Intro Physics Semester I

Name:

Homework 6a: Newtonian Physics: More of the Same: Problems Explicated in Class Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

006 qfull 01010 1 3 0 easy math: generalized Newton's 2nd law

1. Newton's 2nd law in ordinary form is

 $\vec{F}_{\rm net} = m \vec{a}_{\rm cm}$,

where \vec{F}_{net} is the net force on a system (AKA object, AKA body), the system mass is m, and the system acceleration is \vec{a}_{cm} . Newton's 2nd law is the basis for determining the motion of the system center of mass: i.e., as a functions of time acceleration \vec{a}_{cm} , velocity \vec{v}_{cm} , and position \vec{r}_{cm} . Like all general physical laws, Newton's 2nd law is what is eternally true everywhere: i.e., eternally true everywhere in the classical limit. Other features of the system and its environment are contingent on the physical history.

In fact, Newton's 2nd law has to be generalized to allow for inflow (which is outflow if negative) of mass which can change the (system) acceleration $\vec{a}_{\rm cm}$ without a net external force because it changes the (system) momentum. Note, we drop the center-of-mass subscript cm hereafter for formula simplicity. Now momentum (i.e., linear momentum, not angular momentum) for a system is defined

$$\vec{p} = m\vec{v}$$

The generalized Newton's 2nd law is

$$\vec{F}_{\rm net} + \vec{v}_{\rm flow} \frac{dm}{dt} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt} = m\vec{a} + \vec{v} \frac{dm}{dt} ,$$

where dm/dt is the mass inflow rate which can be positive (i.e., actual inflow) or negative (i.e., actual outflow) and $\vec{v}_{\rm flow}$ is the velocity of inflowing mass. Note, we have assumed there is a single $\vec{v}_{\rm flow}$ for simplicity in our discussion. The inflow can change the momentum, and therefore accelerate the system without exerting any net external force. The inflow when not part of the system can exert an external force, but it cannot by definition when part of the system since then it can only exert internal forces. In fact, it may be an analysis choice when to consider the inflow as part of the system.

In this problem, we consider the generalized Newton's 2nd law.

- a) If \vec{F}_{net} and dm/dt are zero, what can one say about the system momentum and acceleration?
- b) If \vec{F}_{net} is zero, $dm/dt \neq 0$, and $\vec{v}_{flow} = \vec{v}$, what can one say about the system momentum and acceleration?
- c) What is the explicit formula for acceleration?
- d) An interesting special case for the generalized Newton's 2nd law is the (Tsiolkovsky) rocket problem (Wikipedia: Tsiolkovsky rocket equation). Say you have a rocket in empty space with $\vec{F}_{net} = 0$. It can still be accelerated by thrust: ejecting burnt exhaust fuel opposite the direction of motion. The speed of the exhausted fuel relative to the rocket is called the exhaust speed v_{ex} which is a parameter of the rocket. The rate of mass flow (i.e., the rate of ejection of exhausted fuel) dm/dt < 0 is also a parameter of the rocket. For our analysis of the rocket problem, we will consider only 1-dimensional motion. Specialize the generalized Newton's second law for the 1-dimentional case of the rocket problem.
- e) What is the velocity of the ejected exhausted fuel in terms of the generalized Newton's 2nd law. What is this velocity in the outside inertial frame the rocket is traveling in, not relative to the rocket? What does it mean if the velocity is positive?
- f) Making use of the part (e) result, write down the formula for the acceleration of the rocket simplified as much as possible.
- g) Solve the part (f) equation (which is a differential equation) for velocity as a function of mass by integration recalling a = dv/dt. Assume the initial mass m_0 and the initial velocity is v_0 . What

happens to velocity as $m \to 0$? Why must the velocity formula actually fail to be physically real if the velocity gets too large? Why is this failure unlikely in practice.

g) Invert formula found in part (g) to find the amount of fuel m_{fuel} needed to achieve a given change in velocity $\Delta v = v - v_0$.

SUGGESTED ANSWER:

- a) In this case, $d\vec{p}/dt = 0$, and thus \vec{p} is a constant and we have conservation of momentum. The system also has $\vec{a} = 0$, and so is unaccelerated.
- b) In this case, $d\vec{p}/dt \neq 0$, and thus \vec{p} is not constant and we do not have conservation of momentum. However, since the inflow has the same velocity as the system, the acceleration is zero: i.e., $\vec{a} = 0$.
- c) Behold:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} + \frac{(\vec{v}_{\text{flow}} - \vec{v})}{m} \frac{dm}{dt} \; .$$

d) Behold:

$$v_{\rm flow} \frac{dm}{dt} = \frac{dp}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt} = ma + v \frac{dm}{dt} \ ,$$

e) The exhausted fuel is the inflow mass, but it is negative since dm/dt < 0. So its velocity is the inflow velocity which is

$$v_{\text{flow}} = v - v_{\text{ex}}$$
.

If $v_{\text{flow}} > 0$, the exhausted fuel is moving in the direction of the rocket, but at a lower velocity. f) Behold:

$$a = -\frac{v_{\rm ex}}{m}\frac{dm}{dt}$$

g) Behold:

1)
$$\frac{dv}{dt} = -\frac{v_{\text{ex}}}{m}\frac{dm}{dt}$$
2)
$$dv = -v_{\text{ex}}\frac{dm}{m}$$
3)
$$v - v_0 = -v_{\text{ex}}\ln\left(\frac{m}{m_0}\right)$$
4)
$$v = v_0 + v_{\text{ex}}\ln\left(\frac{m_0}{m}\right)$$

As $m \to 0$, the velocity $v \to \infty$. If the velocity gets too high, the formula will fail since relativistic effects will arise which our classical calculation does include. This failure is not likely to happen in practice because a rocket can only have so much of its mass in fuel.

h) Behold:

1)
$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right)$$
 2) $\frac{\Delta v}{v_{\text{ex}}} = \ln\left(\frac{m_0}{m}\right)$
3) $\frac{m_0}{m} = e^{\Delta v/v_{\text{ex}}}$ 4) $m_{\text{fuel}} = m_0 - m = m\left(\frac{\Delta v}{v_{\text{ex}}} - 1\right)$

Redaction: Jeffery, 2008jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$\begin{split} c &= 2.99792458 \times 10^8 \,\mathrm{m/s} \approx 2.998 \times 10^8 \,\mathrm{m/s} \approx 3 \times 10^8 \,\mathrm{m/s} \approx 1 \,\mathrm{lyr/yr} \approx 1 \,\mathrm{ft/ns} & \mathrm{exact} \ \mathrm{by} \ \mathrm{definition} \\ e &= 1.602176487(40) \times 10^{-19} \,\mathrm{C} \\ G &= 6.67384(80) \times 10^{-11} \,\mathrm{N} \,\mathrm{m}^2/\mathrm{kg}^2 & (2012, \,\mathrm{CODATA}) \\ g &= 9.8 \,\mathrm{m/s}^2 & \mathrm{fiducial} \ \mathrm{value} \\ k &= \frac{1}{4\pi\varepsilon_0} = 8.987551787 \ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10} \,\mathrm{N} \,\mathrm{m}^2/\mathrm{C}^2 \mathrm{exact} \ \mathrm{by} \ \mathrm{definition} \\ k_{\mathrm{Boltzmann}} &= 1.3806504(24) \times 10^{-23} \,\mathrm{J/K} = 0.8617343(15) \times 10^{-4} \,\mathrm{eV/K} \approx 10^{-4} \,\mathrm{eV/K} \\ m_e &= 9.10938215(45) \times 10^{-31} \,\mathrm{kg} = 0.510998910(13) \,\mathrm{MeV} \\ m_p &= 1.672621637(83) \times 10^{-27} \,\mathrm{kg} = 938.272013(23), \,\mathrm{MeV} \\ \varepsilon_0 &= \frac{1}{\mu_0 c^2} = 8.8541878176 \ldots \times 10^{-12} \,\mathrm{C}^2/(\mathrm{N} \,\mathrm{m}^2) \approx 10^{-11} & \mathrm{vacuum} \ \mathrm{permittivity} \ (\mathrm{exact} \ \mathrm{by} \ \mathrm{definition}) \\ \mu_0 &= 4\pi \times 10^{-7} \,\mathrm{N/A}^2 & \mathrm{exact} \ \mathrm{by} \ \mathrm{definition} \end{split}$$

2 Geometrical Formulae

$$C_{\rm cir} = 2\pi r$$
 $A_{\rm cir} = \pi r^2$ $A_{\rm sph} = 4\pi r^2$ $V_{\rm sph} = \frac{4}{3}\pi r^3$

$$\Omega_{\rm sphere} = 4\pi \qquad d\Omega = \sin\theta \, d\theta \, d\phi$$

3 Trigonometry Formulae

S

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$
$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^{\circ})$$

 $\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$

$$\sin(-\theta) = -\sin(\theta)$$
 $\cos(-\theta) = \cos(\theta)$ $\tan(-\theta) = -\tan(\theta)$

$$\sin(\theta + 90^{\circ}) = \cos(\theta) \qquad \cos(\theta + 90^{\circ}) = -\sin(\theta) \qquad \tan(\theta + 90^{\circ}) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

 $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$$\sin(2a) = 2\sin(a)\cos(a)$$
 $\cos(2a) = \cos^2(a) - \sin^2(a)$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$$

$$\sin^2 \theta = \frac{1}{2} [1 - \cos(2\theta)] \qquad \cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$
$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx}$$
 $\frac{1}{1-x} \approx 1+x$: $(x \ll 1)$

$$\sin\theta\approx\theta\qquad \tan\theta\approx\theta\qquad \cos\theta\approx1-\frac{1}{2}\theta^2\qquad {\rm all \ for \ }\theta<<1$$

5 Quadratic Formula

If
$$0 = ax^2 + bx + c$$
, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2} \left[b + \operatorname{sgn}(b) \sqrt{b^2 - 4ac} \right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
 $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi?$ $\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$
 $\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi?$ $\theta = \cos^{-1}\left(\frac{a_z}{a}\right)$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

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$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_yb_z - b_ya_z, a_zb_x - b_za_x, a_xb_y - b_xa_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \dots$$

$$\int_{a}^{b} f(x) dx = F(x) \Big|_{a}^{b} = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x} dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$
 $v = \frac{dx}{dt}$ $a_{\text{avg}} = \frac{\Delta v}{\Delta t}$ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $g = 9.8 \,\mathrm{m/s^2}$

 $x_{\rm rel} = x_2 - x_1$ $v_{\rm rel} = v_2 - v_1$ $a_{\rm rel} = a_2 - a_1$

$$x' = x - v_{\text{frame}}t$$
 $v' = v - v_{\text{frame}}$ $a' = a$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\rm avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\rm avg} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$$

10 **Projectile Motion**

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos \theta} \qquad y = y_0 + x \tan \theta - \frac{x^2 g}{2v_0^2 \cos^2 \theta}$$

$$x_{\text{for } y \max} = \frac{v_0^2 \sin \theta \cos \theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2 \sin^2 \theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$
$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$
$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$
$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{tan} = r\alpha$$
$$\vec{a}_{centripetal} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{centripetal} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\rm cm} = \frac{\sum_i m_i \vec{r}_i}{m_{\rm total}} = \frac{\sum_{\rm sub} m_{\rm sub} \vec{r}_{\rm cm \ sub}}{m_{\rm total}} \qquad \vec{v}_{\rm cm} = \frac{\sum_i m_i \vec{v}_i}{m_{\rm total}} \qquad \vec{a}_{\rm cm} = \frac{\sum_i m_i \vec{a}_i}{m_{\rm total}}$$
$$\vec{r}_{\rm cm} = \frac{\int_V \rho(\vec{r}) \vec{r} \, dV}{m_{\rm total}}$$
$$\vec{F}_{\rm net} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8 \,\mathrm{m/s^2}$$
$$\vec{F}_{\rm normal} = -\vec{F}_{\rm applied} \qquad F_{\rm linear} = -kx$$
$$f_{\rm normal} = \frac{T}{r} \qquad T = T_0 - F_{\rm parallel}(s) \qquad T = T_0$$

 $F_{\rm f\ static} = \min(F_{\rm applied}, F_{\rm f\ static\ max}) \qquad F_{\rm f\ static\ max} = \mu_{\rm static} F_{\rm N} \qquad F_{\rm f\ kinetic} = \mu_{\rm kinetic} F_{\rm N}$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$
$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$
$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$
$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$
$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

 $\Delta KE = W_{\rm net} \quad \Delta PE_{\rm of \ a \ conservative \ force} = -W_{\rm by \ a \ conservative \ force} \quad \Delta E = W_{\rm nonconservative}$

$$F = -\frac{dPE}{dx}$$
 $\vec{F} = -\nabla PE$ $PE = \frac{1}{2}kx^2$ $PE = mgy$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}}$$
 $\Delta K E_{\text{cm}} = W_{\text{net,external}}$ $\Delta E_{\text{cm}} = W_{\text{not}}$
 $\vec{p} = m\vec{v}$ $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$ $\vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$

$$m\vec{a}_{\rm cm} = \vec{F}_{\rm net \ non-flux} + (\vec{v}_{\rm flux} - \vec{v}_{\rm cm})\frac{dm}{dt} = \vec{F}_{\rm net \ non-flux} + \vec{v}_{\rm rel}\frac{dm}{dt}$$

 $v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right)$ rocket in free space

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t) dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i}$$
 1-d Elastic Collision Expression

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad 1 \text{-d Elastic Collision Expression}$$

 $v_{2'} - v_{1'} = -(v_2 - v_1)$ $v_{rel'} = -v_{rel}$ 1-d Elastic Collision Expressions

17 Rotational Kinematics

$$2\pi = 6.2831853\dots$$
 $\frac{1}{2\pi} = 0.15915494\dots$

$$\frac{180^{\circ}}{\pi} = 57.295779 \dots \approx 60^{\circ} \qquad \frac{\pi}{180^{\circ}} = 0.017453292 \dots \approx \frac{1}{60^{\circ}}$$
$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$
$$\omega = \alpha t + \omega_0 \qquad \Delta \theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha \Delta \theta$$
$$\Delta \theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta \theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p}$$
 $\vec{\tau} = \vec{r} \times \vec{F}$ $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

$$L_z = RP_{xy}\sin\gamma_L$$
 $au_z = RF_{xy}\sin\gamma_{ au}$ $L_z = I\omega$ $au_{z,\text{net}} = I\alpha$

$$I = \sum_{i} m_{i} R_{i}^{2} \qquad I = \int R^{2} \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + m R_{\text{cm}}^{2} \qquad I_{z} = I_{x} + I_{y}$$

$$I_{\rm cyl, shell, thin} = MR^2$$
 $I_{\rm cyl} = \frac{1}{2}MR^2$ $I_{\rm cyl, shell, thick} = \frac{1}{2}M(R_1^2 + R_2^2)$

$$I_{\rm rod,thin,cm} = \frac{1}{12}ML^2 \qquad I_{\rm sph,solid} = \frac{2}{5}MR^2 \qquad I_{\rm sph,shell,thin} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\rm rot} = \frac{1}{2}I\omega^2$$
 $dW = \tau_z \,d\theta$ $P = \frac{dW}{dt} = \tau_z \omega$

$$\Delta K E_{\rm rot} = W_{\rm net} = \int \tau_{z,\rm net} \, d\theta \qquad \Delta P E_{\rm rot} = -W = -\int \tau_{z,\rm con} \, d\theta$$

 $\Delta E_{\rm rot} = K E_{\rm rot} + \Delta P E_{\rm rot} = W_{\rm non, rot} \qquad \Delta E = \Delta K E + K E_{\rm rot} + \Delta P E = W_{\rm non} + W_{\rm rot}$

19 Static Equilibrium

$$\vec{F}_{\mathrm{ext,net}} = 0$$
 $\vec{\tau}_{\mathrm{ext,net}} = 0$ $\vec{\tau}_{\mathrm{ext,net}} = \tau'_{\mathrm{ext,net}}$ if $F_{\mathrm{ext,net}} = 0$

$$0 = F_{\text{net }x} = \sum F_x$$
 $0 = F_{\text{net }y} = \sum F_y$ $0 = \tau_{\text{net}} = \sum \tau$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

 $R_{\text{Earth,mean}} = 6371.0 \,\text{km}$ $R_{\text{Earth,equatorial}} = 6378.1 \,\text{km}$ $M_{\text{Earth}} = 5.9736 \times 10^{24} \,\text{kg}$

 $R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11} \,\mathrm{m} = 1.0000001124 \,\mathrm{AU} \approx 1.5 \times 10^{11} \,\mathrm{m} \approx 1 \,\mathrm{AU}$

 $R_{\rm Sun,equatorial} = 6.955 \times 10^8 \approx 109 \times R_{\rm Earth,equatorial}$ $M_{\rm Sun} = 1.9891 \times 10^{30} \, \rm kg$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle
$$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}})$$
 $\Delta p = \Delta p_{\text{ext}}$
Archimedes principle $F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid $R_V = Av = \text{Constant}$
Bernoulli's equation $p + \frac{1}{2}\rho v^2 + \rho gy = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$
$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi \sqrt{\frac{I}{mgr}} \qquad P = 2\pi \sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\rm T}}{\mu}} \qquad y = f(x \mp vt)$$

 $y = y_{\max} \sin[k(x \mp vt)] = y_{\max} \sin(kx \mp \omega t)$

Period
$$= \frac{1}{f}$$
 $k = \frac{2\pi}{\lambda}$ $v = f\lambda = \frac{\omega}{k}$ $P \propto y_{\max}^2$

$$y = 2y_{\max}\sin(kx)\cos(\omega t)$$
 $n = \frac{L}{\lambda/2}$ $L = n\frac{\lambda}{2}$ $\lambda = \frac{2L}{n}$ $f = n\frac{v}{2L}$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$
$$I = \frac{P}{4\pi r^2} \qquad \beta = (10 \text{ dB}) \times \log\left(\frac{I}{I_0}\right)$$
$$f = n\frac{v}{4L} : n = 1, 3, 5, \dots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$
$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T \, dS - p \, dV$$

$$T_{\rm K} = T_{\rm C} + 273.15 \,{\rm K}$$
 $T_{\rm F} = 1.8 \times T_{\rm C} + 32^{\circ}{\rm F}$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\rm RMS} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$
$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$
$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$