons and electrons is $NZ = 6.88 \times 10^{23}$. The gross positive charge is $NZe = 1.10 \times 10^5\,{\rm C}$.

c) The net positive charge on a penny would be

$$q_{\rm net} = NZe\left(\frac{\delta e}{e}\right) = 0.110\,{\rm C} \ .$$

The net force between two pennies (treated as point charges) would be

$$F = \frac{k q_{\rm net}^2}{r^2} \approx 1.09 \times 10^8\,{\rm N} \ .$$

d) The force found in the part (c) answer is absurd of course. The weight of a kilogram near the Earth's surface is only about 10 N. We see no measurable long range forces between pennies at all. In fact, macroscopic structure might be impossible if the relative proton-electron charge difference were anywhere near as large as 10^{-6}. The proton-electron charge difference if it exists must be much smaller. At present, there is no experimental evidence for a charge difference at all. The universe would be very strange if there were a significant charge difference.

Fortran Code

```
          print*
          amu=1.66e-27
          xmpenny=2.5e-3
          ampenny=63.5
          acount=xmpenny/(amu*ampenny)
          echarge=1.602e-19
          anpenny=29.
          aproton=anpenny*acount
          charge=aproton*echarge
          qnet=charge*1.e-6
          xkcon=8.99e+9
          force=xkcon*qnet**2/1.**2
          print*,'acount,aproton,charge,qnet,force'
          print*,acount,aproton,charge,qnet,force
* 2.37169149E+22  6.87790528E+23  110184.047  0.110184044  109143304.
```

Redaction: Jeffery, 2001jan01

---

022 qfull 00200 3 3 0 tough math: a vector calculation with charges

46. Four point charges are set at the corners of a square 2 meters on a side. Going clockwise from the upper left hand corner, the charges are labeled A (1 C), B (2 C), C (-3 C), and D (4 C). What is the net force exerted on charge B? Remember to specify a direction. **HINTS:** Pick a convenient set of axes and compute the force components along those axes. The components are just scalars. Recall Coulomb's law for the force due to particle 1 on particle 2 is:

$$\vec F_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat r_{12} \ ,$$

where $k = 8.987\ldots \times 10^9\,{\rm N\,m^2/C^2}$ and $\hat r_{12}$ points from particle 1 to particle 2. Draw a diagram.

SUGGESTED ANSWER: I choose the x-axis to be parallel to the AB and DC lines and the y-axis to be parallel to the DA and CB lines. First, we sum the components:

$$F_x = \sum_i F_{x,i} = k \times \left(\frac{1 \times 2}{4} + \frac{2 \times 4}{8} \frac{1}{\sqrt{2}} \right) \approx 1.08 \times 10^{10}$$

and

$$F_y = \sum_i F_{y,i} = k \times \left(-\frac{3 \times 2}{4} + \frac{2 \times 4}{8} \frac{1}{\sqrt{2}} \right) \approx -0.71 \times 10^{10} ,$$

where we have used the fact that $\cos(45^\circ) = \sin(45^\circ) = 1/\sqrt{2}$. Then determine magnitude and direction:

$$F_{\rm net} = \sqrt{F_x^2 + F_y^2} = 1.30 \times 10^{10}\,{\rm N}$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x} \right) \approx \tan^{-1}\left(\frac{-0.71}{1.08} \right) \approx -33^\circ .$$

Because of the ambiguity of the inverse tangent (arctangent) function, one should add π if $F_x < 0$ and $F_y > 0$: i.e., if the force is in the S quadrant of CAST labeling quadrant scheme. On the other hand, one should subtract π if $F_x < 0$ and $F_y < 0$: i.e., if the force is in the T quadrant of CAST labeling quadrant scheme. But in this case, I didn't need to do either of those things.

It's actually very difficult to accumulate a net charge of order a Coulomb. It causes enormous forces as one can see.

Fortran-95 Code

```
          print*
          xk=1.d0/(pi4*permittivity)
          fx=xk*( 1.d0*2.d0/4.d0 + (2.d0*4.d0/8.d0)/sqrt(2.d0) )
          fy=xk*(-3.d0*2.d0/4.d0 + (2.d0*4.d0/8.d0)/sqrt(2.d0) )
          f=sqrt(fx**2+fy**2)
          theta=atan(fy/fx)*raddeg
          print*,'xk,fx,fy,f,theta'
          print*,xk,fx,fy,f,theta
! 8.987551787997911E+9 1.0848934709557558E+10 -7.126168866438265E+9
! 1.2980048807505955E+10 -33.299080491846225
```

Redaction: Jeffery, 2001jan01

---

022 qfull 00210 3 3 0 easy math: 3 charges on a line and equilibrium

Extra keywords: Serway-667-8, suitable for group work

47. Imagine three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and is located **BETWEEN** the two other charges.

a) Is there an equilibrium position for charge q if q_1 and q_2 have different signs? Why or why not?

b) Find the equilibrium position solutions x for charge q. Mathematically what happens to these solutions if q_1 and q_2 have different signs? Mathematically what happens to these solutions if $q_1 = q_2$?

c) Which solution from part (b) is physically allowed by the setup conditions. To save tedium just consider the cases where charges q_1 and q_2 are both greater than 0.

d) When is the equilibrium position stable and when is it unstable. **HINT:** A word argument gives the answer easily.

SUGGESTED ANSWER:

a) No. If the charges have different signs, then charge q will be attract to one of charge q_1 and and charge q_2 and repeled by the other one. Thus, the both forces charges q_1 and q_2 point the same way and those forces never cancel.

b) For the equilibrium case, we have

$$F = k\left[\frac{q_1}{x^2} - \frac{q_2}{(a-x)^2}\right] q = 0 \ ,$$

where q_1 force points formally to the right and q_2 force points formally to the left: the directions actually reverse if $q_1 q$ and $q_2 q$ are negative. We must solve for x. Behold:

$$\begin{aligned}
0 &= \frac{q_1}{x^2} - \frac{q_2}{(a-x)^2} \\
0 &= q_1(a-x)^2 - q_2 x^2 \\
0 &= (q_1 - q_2)x^2 - 2aq_1 x + q_1 a^2 \\
x &= \frac{aq_1 \pm \sqrt{(aq_1)^2 - (q_1 - q_2)q_1 a^2}}{q_1 - q_2} \\
&= a\left(\frac{q_1 \pm \sqrt{q_1 q_2}}{q_1 - q_2}\right) \ ,
\end{aligned}$$

where we have solved a quadratic using the quadratic formula.

If the signs of charges q_1 and q_2 are different there is no real solution since the discriminant is negative. This mathematically result is consistent with what physically must be the case as we argued in the part (a) answer.

If the signs of $q_1 = q_2$, then we didn't actually have a quadratic to solve. The solution in this case is

$$x = \frac{a}{2}$$

which follows from our development above and also just from symmetry: where else could the equilibrium be in this case.

c) Except for the $q_1 = q_2$, we have two solutions for the equilibrium position. But actually only one satisfies our set up conditions: i.e., is a solution that lies between charges q_1 and q_2. Physically, we know that there can be only one equilibrium between charges q_1 and q_2. The force magnitude of one of these charges always grows strong and the other always weaker as one moves away from equilibrium in either direction as lone as one stays between the charges. Thus, there is only one equilibrium point between charges q_1 and q_2 where the forces of charges q_1 and q_2 cancel.

So which of the two quadratic solutions is the physical solution? Well, it depends. It depends on the sign and relative sizes of charges q_1 and q_2. There are four cases.

First, say $q_1 > 0$ and $q_2 > 0$ and $q_1 > q_2$. Consider

$$\frac{q_1 \pm \sqrt{q_1 q_2}}{q_1 - q_2} \ .$$

This ratio has to be in the range $(0, 1)$ to be give a physical solution. The upper case is obviously greater than 1, and so is ruled out. The lower case is

$$\frac{q_1 - \sqrt{q_1 q_2}}{q_1 - q_2} \ .$$

Now $\sqrt{q_1 q_2} < q_1$, and so the numerator is positive. And $\sqrt{q_1 q_2} > q_2$, and so the numerator is smaller than the denominator. Thus the lower case is in the allowed range, and gives the physical solution.

Second, say $q_1 > 0$ and $q_2 > 0$ and $q_1 < q_2$. In this case, let's rewrite the ratio

$$\frac{\pm\sqrt{q_1 q_2} - q_1}{q_2 - q_1} \ .$$

In this case, the lower case makes the numerator negative and so must be ruled out. The lower case is

$$\frac{\sqrt{q_1 q_2} - q_1}{q_2 - q_1} \ .$$

Now $\sqrt{q_1q_2} > q_1$, and so the numerator is positive. And $\sqrt{q_1q_2} < q_2$, and so the numerator is smaller than the denominator. Thus the lower case is in the allowed range, and gives the physical solution.

The third and fourth situation with $q_1 < 0$ and $q_2 < 0$ are straightforward, but we've done enough.

d) If all the charges have the same sign, equilibrium is stable. If q is displaced from equilibrium, the force of the charge it is moving away from weakens in magnitude and the force of the charge it is approaching strengthens in magnitude. Since both forces are repulsive when all, the net force will tend to push the charge back to the equilibrium point. This seems the equilibrium is stable. Small displacements from equilibrium give rise to a restoring force.

If charge q has a different sign from charges q_1 and q_2, then the equilibrium is unstable. Again if q is displaced from equilibrium, the force of the charge it is moving away from weakens in magnitude and the force of the charge it is approaching strengthens in magnitude. But now since the forces are attractive, the net force will tend to pull the charge away from the equilibrium. In fact, if no other forces act the charge will head off toward the charge it is most attracted to and will collide with it. At collision there will be infinite attractive force. Our classical point charge assumption breaks down before this catastrophe eventuates. Since displacement from equilibrium leads to a net force tending to pull q farther from equilibrium, the equilibrium is unstable.

Once electrical potential has been introduced, it is pretty easy determine the stability of the equilibrium in this case. If the charges are all the same sign, the equilibrium is at the bottom of a potential energy well and is stable. If charge q has a different sign, equilibrium is at the top of a potential energy hill and is unstable.

Redaction: Jeffery, 2008jan01

---

022 qfull 00212 3 3 0 tough math: 3 charges on a line and equilibrium 2

Extra keywords: too tough for students? Needs reworking?

48. Image three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and can be located anywhere except at $x = 0$ and $x = a$. This question concerns itself with the equilibrium positions for q along the line. We don't bother with the trivial equilibrium positions are $x = \pm\infty$ in the discussion.

a) Qualitatively where is the (Coulomb force) equilibrium position for charge q if q_1 and q_2 have the same sign? Can there be more than one equilibrium? Is the equilibrium stable or not? Explain your answers.

b) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| > |q_2|$? Explain your answer.

c) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| < |q_2|$? Explain your answer.

d) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| = |q_2|$? Explain your answer.

e) The trouble with find the equilibrium positions in general for the three-charges on a line case, is that the direction of the force changes when one passes a charge. If it wasn't for that, the equation would be a simple quadratic. After a lot of thought, I think a good way to formulate the equation to solve as

$$F = k\left[\frac{d_1q_1}{x^2} - \frac{d_2q_2}{(a-x)^2}\right]q = 0 ,$$

where $d_1 = -1$ for q to the negative of q_1 and positive otherwise and $d_2 = 1$ for q to the negative of q_2 and positive otherwise. This simplifies to

$$\frac{1}{y^2} - \frac{Q}{(1-y)^2} = 0 ,$$

where $y = x/a$ and $Q = (d_2q_2)/(d_1q_1)$. Find the general solutions for y. What is a necessary condition on Q for a physically real solutions? What happens if $q_1 = q_2$ or $q_2 = 0$ (which actually violates are givens for the problem)?

f) Given that q_1 and q_2 have the same signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

f) Given that q_1 and q_2 have different signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

SUGGESTED ANSWER:

a) If the charges have the same sign the equilibrium position for q is between the charges. Whether they both attract or both repel q, they can only cancel if q lies between them.

There is only one equilibrium since as one moves away from q_1 and toward q_2, charge q_1's force magnitude only increases and q_2's only decreases. There can only be one point where they cancel.

The equilibrium is stable if all three charges have the same sign. Remember the individual Coulomb forces fall off with distance from the charges. So if charge q has the same sign as q_1 and q_2 and moves off equilibrium, the charge it moves toward pushes on it toward equilibrium with a force of greater magnitude than the other charge's force which is pushes on it away from equilibrium. If charge q has a different sign from q_1 and q_2 and moves off equilibrium, the charge it moves toward pushes on it away from equilibrium with a force of greater magnitude than the other charge's force which is pushes on it toward equilibrium.

The stability of the equilibria are all much easier to determine using potential plots for system.

b) If q_1 and q_2 have the different signs and $|q_1| > |q_2|$, then equilibrium lies to the positive of q_2. Between the charges q_1 and q_2, the charges push in the same direction and no equilibrium is possible. So the equilibrium must be to the positive of q_2 or to the negative of charge q_1. Now if $|q_1| > |q_2|$, then only to the positive of q_2 can the force of q_2 be equal and magnitude to q_1 force and point opposite to the q_1's force. There must be such a point of balanced forces Now to the positive of q_2, the inverse-square law nature of Coulomb force means that q_2's force must have a greater magnitude than q_1's force when one is close enough to q_2 and thus the net force points in the direction of the charge q_2 force. On the other hand far to the positive of q_2, the two charges look like a point charge of $q_1 + q_2$ which has the sign of charge q_1, and thus creates a force that points opposite to the force of q_2. There must be such a point of balanced forces (i.e., equilibrium) to equilibrium to the positive of q_2.

Now to go beyond the formal answer for the edification of the students.

Is there only one equilibrium to the positive of q_2? Well it seems likely that there is only one since the Coulomb force law falls off in a simple way without oscillations. But there seems no conclusive absolutely simple to show this. One needs some mathematical demonstration and that we do below.

Is the equilibrium stable? Well if q has the same charge from q_2, then if it moves toward q_2 the net force pushes away from q_2 and if it moves away from q_2 the net force pushes it toward q_2. So in this case the equilibrium is stable. But if q has the opposite charge from q_2, then the reverse situation holds, and the equilibrium is unstable.

c) If q_1 and q_2 have the different signs and $|q_2| > |q_1|$, then equilibrium lies to the negative of q_1. There is only one equilibrium and it is stable if q has the same sign as q_1 and unstable otherwise. The arguments are all the same as in the part (b) answer *mutatis mutandis*.

d) This is the simple dipole case and there is no equilibrium. Between the charges the individual forces of the charges always add up and never cancel. To the negative of q_1 the magnitude of the force of the q_1 charge is always larger than that of q_2 because q_1 is closer than q_2. To the positive of q_2 the magnitude of the force of the q_2 charge is always larger than that of q_1 because q_2 is closer than q_1.

e) Behold:

$$
\begin{aligned}
0 &= \frac{1}{y^2} - \frac{Q}{(1-y)^2} \\
0 &= (1-y)^2 - Qy^2 \\
0 &= (1-Q)y^2 - 2y + 1
\end{aligned}
$$

$$y = \frac{2 \pm \sqrt{4y^2 - 4(1-Q)}}{2(1-Q)}$$
$$= \frac{1 \pm \sqrt{Q}}{1-Q} \ .$$

where we have solved a quadratic using the quadratic formula.

So the general solutions are

$$y = \frac{1 \pm \sqrt{Q}}{1-Q} \ .$$

There are no physically real (or mathematically real) solutions if $Q < 0$.

If $q_1 = q_2$, we actually never had a quadratic for the physically real case of $Q > 0$ since then the equation to solve was

$$0 = -2y + 1$$

which gives

$$y = \frac{1}{2} \ .$$

This result is not too surprising since symmetry dictates that the equilibrium would have to be at the geometrically center of the system.

If $q_2 = 0$, then $Q = 0$ and there is no solution to the problem

$$0 = \frac{1}{y^2} \ ,$$

except the trivial one of $y = \pm\infty$.

f) Given that q_1 and q_2 have the same signs, we already know from qualitative analysis that the equilibrium position is between the charges. Thus $d_1 = d_2 = 1$ and $Q > 0$ is obtained.

If $|q_1| > |q_2|$, then $Q < 1$. In this case, the lower case solution gives the actual physical solution:

$$y = \frac{1 - \sqrt{Q}}{1-Q}$$

which we note is between 0 and 1 since $1 > \sqrt{Q} > Q$. The upper case solution gives $y > 1$ which is physically ruled out by the fact that $y < 1$ for the equilibrium being between q_1 and q_2.

If $|q_1| < |q_2|$, then $Q > 1$. In this case, the lower case solution gives the actual physical solution again:

$$y = \frac{1 - \sqrt{Q}}{1-Q} = \frac{\sqrt{Q} - 1}{Q - 1} \ .$$

which we note is between 0 and 1 since. $Q > \sqrt{Q} > 1$. The upper case solution gives $y < 0$ which is physically ruled out by the fact that $y > 0$ for the equilibrium being between q_1 and q_2.

The upshot is that

$$y = \frac{1 - \sqrt{Q}}{1-Q}$$

is valid for all cases where q_1 and q_2 have the same signs.

g) Someday when I've got the strength.

Redaction: Jeffery, 2008jan01

---

022 qfull 00910 1 3 0 easy math: induction charging of a ball

49. A person, who is acting as a grounded conductor, touches a metal ball as a positively charged rod is brought near the ball. Then the person is removed and then the rod is removed. Is the ball charged? Explain.

SUGGESTED ANSWER:

Begorra, yes. The positively charge rod attracts electrons from the ground. The ground is effectively an infinite reservoir of electrons. No matter how many electrons it gains or loses, it stays effectively neutral. When the person is removed, the electrons in the hand are trapped in the ball. This leaves the ball the negatively charged.

Actually ground—the real Earth surface—has can have slight negative charge density of about $-1\,\mathrm{nC/m^2}$ on average, but that is neutral enough for most purposes (Wikipedia: Natural electric field of the Earth).

Redaction: Jeffery, 2008jan01

---

023 qfull 00240 2 5 0 moderate thinking: charge rod, integration

50. There is a thin uniformly charged rod of length L. The total charge is q. The rod's thickness is negligible: it can be treated as a line of charge.

   a) What is the formula for the electric field a distance x away from one end of the rod and along the axis of the rod? **HINTS:** Find the expression for charge per unit length and then divide and conquer by integration. Draw a diagram.

   b) What is the far-field limit (but not infinite-distance limit) approximation to the formula for part (a).

SUGGESTED ANSWER: On tests the instructor could provide a diagram.

a) A time for integration: we find

$$\vec{E} = \int_{-L}^{0} \frac{k(q/L)}{(x-x')^2}\hat{x}\,dx' = \frac{kq}{L}\left[\frac{1}{x-x'}\right]\Bigg|_{-L}^{0}\hat{x} = \frac{kq}{L}\left[\frac{1}{x} - \frac{1}{x+L}\right]\hat{x} = \frac{kq}{x(x+L)}\hat{x}\ ,$$

where $\hat{x}$ points away from the rod along the axis of the rod.

b) In the far-field limit (i.e., to zeroth order in L/x),

$$\frac{1}{x(x+L)} = \frac{1}{x^2(1+L/x)} \approx \frac{1}{x^2}\left(1-\frac{L}{x}\right) \approx \frac{1}{x^2}\ ,$$

and so in the far-field limit,

$$\vec{E} = \frac{kq}{x^2}\hat{x}\ .$$

Thus far from the rod, the rod acts like a point charge. This should be no surprise.

Redaction: Jeffery, 2000oct04

---

023 qfull 00250 3 3 0 tough math: infinite line charge

51. What is the electric field (size and direction) around an infinite line of charge with linear charge density λ. **HINTS:** Divide and conquer: what is the electric field at an arbitrary point in space due to an arbitrary differential bit of the line? Also: by symmetry what must be the direction of the electric field? And lastly, a conversion to angular integration saves nine (of what? cat lives?): $x = r\tan\theta$ where x is the coordinate along the line of charge reaching from $-\infty$ to $+\infty$ and r is the perpendicular distance from your arbitrary point to the line at the $x = 0$ location.

SUGGESTED ANSWER:

```
           *  Point   The x integration ranges
          /|          from -infinity to +infinity in
         / |          the x coordinate and from -\pi/2
        /  |          to \pi/2 in the \theta coordinate
       /   |
      /    |
     /     | r
    /      |
   /       |
  / \theta |
 /         |
/          |
```

```
---------------------------------------------------------- x
 \lambda dx      0
```

By symmetry only the component of the electric field in the direction perpendicular to the line of charge is non-zero. Given that the unit vector in the perpendicular direction is $\hat{r}$, the differential electric field component in that direction is

$$d\vec{E} = \frac{k\lambda}{x^2 + r^2} \cos\theta \hat{r}\, dx \ ,$$

where $\cos\theta = r/\sqrt{x^2 + r^2}$ and the integration must be from $-\infty$ to ∞. The cosine accounts for the fact that only the $\hat{r}$ component is non-zero. The transformation to a θ integration seems hopeful:

$$x = r\tan\theta \qquad \text{and} \qquad dx = r\left[\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos^2\theta}(-\sin\theta)\right] d\theta = \frac{r}{\cos^2\theta}\, d\theta \ .$$

Now the integral becomes

$$\vec{E} = k\lambda \int_{-\pi/2}^{\pi/2} \frac{\cos^2\theta}{r^2} \cos\theta \frac{r}{\cos^2\theta} \hat{r}\, d\theta = k\lambda \int_{-\pi/2}^{\pi/2} \frac{\cos\theta}{r} \hat{r}\, d\theta = 2k\lambda \frac{\sin\theta}{r}\bigg|_0^{\pi/2} \hat{r} = \frac{2k\lambda}{r}\hat{r} \ .$$

The infinite-line-of-charge E-field result is interesting among other things because it gives the electric field of a non-localizable charge distribution: i.e., one that cannot be contained in a finite closed surface. The far-field limit of a localizable charge distribution allows goes as $1/r^2$ where r is the radial coordinate from an origin in the middle of the distribution. The infinite-line-of-charge E-field goes as $1/r$ (where r is the cylindrical radius coordinate from the line of charge) for all r.

The infinite-line-of-charge case is also interesting because it is one of the few cases where the integral form of Gauss's law can be used to find the E-field exactly with almost no effort.

Of course there are really no infinite lines of charge. An infinite line of charge is an idealization. But close to a long line of charge, it can be approximated as an infinite line of charge.

Redaction: Jeffery, 2001jan01

---

023 qfull 00360 1 3 0 hard math: classical hydrogen atom

52. The hydrogen atom cannot really be understood in classical physics: quantum mechanics is required. Nevertheless, one can get some idea of the size of effects from classical estimates.

    NOTE: There are parts a,b,c,d.

    a) Given that the separation of the electron and proton in the hydrogen atom is $a_0 = 0.529177210544(82) \times 10^{-10}\,\text{m}$, what is the magnitude of the electric force between these particles: i.e., the electric force each exerts on the other? Is the force attractive or repulsive?

       The distance a_0 is the Bohr radius and it is the characteristic separation of proton and electron in a hydrogen atom as determined by quantum mechanics.

       HINT: You will need the Coulomb constant $k = 8.9875517862 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2$ and the elementary charge $e = 1.602176634 \times 10^{-19}\,\text{C}$ (exact).

    b) Given that the electrostatic force is the only force between electron and proton (which is not quite true) and that the proton can be regarded as at rest (which is almost true because of its much greater mass), what is the electron's acceleration magnitude and which way does the acceleration point? **HINT:** The electron mass is $9.1093837139(28) \times 10^{-31}\,\text{kg}$

    c) Given that the answer to part (b), what can you suggest as the classical reason why the electron doesn't crash into the proton? **HINT:** Why doesn't the Moon crash into the Earth?

    d) Assuming the motion of the electron is uniform circular motion, what is the electron speed v? What is the ratio v/c, where c is the speed of light? Recall v/c is conventionally designated β in relativity.

SUGGESTED ANSWER:

a) The magnitude of the force is

$$F = \frac{ke^2}{r^2} = 8.24 \times 10^{-8}\,\text{N} \approx 10^{-7}\,\text{N} \ .$$

The force is attractive (i.e., each particle attracts the other with this force) since the charges are unlike.

b) Behold:

$$a = \frac{F}{m} = 9.04 \times 10^{22}\,\mathrm{m/s^2}\ .$$

The acceleration points toward the proton.

c) Classically, one could suggest that the electron is held up by having momentum perpendicular to the line to the proton. The proton keeps pulling the electron in, but the electron keeps missing. The electrostatic force changes the direction of the momentum vector, and thus keeps the electron in an orbit, but it can't eliminate the perpendicular momentum. Literally, the electron is always falling toward the proton, but it keeps missing it. We can abbreviate this answer by saying the electron has angular momentum and for a central force, this angular momentum is conserved.

This classical answer bears some relation to the quantum mechanical truth. Angular momentum exists in quantum mechanics too. But the quantum mechanical solutions are quantized: only a discrete set of energies and angular momenta are allowed, not continuums as in classical physics. Also the quantum mechanical solutions are wave functions that only predict the probability density of an electron which yours like to think of as the density of existence, but no one else uses that expression.

d) From the centripetal acceleration rule

$$a = \frac{v^2}{r}\ ,$$

it follows that

$$v = \sqrt{ar} = 2.19 \times 10^6\,\mathrm{m/s} \qquad \text{and} \qquad \beta = 0.00730\ .$$

From this classical calculation, the electron is seen to be slightly relativistic. In a quantum mechanical treatment, the hydrogen atom is also slightly relativistic.

Fortran-95 Code

```
          print*,'Coulomb constant and the rest'
          pi=3.14159265358979323846264338327950288419716939937510_np
    !
!!23456789a123456789b123456789c123456789d123456789e123456789f123456789g12
    !           ! https://en.wikipedia.org/wiki/Pi#Approximate_value_and_digits 51
digits
          epsilon=8.8541878188e-12_np  !
https://physics.nist.gov/cgi-bin/cuu/Value?ep0 error (14)
          xk=1.0_np/(4.0_np*pi*epsilon)
          print*,'epsilon,xk'
          print*,epsilon,xk
    !   8.85418781880000000030E-0012
    !     123456789b
    !   8.9875517861707986705e+9
    !   8.9875517862e+9 !
https://en.wikipedia.org/wiki/Coulomb%27s_law#Coulomb_constant
          rad_bohr=0.529177210544e-10_np
    !         rad_bohr=0.529177210544(82)\times10^{-10}\,$m,
    !         https://physics.nist.gov/cuu/Constants/Table/allascii.txt nist 2025
          echarge=1.602176634e-19_np !
https://physics.nist.gov/cgi-bin/cuu/Value?e exact in coulombs
          f=xk*echarge**2/(rad_bohr)**2
          emass=9.1093837139e-31_np  !
https://physics.nist.gov/cgi-bin/cuu/Value?me in kg. (28) 2025
          a=f/emass
          v=sqrt(a*rad_bohr)
          clight=2.99792458e8_np
          beta=v/clight
```

```
      print*,'f,a,v,beta'
      print*,f,a,v,beta
! 8.238722427956791E-8 9.044216045944532E+22 2187691.2535971645
! 7.297352535790492E-3
```

Redaction: Jeffery, 2001jan01

---

023 qfull 00530 2 3 0 moderate math: simple electric dipole electric field

53. A simple electric dipole consists of two point charges equal magnitude and opposite in sign (i.e., q and $-q$).

   a) Draw a diagram of a simple electric dipole and sketch in its electric field lines.

   b) Give the general formula of the electric field of the simple dipole in polar coordinates where the radius r is measured from the midpoint between the two charges (which is the conventional origin) and the polar angle θ is measured from the positive side of the axis through the two charges. Give the formula in terms of $\vec{r}$, r (the magnitude of $\vec{r}$), θ, $\vec{a}$ (the vector from the origin to the positive charge q), $-\vec{a}$ (the vector from the origin to the positive charge $-q$), and a (the magnitude of $\vec{a}$ and $-\vec{a}$). **HINT:** You will need to use the law of cosines.

   c) Taylor expand the formula of the electric field of the simple dipole to 1st order in a/r in order to obtain the far-field limit formula for the electric dipole field. Write this formula in terms of the dipole moment

$$\vec{p} = 2q\vec{a}$$

   for the simple dipole.

SUGGESTED ANSWER:

   a) You will have to imagine the diagram. The electric field lines form a butterfly shape in cross section.

   b) Behold:

$$\begin{aligned}
E(\vec{r}\,) &= kq\left(\frac{\vec{r}-\vec{a}}{|\vec{r}-\vec{a}|^3} - \frac{\vec{r}+\vec{a}}{|\vec{r}+\vec{a}|^3}\right) \\
&= kq\left[\frac{\vec{r}-\vec{a}}{(r^2+a^2-2ra\cos\theta)^{3/2}} - \frac{\vec{r}+\vec{a}}{(r^2+a^2+2ra\cos\theta)^{3/2}}\right] \\
&= \frac{kq}{r^2}\left\{\frac{\hat{r}-\vec{a}/r}{[1+(a/r)^2-2(a/r)\cos\theta]^{3/2}} - \frac{\hat{r}+\vec{a}/r}{[1+(a/r)^2+2(a/r)\cos\theta]^{3/2}}\right\} \;,
\end{aligned}$$

   where we have used the fact that $\cos(\pi-\theta) = -\cos\theta$.

   c) Behold:

$$\begin{aligned}
E(\vec{r}\,) &= \frac{kq}{r^2}\left\{\frac{\hat{r}-\vec{a}/r}{[1+(a/r)^2-2(a/r)\cos\theta]^{3/2}} - \frac{\hat{r}+\vec{a}/r}{[1+(a/r)^2+2(a/r)\cos\theta]^{3/2}}\right\} \\
&\approx \frac{kq}{r^2}\left[\left(\hat{r}-\frac{\vec{a}}{r}\right)\left(1+3\frac{a}{r}\cos\theta\right) - \left(\hat{r}+\frac{\vec{a}}{r}\right)\left(1-3\frac{a}{r}\cos\theta\right)\right] \\
&\approx \frac{k(2q)}{r^2}\left(-\frac{\vec{a}}{r}+3\frac{a}{r}\cos\theta\hat{r}\right) \\
&= k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r}-\vec{p}}{r^3}\right] \;,
\end{aligned}$$

   where we have introduced the dipole moment

$$\vec{p} = 2q\vec{a} \;.$$

   We see that the far-field limit electric dipole electric field falls off as $1/r^3$. This a characteristic of dipole electric fields (in the far-field limit).

By the by, we can also write the dipole electric field in terms of the unit vectors of a spherical polar coordinate system with the positive z axis aligned with the dipole moment. Note

$$\vec{p} = p\left[\cos\theta, \cos\left(\theta + \frac{\pi}{2}\right)\right] = p(\cos\theta, -\sin\theta) \ ,$$

where first element is the $\hat{r}$ component, the second is the $\hat{\theta}$ component, and the $\hat{\theta}$ axis is counterclockwise $\pi/2$ from the $\hat{r}$ axis. We can now write

$$E(\vec{r}\,) = k\left(\frac{2p\cos\theta\hat{r} + p\sin\theta\hat{\theta}}{r^3}\right) = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right) \ .$$

Redaction: Jeffery, 2008jan01

---

023 qfull 00580 3 5 0 tough thinking: SHO electric dipole

54. The torque on a dipole moment $\vec{p}$ in an electric field $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E} \ .$$

If we write this formula in scalar form measuring θ from the electric field direction (which is conventional), then

$$\tau = -pE\sin\theta \ ,$$

where p and E are, respectively, the magnitudes of $\vec{p}$ and $\vec{E}$ and the negative sign shows that the torque always tries to align the dipole moment with the field: if $\theta > 0$ the torque points in the negative θ direction; if $\theta < 0$ the torque points in the positive θ direction. The torque of the electric field is the only torque acting on the dipole.

a) From the rotational 2nd law for rigid-body rotation around a fixed axis, find the differential equation for the motion of the dipole in the electric field. Recall this form of the second law is

$$\tau = I\alpha \ ,$$

where τ is **NET** external torque about the fixed axis, I is rotational inertia, and $\alpha = d^2\theta/dt^2$ is angular acceleration.

b) If you make the small angle approximation, what differential equation for a standard kind of motion do you have?

c) For the solution to the approximate equation in the part (b) answer, what is the expression for the angular frequency and period of the motion?

d) Now let's be concrete and consider a water molecule (H_2O) in an applied uniform electric field of strength of 10^3 N/C (like near a charged comb: HRW-521). The dipole moment of a water molecule is about 6.2×10^{-30} C-m (HRW-537). Assume the water molecule is a uniform solid sphere of radius 1 Å (i.e., 10^{-10} m) to calculate its rotational inertia: a pretty so-so approximation. What is the frequency f of the motion? Note frequency f, not angular frequency ω.

SUGGESTED ANSWER:

a) There is only the torque of the electric field in the present case, and so the equation of motion in this case is

$$I\alpha = -pE\sin\theta \ .$$

b) As it stands the equation of motion admits no finite analytic solution. But if we make the small angle approximation ($\sin\theta \approx \theta$, where θ must be in radians) and limit ourselves to small amplitude problems, then we get the simple harmonic oscillator (SHO) equation of motion all over again:

$$\theta'' = -\frac{pE}{I}\theta \ .$$

You should instantly recognize that the general solution is

$$\theta = A\cos(\omega t) + B\sin(\omega t) \ ,$$

where A and B are constants that have to be set by the initial conditions and ω is the angular frequency of the oscillation. Note

$$\omega = \sqrt{\frac{pE}{I}} \ .$$

You can confirm the solution by trial:

$$\theta = A\cos(\omega t) + B\sin(\omega t)$$
$$\theta' = -\omega A\sin(\omega t) + \omega B\cos(\omega t)$$

and

$$\begin{aligned}\theta'' &= -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t) \\ &= -\omega^2\theta = -\frac{pE}{I}\theta \ .\end{aligned}$$

c) We instantly identify $\omega = \sqrt{pE/I}$ and period $P = 2\pi\sqrt{I/(pE)}$. Note we've run right into one those notational confusions. Here ω is the SHO oscillator angular frequency, not the angular velocity of the motion which is $d\theta/dt$. The conventional use of ω for these two different quantities is a nuisance. But we must live with it: there are more quantities that available simple symbols in physics.

d) The rotational inertia with the given approximations is

$$I = (2/5)mr^2 \approx 0.4 \times \left(18. \times 1.7 \times 10^{-27}\right)\left(10^{-10}\right)^2 \approx 1.3 \times 10^{-46}\,\mathrm{kg\,m^2} \ .$$

The frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}} \approx \frac{\sqrt{5\times 10^{19}}}{6.3} \approx 10^9\,\mathrm{Hz}$$

to about 1-digit accuracy.

Redaction: Jeffery, 2001jan01

---

023 qfull 01010 1 3 0 easy math: electrical breakdown and sparks in air

55. Consider the following systems.

a) A rubber rod is charged to a very high negative voltage (i.e., it a very high negative potential). Why will a spark jump if the rod is brought near a sufficiently highly positively charged rod?

b) Will a spark occur if the negative rod is brought near a neutral object? Explain.

SUGGESTED ANSWER:

a) If the combined electrical fields of the two rods exceed the breakdown electrical field of air, the air breaks down and becomes a conductor: ions and free electrons are formed. In this case a current flows between the two rods for a short time. Such sudden, momentary current flows are called electrical discharges although the term electrical discharge is also used for continuous current flow in in gases such as in gas-discharge and fluorescent lamps. If the discharge is strong enough, there is a visible spark. This is caused by accelerated ions and electrons exciting atoms to high energy states by collisions. The atoms decay to low energy states by emitting visible light. If ultraviolet light is produced it can also excite atoms which may then decay emitting visible light too.

For air, the breakdown electrical field is about $3 \times 10^6\,\mathrm{V/m}$. This is a very high field, but small regions of such high field occur pretty commonly: e.g., pulling out the plug of an electrical device.

But what you ask if there is no air and only vacuum? The air can't break down then. Electrons can be pulled straight out metals into vacuum by a strong enough electric field. For room temperature metals, this field is typically higher than about $10^9\,\mathrm{V/m}$. This much higher than for air breakdown. If you heat the metal, electrons can be pulled out more easily as from hot cathodes.

b) Maybe. If the neutral object is a conductor, then charge can redistribute in the neutral object if it is isolated or be pulled up from the ground if grounded and a region with a high enough electrical field for sparking between the charged and neutral object can occur. If the neutral object is an insulator, I don't think you can get a spark unless the neutral object itself breaksdown and conducts. Maybe if it polarizes strongly enough. I don't know. But maybe there is some other way to arrange it an air breakdown with a neutral. There are many odd effects out there that manifest themselves sometimes.

Actually, if the electrical field of the charged object is high enough, there can be corona discharge just from the air alone without another object. At higher charge, the charged object could emit electrons by field emission without another object or even air.

Redaction: Jeffery, 2008jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

---

1 Constants

$$c = 2.997\ 924\ 58 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602\ 176\ 634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition}$$

$$G = 6.674\ 30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987\ 551\ 785\ 972(14) \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}$$

$$k_{\text{Boltzmann}} = 1.380\ 649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\ 333\ 262 \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms}$$

$$m_e = 9.109\ 383\ 7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\ 998\ 950\ 69(16)\,\mathrm{MeV}$$

$$m_p = 1.672\ 621\ 923\ 95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\ 089\ 32(29)\,\mathrm{MeV}$$

$$\varepsilon_0 = 8.854\ 187\ 8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity}$$

$$\mu_0 = 1.256\ 637\ 061\ 27(20)\,\mathrm{N/A^2} \approx \pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}$$

---

2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

---

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

5 Quadratic Formula

If $\quad 0=ax^2+bx+c\ ,\quad$ then $\quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=-\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\frac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\mathrm{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\qquad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\mathrm{for}\ y\ \mathrm{max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\mathrm{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\mathrm{for\ max}} = \frac{\pi}{4} \qquad x_{\mathrm{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\mathrm{tan}} = r\alpha$$

$$\vec{a}_{\mathrm{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\mathrm{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

---

13 Very Basic Newtonian Physics

$$\vec{r}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\mathrm{total}}} = \frac{\sum_{\mathrm{sub}} m_{\mathrm{sub}}\vec{r}_{\mathrm{cm\ sub}}}{m_{\mathrm{total}}} \qquad \vec{v}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\mathrm{total}}} \qquad \vec{a}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\mathrm{total}}}$$

$$\vec{r}_{\mathrm{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\mathrm{total}}}$$

$$\vec{F}_{\mathrm{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\mathrm{m/s^2}$$

$$\vec{F}_{\mathrm{normal}} = -\vec{F}_{\mathrm{applied}} \qquad F_{\mathrm{linear}} = -kx$$

$$F_{\mathrm{f\ static}} = \min(F_{\mathrm{applied}}, F_{\mathrm{f\ static\ max}}) \qquad F_{\mathrm{f\ static\ max}} = \mu_{\mathrm{static}}F_{\mathrm{N}} \qquad F_{\mathrm{f\ kinetic}} = \mu_{\mathrm{kinetic}}F_{\mathrm{N}}$$

$$v_{\mathrm{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\mathrm{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\mathrm{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\mathrm{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\mathrm{drag,lin}} = bv \qquad v_{\mathrm{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\mathrm{T}}}{g} = \frac{m}{b} \qquad v = v_{\mathrm{T}}(1 - e^{-t/\tau})$$

$$F_{\mathrm{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\mathrm{T}} = \sqrt{\frac{mg}{b}}$$

---

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\mathrm{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\text{ deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\text{ rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

---

18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

---

19 **Static Equilibrium**

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

---

20 **Gravity**

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times 10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707\times 10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times 10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned} \text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\ \text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g \\ \text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\ \text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant} \end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\max}\sin[k(x - vt)] = y_{\max}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L}\ :\ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_\text{H}} = \frac{Q_\text{H} - Q_\text{C}}{W} = 1 - \frac{Q_\text{C}}{Q_\text{H}} \qquad \eta_{\text{heating}} = \frac{Q_\text{H}}{W} = \frac{Q_\text{H}}{Q_\text{H} - Q_\text{C}} = \frac{1}{1 - Q_\text{C}/Q_\text{H}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_\text{C}}{W} = \frac{Q_\text{H} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_\text{C}}{T_\text{H}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_\text{C}/T_\text{H}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_\text{C}/T_\text{H}}{1 - T_\text{C}/T_\text{H}}$$

25 Electrostatics

$$\vec{F}_{1 \text{ on } 2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \dots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_\text{i}^\text{f} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\,d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_iV_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

---

26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J}\cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma\vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R}e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}}\left(1 - e^{-t/\tau}\right)$$

---

27 Magnetic Fields and Forces

$$\mu_0 = 1.256\ 637\ 061\ 27(20)\,\text{N/A}^2 \approx \pi\times 10^{-7}\,\text{N/A}^2 \qquad 1\,\text{N/A}^2 = 1\,\text{T}\cdot\text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad d\vec{F} = I\,d\vec{s}\times\vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu}\times\vec{B} \qquad U = -\vec{\mu}\cdot\vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I\theta}{4\pi r} \qquad \vec{F}_{1\text{ on }2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r}\hat{r}_{2\text{ to }1}$$

$$B_{\text{solenoid}} = \mu_0 nI \qquad B_{\text{toroid}} = \frac{\mu_0 NI}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}\hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0\vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

---

28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2}LI^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con}-\text{axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0\cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_\pm = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0\cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right] e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

---

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0\cos(\omega t) \qquad I = I_0\cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2}I_0V_0\cos\phi = I_{\text{rms}}V_{\text{rms}}\cos\phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \qquad I_{\text{p}}N_{\text{p}} = I_{\text{s}}N_{\text{s}} \qquad I_{\text{p}}\mathcal{E}_{\text{p}} = I_{\text{s}}\mathcal{E}_{\text{s}} \qquad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

---

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B}\cdot d\vec{A} = 0 \qquad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0 \oint \vec{J}\cdot d\vec{A} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}}\cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2}I_{\text{inc}}$$