Introductory Physics: Calculus-Based **Name:**

Homework 02_05: Electric Charge and Electric Fields: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Multiple Choice Questions:

1. "Let's play *Jeopardy*! For \$100, the answer is: The equations that summarize most of classical electromagnetism:

$$1) \quad \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \qquad 2) \quad \nabla\cdot\vec{B}=0 \qquad 3) \quad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad 4) \quad \nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

which are the vacuum form. To complete summary, one also needs the Lorentz force formula

$$\vec{F}=q\left(\vec{E}+\vec{v}\times\vec{B}\right) \ .$$

What are ____________, Alex?

a) Ehrenfest's equations b) Friedmann's equations c) Maxwell's equations
d) Maxwell's relations e) the Schrödinger and Dirac equations

2. Electric charge is:

a) a fundamental property of matter: it comes in invariant quantized amounts of size $\pm e$.
b) a derived property of matter: it comes in somewhat variable quantized amounts of size $\pm e$ more or less.
c) a fundamental property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
d) a derived property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
e) amber.

3. Who named positive and negative electric charge?

a) George Washington (1732–1799). b) John Adams (1735–1826).
c) Benedict Arnold (1741–1801). d) Aaron Burr (1756–1836).
e) Benjamin Franklin (1706–1790).

4. The absolute value of the smallest nonzero physical amount of electric charge (not considering quarks) is:

a) e or $1.602\ldots\times10^{-19}$ C. b) 1 C. c) indefinitely small. d) indefinitely large.
e) indeterminate.

5. "Let's play *Jeopardy*! For \$100, the answer is: It is usually approximately electrically neutral."

What is ____________, Alex?

a) an electron b) a proton c) highly charged matter
d) highly negatively charged matter e) ordinary, terrestrial matter above the atomic scale

6. Why don't positive and negative charge largely neutralize each other at the microscopic level just like they mostly do at the macroscopic level? The short answer is it is forbidden in most circumstances by ____________.

a) classical physics b) James Clerk Maxwell c) Michael Faraday d) Ben Franklin
e) quantum mechanics

7. Ordinary matter is positively charged whenever protons:

a) outnumber neutrons. b) outvote electrons. c) outnumber electrons.
d) are fewer than electrons. e) are absent.

8. Electrons in an atom are:

a) permanently bound to the atom. b) not permanently bound to the atom.
c) confined to the nucleus. d) more massive than the nuclues. e) green.

9. Which of electrons, protons, neutrons are easier to remove from an atom?

a) Protons. b) They are all irremovable. c) They are equally easily removable.
d) Neutrons. e) Electrons.

10. Given the mass of a proton is approximately 1.67×10^{-24} g and the elementary charge is approximately 1.602×10^{-19} C, what approximately is the total charge of 1 g of protons?

a) 1.67×10^{-24} C. b) 1.602×10^{-19} C. c) 10^5 C. d) 2.5×10^{-43} C. e) zero.

11. There are four main charge conduction categories for materials:

a) insulator, conductor, demiconductor, orchestra conductor.
b) insulator, conductor, semiconductor, infraconductor.
c) insulator, conductor, semiconductor, Superman-conductor.
d) insulator, conductor, semiconductor, superconductor.
e) insulator, conductor, semiconductor, demisemiconductor.

12. "Let's play *Jeopardy*! For \$100, the answer is: These materials do **NOT** allow an electric current to flow through them easily."

What are ____________, Alex?

a) insulators b) conductors c) vacuum states d) solids e) crystals

13. The electricity and magnetism are now understood to be a united set of phenomena which we call:

a) magnoelectrism. b) magmaelectrism. c) electromagi-ism. d) electromagnetism.
e) electromagnanimousism.

14. What are contact forces?

a) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
b) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
c) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
d) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
e) The forces holding contact lenses in place. They're not for the squeamish.

15. The formula

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, is ________ law.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

16. The formula

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} \ ,$$

where $k \approx 8.99\ldots \times 10^9\,\mathrm{N \cdot m^2/C^2}$, is ________ law in scalar form.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

17. “Let’s play *Jeopardy*! For \$100, the answer is:

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987\,551\,785\,972(14)\times 10^9\,\mathrm{N\cdot m/C^2} \approx 8.99\times 10^9\,\mathrm{N\cdot m/C^2} \approx 9\times 10^9\,\mathrm{N\cdot m/C^2} \approx 10^{10}\,\mathrm{N\cdot m/C^2}\ ,$$

where ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, and c is the vacuum light speed. The k constant is an exact rational number because of the way the basic units are defined.

What is ____________, Alex?

a) Boltzmann’s constant b) Planck’s constant c) Avogadro’s number d) Fred’s constant e) Coulomb’s constant

18. Coulomb’s law is a:

a) societal law. b) law of thermodynamics. c) law of motion. d) conservation law. e) force law.

19. The electrostatic force and gravity are both ____________ force laws.

a) inverse-linear b) inverse-square c) inverse-cube d) linear e) quadratic

20. For two **POINT** masses to exert a Coulomb force on each other:

a) both must be uncharged. b) both must be charged. c) one must charged and the other uncharged. d) their charges must both be greater than 1 C. e) their charges must both be less than 1 C.

21. Can two finite bodies each electrically neutral overall ever attract or repel each other?

a) No.
b) Always.
c) Always and no.
d) **YES**. For example, consider two small, non-conducting balls attached by a non-conducting bar: give one ball a positive charge (uniformly spread) and the other ball a negative charge (uniformly spread) of the same magnitude. The structure is an electric dipole that is overall neutral. Now consider a second identical dipole. Align the two with unlike ends closest and then with like ends closest. The distance between the balls is a fixed distance a in both cases. The force between the two unlike ends is attractive and between the like ends is repulsive. What of the other forces between the balls? We can make the bars as long as we like. The other forces between the balls get smaller and smaller as we make the bars longer and longer. Eventually the other forces become negligible and the closest ball forces dominate.
e) They can **REPEL**, but never **ATTRACT**. That is the valid conclusion of answer (d).

22. If the distance between charges is changed by a **MULTIPLICATIVE** factor of 1/4, the magnitude of electric force between the charges changes by a **MULTIPLICATIVE** factor of:

a) 16. b) 4. c) 1. d) 1/4. e) 1/16.

23. Coulomb’s law and the point-mass gravitational formula in scalar form are, respectively:

$$F_{\mathrm{C}} = \frac{kq_1q_2}{r^2} \qquad \text{and} \qquad F_{\mathrm{G}} = \frac{Gm_1m_2}{r^2}\ ,$$

where $k = 8.98755179\times 10^9\,\mathrm{N\,m^2/C^2}$, $G = 6.67430(15)\times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25), q stands for charge, m for mass, 1 and 2 for particles 1 and 2, and r is the distance between the particles. The mass of electrons is 9.1093826×10^{-31} kg and their electric charge is $1.60217653\times 10^{-19}$ C. Approximately, what is the ratio of the magnitude of gravitational force to the magnitude of the electrical force between two electrons (i.e., $F_{\mathrm{G}}/F_{\mathrm{C}}$) for any distance r?

a) 1. b) 10. c) 3×10^{-43}. d) 3×10^{43}. e) 10^{-19}.

24. Charge separation in a object is called:

a) rasterization. b) pasteurization. c) polarization. d) north polarization. e) miniaturization.

25. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged rod is brought from infinity to near B, but not near A. The balls are then separated and the rod is put back at infinity. The charges on A and B are, respectively:

    a) positive and positive. b) negative and negative. c) negative and positive.
    d) positive and negative. e) zero and zero.

26. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged conducting rod is brought from infinity to near B, but not near A. The rod touches B. Then the rod is then removed and the balls are separated. The charges on A and B are ____________.

    a) both positive b) both negative c) negative and positive d) positive and negative
    e) both zero.

27. The electric field is a:

    a) vector field that emanates from charge (but not only charge) and causes the electric force.
    b) scalar field that emanates from charge (but not only charge) and causes the electric force.
    c) scalar field that emanates from current (but not only current) and causes the electric force.
    d) vector field that emanates from current (but not only current) and causes the magnetic force.
    e) scalar field that emanates from current (but not only current) and causes the magnetic **FARCE**.

28. A curve through space that is tangent to the electric field vector at each point is a/an:

    a) streamline. b) stream curve. c) electric field line. d) magnetic field line.
    e) straight line always.

29. The force on a point charge q caused by an electric field $\vec{E}$ is given by:

    a) $\vec{F} = m\vec{a}$. b) $\vec{F} = \dfrac{\vec{E}}{q}$. c) $\vec{\tau} = \vec{p} \times \vec{E}$. d) $\vec{F} = \dfrac{\left(\vec{E}\right)^2}{q}$. e) $\vec{F} = q\vec{E}$.

30. Say that you have charge distribution of point charges with net charge q in an external electric field $\vec{E}$. Since it is external, $\vec{E}$ does not include contributions from the charge distribution. Also $\vec{E}$ is **NOT** necessarily uniform. The electric force on the distribution due to $\vec{E}$ is

$$\vec{F} = \sum_i q_i \vec{E}_i \ ,$$

    where q_i is the charge of one member of the distribution and $\vec{E}_i$ is the external electric at the location of charge q_i. The force $\vec{F}$ is ____________ on the distribution and in the absence of any other net external force dictates ____________ of the distribution.

    a) the net electric force; individual charge accelerations
    b) not the net electric force; center-of-mass acceleration
    c) the net electric force; center-of-mass acceleration
    d) not the net electric force; individual charge accelerations
    e) half the net electric force; individual charge accelerations

31. A self-propagating electromagnetic field is:

    a) a static electric field. b) a static magnetic field. c) a grass field. d) light.
    e) sound.

32. The electric field about a point charge or any spherically symmetric charge distribution is given by

$$\vec{E} = \frac{kq}{r^2}\hat{r} \ ,$$

    where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, r is the radius from the center of system, q is the charge inside of a shell of size r, and $\hat{r}$ is a/an:

    a) radially outward pointing unit vector. b) a radially inward pointing unit vector.
    c) an azimuthal angle unit vector. d) a polar angle unit vector. e) an r with tilde on it.

33. "Let's play *Jeopardy*! For \$100, the answer is: The formula for a discrete set of charges is

$$E(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \ ,$$

where the sum is over all charges q_i, $\vec{r}_i$ is the charge location, $\vec{r}$ is the point of evaluation, and $\vec{r}$ is the point of evaluation."

What is the discrete charge electric field formula for ____________, Alex?

a) an overall neutral charge distribution b) a point charge c) a dipole charge distribution.
d) an arbitrary charge distribution e) a huge charge distribution

34. Say you had a static distribution of charge that gave rise to an electric field $\vec{E}$ and a test charge q (which not counted in the distribution). Say you magically changed all the charge signs in the distribution, but not q sign. The force on the charge is now ____________. Now say you magically changed all the charge signs including changing q to $-q$. The force on the charge is now ____________. Note we do **NOT** redefine the values assigned to the symbols $\vec{E}$ and q.

a) $q\vec{E}$; $q\vec{E}$ b) $-q\vec{E}$; $-q\vec{E}$ c) $-q\vec{E}$; $q\vec{E}$ d) $q\vec{E}$; $-q\vec{E}$ e) $-\vec{E}$; $\vec{E}$

35. The electric field around a localized distribution of charge (with a non-zero net charge) in the far-field limit looks like the electric field of a/an:

a) dipole. b) point charge. c) infinite plane of charge. d) infinite cylinder of charge.
e) infinite line of charge.

36. The quantity

$$\vec{p} = \int \vec{r}\rho(\vec{r}\,)\,dV$$

for a continuous distribution (and where the integral is over the whole distribution) or

$$\vec{p} = \sum_i \vec{r}_i q_i$$

for a discrete distribution (and where the sum is over the whole distribution) is called:

a) an electric dipole moment. b) an electric monopole moment.
c) a magnetic dipole moment. d) a magnetic monopole moment.
e) an electric field moment.

37. "Let's play *Jeopardy*! For \$100, the answer is:

$$E(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] \ ."$$

What is the ____________ moment electric field in the far-field limit, Alex?

a) monopole b) dipole c) quadrupole d) octupole e) hexacontatetrapole

38. The far-field electric field of an electric dipole falls off as:

a) $1/r$. b) $1/r^2$. c) $1/r^3$. d) $1/r^4$. e) $1/r^5$.

39. The pattern of the electric field lines about a dipole viewed in a cross section plane in which the dipole axis lies is sort of:

a) butterfly-like. b) ladybug-like. c) dragonfly-like. d) aphid-like.
e) praying-mantis-like.

40. The permanent or induced dipoles at the microscopic level are:

a) uncommon. b) common. c) never found. d) never found on Earth.
e) never found in space.

41. The torque on an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$ is:

a) $U = \vec{p} \cdot \vec{E}$. b) $U = -\vec{p} \cdot \vec{E}$. c) $\tau = \vec{p} \times \vec{E}$. d) $\tau = -\vec{p} \times \vec{E}$. e) $\tau = -\vec{p} \cdot \vec{E}$.

42. An electric dipole (with moment $\vec{p}$) in an electric field $\vec{E}$ has a potential energy of alignment:

a) $U = \vec{p} \cdot \vec{E}$. b) $U = -\vec{p} \cdot \vec{E}$. c) $\tau = \vec{p} \times \vec{E}$. d) $\tau = -\vec{p} \times \vec{E}$. e) $\tau = -\vec{p} \cdot \vec{E}$.

43. The equation of angular motion for a rigid dipole (of dipole moment $\vec{p}$ and rotational inertia I) in a uniform electric field $\vec{E}$ is

$$I\alpha = I\frac{d^2\theta}{dt^2} = -pE\sin\theta \ ,$$

where θ is the angle of the dipole moment from the electric field direction. This innocent-looking differential equation (DE) has no exact analytic solution—that gosh-darn transcendental function of the solution θ is pretty intractable. But one can make the small angle approximation

$$\sin\theta \approx \theta$$

(with θ in radians of course) which is exact to 2nd order in small θ since the Taylor's expansion for $\sin\theta$ is

$$\sin\theta = \sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{\theta^{2\ell+1}}{(2\ell+1)!}$$

(Ar-264). The angle θ in radians is larger than $\sin(\theta)$ at 10° by 0.51 %, 20° by 2.1 %, 30° by 4.7 %, 45° by 11 %, and 60° by 21 %. So the small angle approximation is pretty good even for not-so-small angles. With the small angle approximation, the DE becomes

$$I\frac{d^2\theta}{dt^2} = -pE\theta$$

which is the simple harmonic oscillator DE. The general solution of this linear 2nd order DE (linear because the solution function and its derivatives appear linearly and 2nd order because the highest derivative is 2n order) is

$$\theta = A\cos(\omega t) + B\sin(\omega t) \ ,$$

where ω here is not the angular velocity (not $d\theta/dt$), but the angular frequency and is given by

$$\omega = \sqrt{\frac{pE}{I}} \ .$$

We see that the dipole oscillation is simple harmonic oscillation in the small angle approximation. The period of the solution is

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}} \ .$$

The period increases as I increases (increasing resistance to angular acceleration) and decreases as pE increases (increasing torque magnitude per angular displacement from equilibrium). The constants A and B are independent of ω and P and set by the initial conditions. Say at time $t = 0$, the dipole is at $\theta = 0$ and has angular velocity $\omega\theta_0$. The solution for this special case is:

a) $\theta_0\sin(\omega t)$. b) $\theta_0\cos(\omega t)$. c) $\omega\theta_0\sin(\omega t)$. d) $(\theta_0/\omega)\sin(\omega t)$. e) ωt.

Full Answer Problems:

44. A typical current is 1 A (A for ampere) or 1 Coulomb per second. Given that the elementary charge is $e = 1.602 \times 10^{-19}$ C, how many elementary charges flow past a point in 1 second if the current is 1 A?

45. A copper penny weighs about 2.5 g (by actual, but not terribly accurate, measurement with a mail weighing scale). It is a pure copper penny, not one of those new-fangled things with zinc cores.

a) Given that copper's atomic mass is 63.5 in atomic mass units (i.e., in units of 1.66×10^{-27} kg), how many copper atoms are there in the penny? **HINT:** If you have 10 kg of eggs and each egg weighs 50 g, then you how many eggs?

b) Given that a copper atom has 29 protons (i.e., atomic number Z=29), how much gross positive charge in Coulombs does the penny have?

c) Imagine (contrafactually as they say in Washington) that the absolute value of the elementary proton charge was 1.000001 times larger than the absolute value of the elementary electron charge (i.e., there was a fractional charge difference of $(\delta e/e) = 10^{-6}$), what would be the force between two pennies a distance 1 meter apart assuming that the pennies would be neutral if there were no fractional charge difference.

d) Is a charge difference between proton and electron of the order suggested in the part (c) question likely? Explain.

46. Four point charges are set at the corners of a square 2 meters on a side. Going clockwise from the upper left hand corner, the charges are labeled A (1 C), B (2 C), C (−3 C), and D (4 C). What is the net force exerted on charge B? Remember to specify a direction. **HINTS:** Pick a convenient set of axes and compute the force components along those axes. The components are just scalars. Recall Coulomb's law for the force due to particle 1 on particle 2 is:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$ and $\hat{r}_{12}$ points from particle 1 to particle 2. Draw a diagram.

47. Imagine three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and is located **BETWEEN** the two other charges.

a) Is there an equilibrium position for charge q if q_1 and q_2 have different signs? Why or why not?

b) Find the equilibrium position solutions x for charge q. Mathematically what happens to these solutions if q_1 and q_2 have different signs? Mathematically what happens to these solutions if $q_1 = q_2$?

c) Which solution from part (b) is physically allowed by the setup conditions. To save tedium just consider the cases where charges q_1 and q_2 are both greater than 0.

d) When is the equilibrium position stable and when is it unstable. **HINT:** A word argument gives the answer easily.

48. Image three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and can be located anywhere except at $x = 0$ and $x = a$. This question concerns itself with the equilibrium positions for q along the line. We don't bother with the trivial equilibrium positions are $x = \pm\infty$ in the discussion.

a) Qualitatively where is the (Coulomb force) equilibrium position for charge q if q_1 and q_2 have the same sign? Can there be more than one equilibrium? Is the equilibrium stable or not? Explain your answers.

b) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| > |q_2|$? Explain your answer.

c) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| < |q_2|$? Explain your answer.

d) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| = |q_2|$? Explain your answer.

e) The trouble with find the equilibrium positions in general for the three-charges on a line case, is that the direction of the force changes when one passes a charge. If it wasn't for that, the equation would be a simple quadratic. After a lot of thought, I think a good way to formulate the equation to solve as

$$F = k\left[\frac{d_1 q_1}{x^2} - \frac{d_2 q_2}{(a-x)^2}\right] q = 0 ,$$

where $d_1 = -1$ for q to the negative of q_1 and positive otherwise and $d_2 = 1$ for q to the negative of q_2 and positive otherwise. This simplifies to

$$\frac{1}{y^2} - \frac{Q}{(1-y)^2} = 0 ,$$

where $y = x/a$ and $Q = (d_2q_2)/(d_1q_1)$. Find the general solutions for y. What is a necessary condition on Q for a physically real solutions? What happens if $q_1 = q_2$ or $q_2 = 0$ (which actually violates are givens for the problem)?

f) Given that q_1 and q_2 have the same signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

f) Given that q_1 and q_2 have different signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

49. A person, who is acting as a grounded conductor, touches a metal ball as a positively charged rod is brought near the ball. Then the person is removed and then the rod is removed. Is the ball charged? Explain.

50. There is a thin uniformly charged rod of length L. The total charge is q. The rod's thickness is negligible: it can be treated as a line of charge.

 a) What is the formula for the electric field a distance x away from one end of the rod and along the axis of the rod? **HINTS:** Find the expression for charge per unit length and then divide and conquer by integration. Draw a diagram.

 b) What is the far-field limit (but not infinite-distance limit) approximation to the formula for part (a).

51. What is the electric field (size and direction) around an infinite line of charge with linear charge density λ. **HINTS:** Divide and conquer: what is the electric field at an arbitrary point in space due to an arbitrary differential bit of the line? Also: by symmetry what must be the direction of the electric field? And lastly, a conversion to angular integration saves nine (of what? cat lives?): $x = r\tan\theta$ where x is the coordinate along the line of charge reaching from $-\infty$ to $+\infty$ and r is the perpendicular distance from your arbitrary point to the line at the $x = 0$ location.

52. The hydrogen atom cannot really be understood in classical physics: quantum mechanics is required. Nevertheless, one can get some idea of the size of effects from classical estimates.

 NOTE: There are parts a,b,c,d.

 a) Given that the separation of the electron and proton in the hydrogen atom is $a_0 = 0.529177210544(82) \times 10^{-10}\,\mathrm{m}$, what is the magnitude of the electric force between these particles: i.e., the electric force each exerts on the other? Is the force attractive or repulsive?

 The distance a_0 is the Bohr radius and it is the characteristic separation of proton and electron in a hydrogen atom as determined by quantum mechanics.

 HINT: You will need the Coulomb constant $k = 8.9875517862 \times 10^9\,\mathrm{N\,m^2/C^2}$ and the elementary charge $e = 1.602176634 \times 10^{-19}\,\mathrm{C}$ (exact).

 b) Given that the electrostatic force is the only force between electron and proton (which is not quite true) and that the proton can be regarded as at rest (which is almost true because of its much greater mass), what is the electron's acceleration magnitude and which way does the acceleration point? **HINT:** The electron mass is $9.1093837139(28) \times 10^{-31}\,\mathrm{kg}$

 c) Given that the answer to part (b), what can you suggest as the classical reason why the electron doesn't crash into the proton? **HINT:** Why doesn't the Moon crash into the Earth?

 d) Assuming the motion of the electron is uniform circular motion, what is the electron speed v? What is the ratio v/c, where c is the speed of light? Recall v/c is conventionally designated β in relativity.

53. A simple electric dipole consists of two point charges equal magnitude and opposite in sign (i.e., q and $-q$).

 a) Draw a diagram of a simple electric dipole and sketch in its electric field lines.

 b) Give the general formula of the electric field of the simple dipole in polar coordinates where the radius r is measured from the midpoint between the two charges (which is the conventional origin) and the polar angle θ is measured from the positive side of the axis through the two charges. Give the formula in terms of $\vec{r}$, r (the magnitude of $\vec{r}$), θ, $\vec{a}$ (the vector from the origin to the positive charge q), $-\vec{a}$ (the vector from the origin to the positive charge $-q$), and a (the magnitude of $\vec{a}$ and $-\vec{a}$). **HINT:** You will need to use the law of cosines.

c) Taylor expand the formula of the electric field of the simple dipole to 1st order in a/r in order to obtain the far-field limit formula for the electric dipole field. Write this formula in terms of the dipole moment

$$\vec{p} = 2q\vec{a}$$

for the simple dipole.

54. The torque on a dipole moment $\vec{p}$ in an electric field $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E} \ .$$

If we write this formula in scalar form measuring θ from the electric field direction (which is conventional), then

$$\tau = -pE \sin\theta \ ,$$

where p and E are, respectively, the magnitudes of $\vec{p}$ and $\vec{E}$ and the negative sign shows that the torque always tries to align the dipole moment with the field: if $\theta > 0$ the torque points in the negative θ direction; if $\theta < 0$ the torque points in the positive θ direction. The torque of the electric field is the only torque acting on the dipole.

a) From the rotational 2nd law for rigid-body rotation around a fixed axis, find the differential equation for the motion of the dipole in the electric field. Recall this form of the second law is

$$\tau = I\alpha \ ,$$

where τ is **NET** external torque about the fixed axis, I is rotational inertia, and $\alpha = d^2\theta/dt^2$ is angular acceleration.

b) If you make the small angle approximation, what differential equation for a standard kind of motion do you have?

c) For the solution to the approximate equation in the part (b) answer, what is the expression for the angular frequency and period of the motion?

d) Now let's be concrete and consider a water molecule (H_2O) in an applied uniform electric field of strength of 10^3 N/C (like near a charged comb: HRW-521). The dipole moment of a water molecule is about 6.2×10^{-30} C-m (HRW-537). Assume the water molecule is a uniform solid sphere of radius 1 Å (i.e., 10^{-10} m) to calculate its rotational inertia: a pretty so-so approximation. What is the frequency f of the motion? Note frequency f, not angular frequency ω.

55. Consider the following systems.

a) A rubber rod is charged to a very high negative voltage (i.e., it a very high negative potential). Why will a spark jump if the rod is brought near a sufficiently highly positively charged rod?

b) Will a spark occur if the negative rod is brought near a neutral object? Explain.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.997\,924\,58 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition}$$

$$G = 6.674\,30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987\,551\,785\,972(14) \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}$$

$$k_{\mathrm{Boltzmann}} = 1.380\,649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\,333\,262 \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms}$$

$$m_e = 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\,998\,950\,69(16)\,\mathrm{MeV}$$

$$m_p = 1.672\,621\,923\,95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\,089\,32(29)\,\mathrm{MeV}$$

$$\varepsilon_0 = 8.854\,187\,8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity}$$

$$\mu_0 = 1.256\,637\,061\,27(20)\,\mathrm{N/A^2} \approx \pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}$$

---

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

---

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1 - x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan\left(a_x, a_y\right) + n\pi = \arctan_{\text{computational}}\left(\dfrac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\mathrm{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \dfrac{\pi}{2} + n\pi \qquad \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\dfrac{\pi}{2} + n\pi \qquad \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\mathrm{for}\ y\ \mathrm{max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\mathrm{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\mathrm{for\ max}} = \frac{\pi}{4} \qquad x_{\mathrm{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

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15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

---

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

---

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\ \text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\ \text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\mathrm{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\mathrm{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\mathrm{cm}} + mR_{\mathrm{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\mathrm{cyl,shell,thin}} = MR^2 \qquad I_{\mathrm{cyl}} = \frac{1}{2}MR^2 \qquad I_{\mathrm{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\mathrm{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\mathrm{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\mathrm{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta K_{\mathrm{rot}} = W_{\mathrm{net}} = \int \tau_{z,\mathrm{net}}\, d\theta \qquad \Delta U_{\mathrm{rot}} = -W = -\int \tau_{z,\mathrm{con}}\, d\theta$$

$$\Delta E_{\mathrm{rot}} = K_{\mathrm{rot}} + \Delta U_{\mathrm{rot}} = W_{\mathrm{non,rot}} \qquad \Delta E = \Delta K + K_{\mathrm{rot}} + \Delta U = W_{\mathrm{non}} + W_{\mathrm{rot}}$$

---

19 **Static Equilibrium**

$$\vec{F}_{\mathrm{ext,net}} = 0 \qquad \vec{\tau}_{\mathrm{ext,net}} = 0 \qquad \vec{\tau}_{\mathrm{ext,net}} = \tau'_{\mathrm{ext,net}} \quad \text{if } F_{\mathrm{ext,net}} = 0$$

$$0 = F_{\mathrm{net}\ x} = \sum F_x \qquad 0 = F_{\mathrm{net}\ y} = \sum F_y \qquad 0 = \tau_{\mathrm{net}} = \sum \tau$$

---

20 **Gravity**

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707\times10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

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21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

---

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_\mathrm{T}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_\mathrm{max}\sin[k(x - vt)] = y_\mathrm{max}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\mathrm{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_\mathrm{max}^2$$

$$y = 2y_\mathrm{max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_\mathrm{medium} = \frac{f_0}{1 - v_0/v_\mathrm{medium}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_\mathrm{K} = T_\mathrm{C} + 273.15\,\mathrm{K} \qquad T_\mathrm{F} = 1.8 \times T_\mathrm{C} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_\mathrm{avg} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_\mathrm{RMS}^2\right)$$

$$v_\mathrm{RMS} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_\text{sound} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma k T}{m}}$$

$$\varepsilon = \frac{W}{Q_\text{H}} = \frac{Q_\text{H} - Q_\text{C}}{W} = 1 - \frac{Q_\text{C}}{Q_\text{H}} \qquad \eta_\text{heating} = \frac{Q_\text{H}}{W} = \frac{Q_\text{H}}{Q_\text{H} - Q_\text{C}} = \frac{1}{1 - Q_\text{C}/Q_\text{H}} = \frac{1}{\varepsilon}$$

$$\eta_\text{cooling} = \frac{Q_\text{C}}{W} = \frac{Q_\text{H} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_\text{heating} - 1$$

$$\varepsilon_\text{Carnot} = 1 - \frac{T_\text{C}}{T_\text{H}} \qquad \eta_\text{heating,Carnot} = \frac{1}{1 - T_\text{C}/T_\text{H}} \qquad \eta_\text{cooling,Carnot} = \frac{T_\text{C}/T_\text{H}}{1 - T_\text{C}/T_\text{H}}$$

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25 Electrostatics

$$\vec{F}_\text{1 on 2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_\text{all space} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,')\, d\vec{r}\,' \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q_\text{enc}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p} \cdot \vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_\text{i}^\text{f} \vec{E} \cdot d\vec{s} \qquad dV = -\vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad V(\vec{r}\,) = \int_\text{all space} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\, d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_iV_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r})\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J}\cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma\vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R}e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}}\left(1 - e^{-t/\tau}\right)$$

27 Magnetic Fields and Forces

$$\mu_0 = 1.256\ 637\ 061\ 27(20)\,\text{N/A}^2 \approx \pi\times 10^{-7}\,\text{N/A}^2 \qquad 1\,\text{N/A}^2 = 1\,\text{T}\cdot\text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad d\vec{F} = I\,d\vec{s}\times\vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu}\times\vec{B} \qquad U = -\vec{\mu}\cdot\vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I\theta}{4\pi r} \qquad \vec{F}_{1\text{ on }2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r}\hat{r}_{2\text{ to }1}$$

$$B_{\text{solenoid}} = \mu_0 nI \qquad B_{\text{toroid}} = \frac{\mu_0 NI}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}\hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0\vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2}LI^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con}-\text{axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0 \cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_\pm = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0 \cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right] e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0 \cos(\omega t) \qquad I = I_0 \cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2}I_0 V_0 \cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$\frac{\mathcal{E}_\text{p}}{N_\text{p}} = \frac{\mathcal{E}_\text{s}}{N_\text{s}} \qquad I_\text{p} N_\text{p} = I_\text{s} N_\text{s} \qquad I_\text{p}\mathcal{E}_\text{p} = I_\text{s}\mathcal{E}_\text{s} \qquad \frac{\mathcal{E}_\text{p}}{\mathcal{E}_\text{s}} = \frac{N_\text{p}}{N_\text{s}} = \frac{I_\text{s}}{I_\text{p}}$$

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{cB_{\text{max}}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\text{max}}^2}{2} = \frac{B_{\text{max}}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2 \theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$