Introductory Physics: Calculus-Based **Name:**

Homework 14: Fluids: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Multiple Choice Questions:

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015 qmult 00100 1 1 3 easy memory: fluid defined

1. A ____________ is a substance that continuously deforms under an applied shear stress. In other words a ____________ can't resist a shear stress. At least ideally, it can't. In reality, some small resistance does exist for those ____________s that are liquids. A shear stress is a force per unit area applied tangentially to material surface or layer. A normal stress, on the other hand, is applied perpendicularly (or normal) to a surface or layer.

   a) rope b) crystal c) fluid d) solid e) ice

   SUGGESTED ANSWER: (c)

   Wrong answers:

   a) An ideal rope can't resist any deforming force except one that tries to extend it. But a rope is not a substance.

   Redaction: Jeffery, 2001jan01

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015 qmult 00110 1 1 3 easy memory: liquids and gases

2. Fluids are either:

   a) solids or liquids. b) solids or gases. c) liquids or gases. d) crystals or gases.
   e) ices or metals.

   SUGGESTED ANSWER: (c)

   Wrong answers:

   a) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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015 qmult 00120 1 4 5 easy deducto-memory: solids defined

3. "Let's play *Jeopardy*! For \$100, the answer is: They are materials in which the atoms or molecules are rigidly (but not perfectly rigidly) bonded together by chemical bonds. They resist shear and normal stresses. The atoms or molecules are essentially touching in these materials. Of course, atoms and molecules have no hard edges, but they do have regions of strong interaction and it is these regions that touch. Atoms and molecules strongly resist compression, and so compressing these materials from their zero-pressure state takes relatively large amounts of force for relatively small compression."

   What is ____________, Alex?

   a) ices b) crystals c) gases d) liquids e) solids

   SUGGESTED ANSWER: (e)

   Wrong answers:

   a) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2008jan01

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015 qmult 00130 1 4 4 easy deducto-memory: liquids defined

4. "Let's play *Jeopardy*! For \$100, the answer is: They are materials in which the atoms or molecules are bonded together by chemical bonds, but the bonds are not rigid and are interchangeable. They have very weak resistance to shear stresses. The atoms or molecules are essentially touching in these

materials. Of course, atoms and molecules have no hard edges, but they do have regions of strong interaction and it is these regions that touch. Atoms and molecules strongly resist compression, and so compressing these materials from their zero-pressure state takes relatively large amounts of force for relatively small compression. But actually these materials usually do not exist in zero-pressure state since they usually rapidly evaporate if the pressure gets too low.

What is ____________, Alex?

a) ices b) crystals c) gases d) liquids e) solids

SUGGESTED ANSWER: (d)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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015 qmult 00200 1 1 1 easy memory: density defined
Extra keywords: physci

5. Density (with standard physics symbol the Greek rho ρ) unqualified is conventionally take to be the ratio of:

a) mass over volume. b) volume over mass. c) weight over volume.
d) weight over mass. e) mass over weight.

SUGGESTED ANSWER: (a)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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015 qmult 00202 1 1 2 easy memory: density symbol

6. The virtually universal symbol for density is:

a) α (the small Greek alpha). b) ρ (the small Greek rho). c) ω (the small Greek omega).
d) ζ (the small Greek zeta). e) β (the small Greek beta).

SUGGESTED ANSWER: (b)

Wrong answers:
c) This symbol is used for angular velocity and angular frequency usually.

Redaction: Jeffery, 2008jan01

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015 qmult 00210 1 3 3 easy math: g/cm**3 to kg/m**3
Extra keywords: physci

7. The MKS unit of density is the $\mathrm{kg/m^3}$, but this unit is often inconveniently small for many ordinary terrestrial densities. Thus, these terrestrial densities in $\mathrm{kg/m^3}$ are inconveniently and unmemorably large numbers. Consequently, the CGS unit of density is often used for convenience: $\mathrm{g/cm^3}$ (grams per cubic centimeter). Many common terrestrial substances are of order a few $\mathrm{g/cm^3}$. Now $1\,\mathrm{g/cm^3}$ equals:

a) $1\,\mathrm{kg/m^3}$. b) $0.001\,\mathrm{kg/m^3}$. c) $1000\,\mathrm{kg/m^3}$. d) $10^6\,\mathrm{kg/m^3}$. e) $0.5\,\mathrm{kg/m^3}$.

SUGGESTED ANSWER: (c) The conversion calculation is

$$1\,\mathrm{g/cm^3} = 1\,\mathrm{g/cm^3}\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(\frac{10^2\,\mathrm{g}}{1\,\mathrm{m}}\right)^3 = 1000\,\mathrm{kg/m^3}\ .$$

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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015 qmult 00410 1 4 1 easy deducto-memory: Earth atmosphere pressure

Extra keywords: physci KB-129-3

8. The pressure of the Earth's atmosphere at any level is caused by:

   a) the weight of the overlying air mass. b) respiration by living things.
   c) evaporation of sea water. d) glaciers. e) squid.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   e) As Lurch would say: "Aaaarh."

   Redaction: Jeffery, 2001jan01

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001 qmult 00420 1 1 2 easy memory: constant air pressure

9. Because of its low density, ____________ pressure varies slowly with height and can usually can be taken as a constant over changes of height of a few meters or even hundreds of meters depending on how accurate you want to be.

   a) water b) air c) mercury d) iron e) lead

   SUGGESTED ANSWER: (b)

   Wrong answers:
   e) Now is this likely?

   Redaction: Jeffery, 2008jan01

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015 qmult 00510 2 4 2 moderate deducto-memory: pressure with depth
Extra keywords: physci?

10. The expression for pressure of an incompressible fluid (of density ρ) as a function of depth y (measured positive downward) below a surface is:

   a) $P = P_{\text{surface}} + y$. b) $P = P_{\text{surface}} + \rho g y$. c) $P = \frac{1}{2} P_{\text{surface}} + \rho g y$.
   d) $P = \rho g y / P_{\text{surface}}$. e) $P = P_{\text{surface}}$.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   d) The left hand side is dimensionless.
   e) Everyone knows pressure increases somehow with depth.

   Redaction: Jeffery, 2001jan01

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015 qmult 00520 2 3 5 moderate math: pressure at depth 25 m
Extra keywords: physci

11. The expression for the pressure of an incompressible fluid with depth is

$$P = P_{\text{surface}} + \rho g y ,$$

where is P_{surface} is surface pressure, ρ is fluid density, $g = 9.8\text{N/kg}$ is the force per unit mass due to gravity near the Earth's surface, and y is depth measured downward from the surface. Air pressure and water density near the Earth's surface are to good approximation, respectively, 10^5 Pa (almost 1 atm) and $1000\,\text{kg/m}^3$. The pressure in atmospheres at 25 m in depth is about:

   a) 3.5×10^5 atm. b) 10^5 atm. c) 1 atm. d) 2.5 atm. e) 3.5 atm.

   SUGGESTED ANSWER: (e) Behold

$$P = P_{\text{surface}} + \rho g y \approx 10^5 + 10^3 \times 10 \times 25 = 3.5 \times 10^5\,\text{Pa} \approx 3.5\,\text{atm} .$$

   Wrong answers:
   a) You forgot to convert to atmospheres.

   Redaction: Jeffery, 2001jan01

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015 qmult 00700 1 1 4 easy memory: Archimedes's principle

Extra keywords: physci

12. Archimedes's principle is that buoyant force on an object surrounded by a fluid is upward and equal in magnitude to the:

a) weight of the object.
b) weight of the object submerged beneath the surface of the fluid.
c) weight of the fluid inside the object.
d) weight of the fluid displaced by the object.
e) ratio of the density of the fluid to that of the object.

SUGGESTED ANSWER: (d)

Wrong answers:

e) A ratio has no units and cannot be a force.

Redaction: Jeffery, 2001jan01

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015 qmult 00710 1 1 4 easy memory: buoyant force magnitude

13. The formula for the magnitude of the buoyant force is:

a) $F_{\text{buoy}} = m_{\text{dis}}/g$. b) $F_{\text{buoy}} = g/m_{\text{dis}}$. c) $F_{\text{buoy}} = 1/(m_{\text{dis}}g)$. d) $F_{\text{buoy}} = m_{\text{dis}}g$.
e) $F_{\text{buoy}} = m_{\text{dis}}^2 g$.

SUGGESTED ANSWER: (d)

Redaction: Jeffery, 2008jan01

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015 qmult 00950 1 5 5 easy easy thinking: paper and lift

14. Take this quiz and ...—no, no not that. Take this quiz—or some single sheet of paper if you arn't in a quiz *mise en scène*—in your fingers with your fingers on either side of one of narrow ends. Hold this end **JUST BELOW** your lips and blow a strong gust.

a) Nothing happens, because you've blown too hard.
b) Nothing happens, because you've blown too softly and you've never succeeded in blowing up a balloon in your life.
c) You spit.
d) The instructions are unintelligible.
e) The paper rises because you've created a high-speed, low-pressure zone above the paper. This is the Bernoulli lift which is part of aerodynamic lift by which airplanes fly. Of course, if you put the paper above your lips and blow the paper rises too. This time the rise is caused by the reaction lift which is the other part of aerodynamic lift. The blown air is deflected down by the paper, but for every force there is an equal and opposite force and so the air pushes up on the paper too.

SUGGESTED ANSWER: (e)

The experiment should work and the answer obeys the longest-answer-is-right rule. How can anyone miss. You only have to be a real blowhard to do it.

The trick is an example of Bernoulli lift. Moving air above the paper is at lower pressure than stationary air below the paper as the Bernoulli equation suggests. (We derived the Bernoulli equation for incompressible fluids, but Bernoulli-like behavior for compressible fluids is to be expected.) The situation is actually pretty complex: the low pressure zone above the paper causes the high pressure below the paper to push the paper up. But the low pressure zone also causes air above the low pressure zone to lose pressure support and fall down, but I guess the downfalling air gets entrained by the blowing air. Oh, well someone probably knows exactly what everything is doing.

Wrong answers:

a) This seems to be distinctly wrong.
b) This could well be true.
c) It's been known to happen. Best not to aim at anyone.
d) Well I tried my best, but, as we say in science, one picture is worth 10^3 words.

Redaction: Jeffery, 2001jan01

Full Answer Problems:

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015 qfull 00310 1 3 0 easy math: surface floating at interface of two fluids

15. This is an Archimedes' principle question. Consider a body (mass m, mean density ρ, volume V) surface floating at the interface of two fluids: lower fluid 1 (mean density ρ_1, embedded body component volume V_1; upper fliud 2 (mean density ρ_2 (with $\rho_2 \leq \rho_1$, of course), embedded body component volume V_2.

   a) Write down equation that the body component volumes must satisfy.

   b) Write down the formula for the buoyancy force $F_{\rm B}$ on the body.

   c) Write down the vertical Newton's 2nd law that the body must satisfy substituing in the buoyancy force $F_{\rm B}$ formula.

   d) From the equations found in parts (a) and (c) solve for V_1/V as a function of the densities Write down the vertical Newton's 2nd law that the body must satisfy substituing in the buoyancy force $F_{\rm B}$ formula.

   e) Write down the special cases of V_1/V formula from part (d) for $\rho_2 = 0$ or negiligible, $\rho = \rho_1$, $\rho = \rho_2$, $\rho = w\rho_1 + (1-w)\rho_2$ for $w \in [0,1]$, and $\rho = w\rho_1 + (1-w)\rho_2$ for $w \in [0,1]$ with $\rho_1 \to \rho_2$. What do the cases $\rho = \rho_1$ and $\rho = \rho_2$ imply about the position of the body?

   f) Why is the case of $\rho_1 = \rho_2$ for the V_1/V formula physically meaningless if you hold fixed the value of the numerator? **HINT:** There are physical limits on V_1, V_2, and ϕ not incorporated in the equations from which the formula was derived.

SUGGESTED ANSWER:

a) Behold:

$$V = V_1 + V_2 \ .$$

b) Behold:

$$F_{\rm B} = (\rho_1 V_1 + \rho_2 V_2)g \ .$$

c) Behold:

$$0 = (\rho_1 V_1 + \rho_2 V_2)g - mg = (\rho_1 V_1 + \rho_2 V_2 - \rho V)g \ .$$

d) Behold:

$$1)\quad V_2 = V - V_1 \qquad 2)\quad 0 = \rho_1 V_1 + \rho_2 V_2 - \rho V \qquad 3)\quad 0 = \rho_1 V_1 + \rho_2 (V - V_1) - \rho V$$

$$4)\quad -(\rho_1 - \rho_2)V_1 = -(\rho - \rho_2)V \qquad 5)\quad \frac{V_1}{V} = \frac{\rho - \rho_2}{\rho_1 - \rho_2} \qquad 6)\quad \frac{V_2}{V} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2} \ .$$

e) Behold:

$$\frac{V_1}{V} = \begin{cases} \dfrac{\rho - \rho_2}{\rho_1 - \rho_2} & \text{in general.} \\ \dfrac{\rho}{\rho_1} & \text{for } \rho_2 = 0 \text{ or negiligible.} \\ 1 & \text{for } \rho = \rho_1. \\ 0 & \text{for } \rho = \rho_2. \\ \dfrac{w\rho_1 + (1-w)\rho_2 - \rho_2}{\rho_1 - \rho_2} = w & \text{for } \rho = w\rho_1 + (1-w)\rho_2 \text{ for } w \in [0,1]. \\ w & \rho = w\rho_1 + (1-w)\rho_2 \text{ for } w \in [0,1] \\ & \quad \text{with } \rho_1 \to \rho_2. \end{cases}$$

When $\rho = \rho_1$ ($\rho = \rho_2$), the body is embedded in fluid 1 (fluid 2) and is just touching the fluid 2 (fluid 1) assuming the condition of it being at the inferface is maintained: otherwise it could floating neutrally anywhere in fluid 1 (fluid 2).

f) The equations from which the V_1/V formula was derived do not incorparate the facts that $V_1 \leq V$, $V_2 \leq V$, and $\rho \in [\rho_2, \rho_1]$. In fact, the numerator of the V_1/V will always be less than the denominator. If you contrain ρ to be a particular average of ρ_1 and ρ_2 as we did in part (e), you find that $V_1/V \leq 1$ including in the limit of $\rho_2 \to \rho_1$ where, in fact, V_1 and V_2 are indeterminate. If you look back at part (c), you will see that if $\rho_1 = \rho_2$, there is only the contraint $V = V_1 + V_2$ which the same as the part (a) constraint. So with one constraint you cannot solve for two unknowns. If the two fluids were actually the same fluid, there would be no meaning to V_1 and V_2. If the two fluids were different and immisciblle, but just having the same density, then there would be a meaning to V_1 and V_2, but that would be determined by the history of the body and maybe its actual internal properties, not by a simple buoyancy calculation.

Redaction: Jeffery, 2008jan01

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015 qfull 00350 2 5 0 mod thinking: archimedes principle, Tasmanian devil

16. We've all heard of Archimedes and King Hieron II's crown. But modern scientists and technologists are seldom called on to hallmark crowns. Let's apply Archimedes's principle to a real world situation.

a) You wish to accurately determine the density of your Tasmanian devil (Sarcophilus harrisii). You've already determined her mass to be 10.0 kg—between master and pet, what are a few scratches and one little bite to the bone? (Female Tasmanian devils have an average mass of about 6 kg, and so your devil is huge.) But now you need to determine her volume. What do you do? **HINT:** You'll need a tank of water with a water-level scale and some persuasion.

b) But Gris-Gris doesn't submerge, but just floats indignantly. By some miracle, you measure her unsubmerged volume to be $V_2 = 0.00100\,\mathrm{m}^3$. You know that the air density is $\rho_2 = 1.204\,\mathrm{kg/m}^3$ (at 20°C and 1 atm) and the water density is $\rho_1 = 998.29\,\mathrm{kg/m}^3$ (at 20°C and 1 atm). Note,

$$\frac{V_2}{V} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2} \ ,$$

where ρ is the floater's density. Solve for her density.

c) Now you say? And Gris-Gris says?

NOTE: To find out about Tasmanian devils including what they really sound like (but not what their density is) try the Tasmanian devil page:
`https://www.youtube.com/watch?v=_Ku_Wd5CFQw`

SUGGESTED ANSWER:

a) You her put in a tank of water (in a humane manner) and hope that she submerges fully so that you can measure her total volume. Her volume is measured from the increase in water volume of the tank measured off the water-level scale on the tank side.

b) We have two equations:

$$\rho = \frac{m}{V} \qquad \text{and} \qquad \frac{V_2}{V} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2}$$

and two unknowns ρ and V. We can solve for both unknowns, but we only want density. Behold:

$$1)\quad \rho = \frac{m}{V} \qquad 2)\quad V = \frac{m}{\rho} \qquad 3)\quad \frac{V_2}{m/\rho} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2} \qquad 4)\quad \frac{V_2(\rho_1 - \rho_2)}{m} = \frac{\rho_1}{\rho} - 1$$

$$5)\quad \rho = \frac{\rho_1}{V_2(\rho_1 - \rho_2)/m + 1} = (0.90777684\ldots)\,\mathrm{g/cm}^3 \qquad 6)\quad V = \frac{m}{\rho} = 11015.923\,\mathrm{cm}^3 \ .$$

c) You say Eureka! Gris-Gris says @#$+@!!!

Fortran-95 Code

```
print*
print*,'Tasmanian devil density'
xm=10.0_np
```

```
          v2=0.001_np
          rho1=998.29_np !
https://www.vip-ltd.co.uk/Expansion/Density_Of_Water_Tables.pdf
          rho2=1.204_np  ! https://en.wikipedia.org/wiki/Density_of_air
          rho=rho1/(v2*(rho1-rho2)/xm+1.0_np)
          v=xm/rho
          print*,'rho in g/cm**, v in cm**3'
          print*,rho*1.0e-3_np,v*1.0e+6_np
    !  0.90777684197431937873        11015.923228721113103
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Redaction: Jeffery, 2001jan01

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015 qfull 00410 2 3 0 moderate thinking: Guericke, hemispheres, horses

Extra keywords: Clydesdales. For calculus-based courses only until reevaluated.

17. Otto von Guericke (1602–1686) invented the air pump and wrote a book describing his invention and experiments that could be done with it: *Experimenta nova Magdeburgica de vacuo spatio* (1672). A famous illustration shows two teams of eight horses trying to separate a pair of joined brass hemispheres. Let the line through the horses and hemispheres be the x-axis. The hemispheres had been evacuated and external air pressure held them together. The hemispheres were about a foot in diameter. The total horse force needed **TO JUST SEPARATE** separate the hemispheres was about 7500 N. **NOTE:** The diameter and total horse force values are very approximate, and so any values calculated using them will be likewise approximate. Also uncertainties tend to be large for the difference of nearly equal values.

   a) Consider the x-direction forces on one hemisphere (the one that's roundy in the positive x-direction). Four forces act: the external air force, the internal air force (the vacuum's not complete), the normal force of the rim of the other hemisphere, and the pull force of the horses. For equilibrium

   $$0 = F_{\rm int} + F_{\rm ext} + F_{\rm N} + F_{\rm horse} \ .$$

   Note that the internal pressure force in the x-direction on a differential bit of sphere area dA is

   $$dF_{\rm int} = P_{\rm int}\, dA\hat n \cdot \hat x \ ,$$

   where $\hat n$ is the normal to the surface area. A bit of insight shows that

   $$F_{\rm int} = P_{\rm int}\pi R^2 \ ,$$

   where R is the radius of the hemisphere. What is the expression for $F_{\rm ext}$ in terms of external pressure? Get the sign of $F_{\rm ext}$ right.

   b) Given $R = 0.5$ ft (ft is foots, foots), $P_{\rm ext} = 1.01\times10^5$ Pa, and $F_{\rm horse} = 7500$ N, what is $P_{\rm int}$? Assume the hemispheres have negligible thickness.

   c) Would the horse force have to be any different to pull a hemisphere off a wall if the internal pressure were the same as in the two hemisphere experiment? Explain.

   d) Von Guericke's horses couldn't separate the hemispheres. As natural-born cowgirls and cowboys what would you say: did he use Clydesdales or ponies? Explain.

NOTE: My sources for this question are Cardwell (p. 98 and 117), HRW, p. 341, and the Clydesdale page:

http://www.imh.org/imh/bw/clyde.html

SUGGESTED ANSWER:

a) Clearly,

$$dF_{\rm ext} = P_{\rm ext}\, dA\hat n \cdot (-\hat x) \ ,$$

and so

$$F_{\rm ext} = -P_{\rm ext}\pi R^2 \ .$$

b) When the hemispheres are just on the verge of separation $F_{\rm N} = 0$. Thus

$$P_{\rm int} = P_{\rm ext} - \frac{F_{\rm horse}}{\pi R^2} \approx 10^5 - \frac{7500}{\pi \times 0.023} \approx 10^5 - 1.1\times10^5 \approx 0\,{\rm Pa} \ .$$

I've used the fact that there's no such thing as negative pressure. My values for the input parameters are of only low accuracy, so a slightly negative result isn't a worry. The actual internal pressure was just small compared to external pressure.

c) Nope. The force situation for one hemisphere on a wall is exactly the same as it is for one hemisphere in the two hemisphere situation. In the wall case, the wall provides the normal force on the rim. But just trying to pull a hemisphere off a wall wouldn't have been so dramatic.

d) Well 7500 N divided by 8 horses is about 940 newtons per horse or about 200 pounds per horse. A typical Clydesdale stands 17 hands (5'8" at the withers) and weighs 2000 , or so the Clydesdale page tells me. I think von Guericke's horses were closer to ponies.

Redaction: Jeffery, 2001jan01

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015 qfull 00610 1 3 0 easy math: Pascal's principle, lifting a car

Extra keywords: Doesn't use Pascal's principle really, but conventional with it.

18. You have a static water-based hydraulic system with two circular pistons. Piston 1 is 1.0 cm in **DIAMETER** and you push on it with 2.0 N of force with your finger. Piston 2 is vertical and supporting a car of mass 2.0×10^3 kg against gravity. The piston 2 bottom is 2.0 m above the location of the piston 1 bottom. Air pressure can be considered constant with height.

a) What expression for the pressure of the 2nd piston as a function of the pressure of the 1st piston.

d) What is the **DIAMETER** of the 2nd piston?

SUGGESTED ANSWER:

a) The pressure difference between the two levels is only due to the height of water (which can also be called the head of water). Piston 2 is higher and so its pressure is reduced by the pressure caused by the head of water. Thus

$$P_2 = P_1 - \rho g y \ ,$$

where we measure y upward from a zero level set at the 1st piston height and ρ is water density.

b) Since

$$P_2 = P_1 - \rho g y \ ,$$

we find

$$\frac{F_2}{(\pi/4)d_2^2} + P_{\text{air}} = \frac{F_1}{(\pi/4)d_1^2} + P_{\text{air}} - \rho g y \ ,$$

where F_1 is the finger force exerted on piston 1, F_2 is the force exerted on piston 2 by the weight of the car, d_1 is the diameter of piston 1, d_2 is the diameter of piston 2, and p_{air} is the air pressure approximated as a constant.

Notice I'm thinking of the pressure caused by the finger and the weight of the car as not including air pressure. This is usually the best way to think of things. To understand this let us consider things very generally. Say a static system is embedded in an ambient medium of constant pressure. By Pascal's principle, the ambient pressure is communicated everywhere throughout the system in some way and canceled. The cancellation is usually by some increased internal pressure in the system that results from the compression by the ambient medium. This does not mean the system is necessarily at constant pressure at the ambient medium pressure. Pockets in the system encased in material that doesn not respond much to ambient pressure can have very different pressures. Nevertheless, anything not perfectly rigid or enclosed by an effectively perfectly rigid casing responds to ambient pressure. The ambient pressure can be neglected insofar as the internal changes to the system caused by the ambient pressure are negligible or just considered givens. For instance a lump of steel in an ambient medium is compressed a bit by the ambient medium pressure. But for ordinary Earth surface pressures, this compression is usually negligible for many purposes. But the compression creates a compensating pressure that holds the steel up under the ambient pressure. We can often just neglect ambient pressure, compression, and compensating pressure. If the system changes, then the effect of the ambient medium pressure is usually still canceled through the change as long as the change is slow on the time scale needed for sound waves to cancel local pressure variations that the change causes.

Now one complication to the above picture is the change in ambient pressure with height. (We are limiting our discussion to the Earth's surface.) As long as these changes are small relative to other effects in the system, they can be neglected. If the ambient medium is air, then for most human purposes air pressure variation can be neglected. The scale height for change of an amount p in air pressure near the Earth's surface is

$$\frac{P}{|dP/dr|} = \frac{p}{\rho g} \approx \frac{10^5}{1.21 \times 9.8} \approx 8000\,\mathrm{m}\ .$$

Thus, only a 1 % change in pressure can be expected over 80 m. The first-order way to account for the ambient medium pressure change with height is by introducing the buoyant force which gives the net force on a system due to this change. In the case, of air the buoyant force of air is of order a thousand or less than the weight of a solid or liquid object because their density is of order a thousand for more denser. Thus, the buoyant force of air can be neglected in many cases.

To turn to our present case, we exclude pressure force from the finger force. This is because most means for measuring the finger force will not include pressure force pushing on it. Say you press the finger on a spring scale and measure a force. Does that measured force include the air pressure force? No. The scale zero-point already accounts for the pressure force of air. Usually because the spring scale has air inside its casing the air pressure force is partially canceled no matter what it is. Actually, the extension of the spring must vary a little with air pressure because of compression, but this is probably so tiny for normal Earth surface pressure variations that no correction for the varying zero-point of the scale is needed for ordinary scales which are not used for high-precision work. Similarly almost all measures of force don't include air pressure or ambient pressure of any kind explicitly since this pressure usually cancels out nearly exactly.

We note that the air pressure forces do act on the two pistons even if no air directly touches them. The air pressure is communicated through the finger and car body. This is because this must force must be communicated through the finger and car according to Pascal's principle. Of course, as we just argued above such pressure forces can usually be neglected since they just cancel out. They do, in fact, cancel out in the present case because we can neglect the variation in air pressure over the small height changes in our hydraulic system.

Now we finally complete the calculation. After canceling out the air pressure and rearranging, we find

$$\begin{aligned} d_2 &= \sqrt{\frac{F_2}{F_1/d_1^2 - \rho g y \pi/4}} = \sqrt{\frac{mg}{F_1/d_1^2 - \rho g y \pi/4}} \\ &\approx \sqrt{\frac{20000}{2/10^{-4} - 10^3 \times 15}} \approx \sqrt{\frac{20000}{5 \times 10^3}} \approx \sqrt{4} \\ &\approx 2.0\,\mathrm{m}\ , \end{aligned}$$

to about 2-digit accuracy

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{circa 2025)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\mathrm{tan}} = r\alpha$$

$$\vec{a}_{\mathrm{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\mathrm{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\mathrm{total}}} = \frac{\sum_{\mathrm{sub}} m_{\mathrm{sub}}\vec{r}_{\mathrm{cm\ sub}}}{m_{\mathrm{total}}} \qquad \vec{v}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\mathrm{total}}} \qquad \vec{a}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\mathrm{total}}}$$

$$\vec{r}_{\mathrm{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\mathrm{total}}}$$

$$\vec{F}_{\mathrm{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\mathrm{m/s^2}$$

$$\vec{F}_{\mathrm{normal}} = -\vec{F}_{\mathrm{applied}} \qquad F_{\mathrm{linear}} = -kx$$

$$f_{\mathrm{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\mathrm{parallel}}(s) \qquad T = T_0$$

$$F_{\mathrm{f\ static}} = \min(F_{\mathrm{applied}}, F_{\mathrm{f\ static\ max}}) \qquad F_{\mathrm{f\ static\ max}} = \mu_{\mathrm{static}}F_{\mathrm{N}} \qquad F_{\mathrm{f\ kinetic}} = \mu_{\mathrm{kinetic}}F_{\mathrm{N}}$$

$$v_{\mathrm{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\mathrm{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\mathrm{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\mathrm{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\mathrm{drag,lin}} = bv \qquad v_{\mathrm{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\mathrm{T}}}{g} = \frac{m}{b} \qquad v = v_{\mathrm{T}}(1 - e^{-t/\tau})$$

$$F_{\mathrm{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\mathrm{T}} = \sqrt{\frac{mg}{b}}$$

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14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

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15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\rm net} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\rm net} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2\rho\,dV \qquad I_{\text{parallel axis}} = I_{\rm cm} + mR_{\rm cm}^2 \qquad I_z = I_x + I_y$$

$$I_{\rm cyl,shell,thin} = MR^2 \qquad I_{\rm cyl} = \frac{1}{2}MR^2 \qquad I_{\rm cyl,shell,thick} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\rm rod,thin,cm} = \frac{1}{12}ML^2 \qquad I_{\rm sph,solid} = \frac{2}{5}MR^2 \qquad I_{\rm sph,shell,thin} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\rm rot} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\,d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta K_{\rm rot} = W_{\rm net} = \int \tau_{z,\rm net}\,d\theta \qquad \Delta U_{\rm rot} = -W = -\int \tau_{z,\rm con}\,d\theta$$

$$\Delta E_{\rm rot} = K_{\rm rot} + \Delta U_{\rm rot} = W_{\rm non,rot} \qquad \Delta E = \Delta K + K_{\rm rot} + \Delta U = W_{\rm non} + W_{\rm rot}$$

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19 **Static Equilibrium**

$$\vec{F}_{\rm ext,net} = 0 \qquad \vec{\tau}_{\rm ext,net} = 0 \qquad \vec{\tau}_{\rm ext,net} = \tau'_{\rm ext,net} \quad \text{if } F_{\rm ext,net} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \ \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_\mathrm{T}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_\mathrm{max}\sin[k(x \mp vt)] = y_\mathrm{max}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_\mathrm{max}^2$$

$$y = 2y_\mathrm{max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L}\ :\ n = 1, 3, 5, \ldots \qquad f_\mathrm{medium} = \frac{f_0}{1 - v_0/v_\mathrm{medium}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_\mathrm{K} = T_\mathrm{C} + 273.15\,\mathrm{K} \qquad T_\mathrm{F} = 1.8 \times T_\mathrm{C} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_\mathrm{avg} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_\mathrm{RMS}^2\right)$$

$$v_\mathrm{RMS} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$