Intro Physics Semester I — Name:

Homework 6: Newtonian Physics: More of the Same: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

1. "Let's play *Jeopardy*! For $100, the answer is: The macroscopic binding force between smooth surfaces that is parallel to the surfaces."

   What is ____________, Alex?

   a) the linear restoring force b) the spring force c) the tension force
   d) the normal force e) friction

2. The magnitude of the kinetic friction force between a body and a surface equals a coefficient of friction times:

   a) the area of macroscopic contact between the body and the surface.
   b) the magnitude of the normal force acting on the body.
   c) the mass of the body.
   d) the density of the body.
   e) the density of the air surrounding the body.

3. Which is larger: the coefficient of static or kinetic friction?

   a) They are always equal.
   b) Neither. The larger depends on the materials involved and its about a 50-50 split on which is larger.
   c) The kinetic coefficient is always larger.
   d) The kinetic coefficient is usually larger.
   e) The static coefficient is almost always (always?) larger.

4. The friction between sliding surfaces tends to change macroscopic kinetic energy into:

   a) potential energy. b) rest mass energy. c) thermal or heat energy.
   d) magnetic energy. e) nothing.

5. You are pushing a coffee cup across a table (just an ordinary table, not an imaginary frictionless table) at a **CONSTANT** velocity. The magnitudes of the push force and frictional force are $|F_{\rm p}|$ and $|F_{\rm f}|$, respectively.

   a) $|F_{\rm f}| > |F_{\rm p}|$ and this is why the cup does not accelerate.
   b) The $|F_{\rm f}| < |F_{\rm p}|$, but nevertheless the frictional force prevents any acceleration.
   c) There is no frictional force when you push the cup at a constant velocity. Thus, $|F_{\rm f}| = 0$ and clearly then $|F_{\rm p}| > |F_{\rm f}|$.
   d) The $|F_{\rm f}|$ must **EQUAL** $|F_{\rm p}|$ in order for there to be no acceleration.
   e) The $|F_{\rm f}|$ must be **TWICE** $|F_{\rm p}|$ in order for there to be no acceleration.

6. Will will now determine the coefficient of static friction $\mu_{\rm st}$ from an empirical measurement using an ajustable incline. The adjustable angle of incline from the horizontal is θ.

   a) Write down Newton's 2nd law for a block sitting at rest on the incline for two directions: perpendicular to the incline and parallel to it. The only forces are the gravity, the normal force, and friction.

   b) The angle of adjustable incline is increased just to the slipping point for the block. Give the parallel 2nd law equation just before slipping occurs.

   c) Solve for the $\mu_{\rm st}$.

7. The formula

$$F_{r,{\rm net}}\hat r + F_{\theta,{\rm net}}\hat\theta = m\left[\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat r + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat\theta\right]$$

is:

a) Newton's 2nd law in spherical polar coordinates. b) Newton's 2nd law in polar coordinates.
c) the centripetal force formula. d) the centripetal acceleration formula.
e) the simple harmonic oscillator formula.

8. Uniform circular motion is motion in a/an:

   a) circle at a constant **VELOCITY**. b) oval at a constant **VELOCITY**.
   c) oval at a constant **SPEED**. d) circle at a constant **SPEED**.
   e) circle at a nonconstant **SPEED**.

9. The formula for the magnitude of centripetal acceleration is:

$$a_{\text{centripetal}} = \frac{v^2}{r} \ ,$$

   where v is the speed of a uniform circular motion and r is the radius of the motion. Say $v = 10\,\text{m/s}$ and $r = 5\,\text{m}$, what is $a_{\text{centripetal}}$?

   a) $20\,\text{m/s}^2$. b) $10\,\text{m/s}^2$. c) $5\,\text{m/s}^2$. d) $100\,\text{m/s}^2$. e) $15\,\text{m/s}^2$.

10. The centripetal force is:

    a) a mysterious force that **APPEARS** whenever an object goes into uniform circular motion.
    b) a mysterious force that tries to throw you **OFF** playground merry-go-rounds.
    c) in fact $m\vec{a}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. It is **NOT** a mysterious force that appears whenever you have uniform circular motion: it is a force requirement to be satisfied for uniform circular motion. Particular physical forces (e.g., gravity, tension force, and normal force) must act (sometimes in combination) to give a centripetal force which then causes uniform circular motion.
    d) in fact $m\vec{a}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. The force itself is **ALWAYS** a field force emanating from the center of motion that pulls on the circling object atom by atom.
    e) a mysterious force that **DISAPPEARS** whenever an object goes into circular motion.

11. There is a hump on the road with a cylindrical shape. The radius of the hump is $14.7\,\text{m}$. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

    a) $12\,\text{m/s}$. b) $14.7\,\text{m/s}$. c) $144\,\text{m/s}$. d) $10\,\text{m/s}$. e) $10.4\,\text{m/s}$.

12. The banking angle formula is ____________. **HINT:** Use dimensional analysis. Ask yourself what formula has reasonable limiting behavior when input values go to extremes. Or just derive it using the centripetal force formula and the 2nd law.

    a) $\theta = \tan^{-1}\left(\frac{v}{rg}\right)$ b) $\theta = \tan^{-1}\left(\frac{rg}{v^2}\right)$ c) $\theta = \tan^{-1}\left(\frac{v^2 r}{g}\right)$ d) $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$
    e) $\theta = \tan^{-1}\left(\frac{g}{v^2 r}\right)$

13. The drag force (the resistive force of a fluid: e.g., air drag) is an example of a/an:

    a) linear (or Hooke's law) force in all cases. b) gravitational force. c) tension force.
    d) contact force. e) animal force.

14. In general the drag force on an object moving through a fluid is complex, but there are two well-known drag force laws that have wide application in different physical regimes: the drag equation law and the Stokes law. They both depend linearly on a power of the relative velocity opposite the direction of the motion of the object (i.e., the flow speed). The powers for the drag equation law and the Stokes law are, respectively:

    a) 1 and 2. b) 2 and 1. c) 2 and 3. d) 1 and 3. e) 3/2 and 5/2

15. In what situations, if any, can a body move in a circular path at constant speed without a centripetal force?

    a) None. b) In certain special non-inertial frames. c) In all non-inertial frames.
    d) In all inertial frames. e) Always.

16. Is-Hilda the ladybug is on a vinyl record spinning at 78 rpm. She starts at the center and six-leggedly walks radially outward. Her radial velocity (caused by her walking) is negligible compared to her tangential velocity (caused by the record motion). At 6 cm from the center, Is-Hilda suddenly slides off the record. The record counterfactually is a smooth surface—and isn't corrugated—and Is-Hilda is an ideal ladybug without sticky feet. What is the static friction coefficient between her and the record? What is the kinetic friction coefficient if you can determine it?

17. There is a conical pendulum of length ℓ with point mass bob of mass m. The bob is executing uniform circular motion with velocity v and radius r (which is measured from the axis of rotation not along the length of the pendulum). The pendulum has an ideal rope with tension T. Note, an ideal rope is massless and can only exert a tension force which is uniform along the rope and only turns on when the rope is taut. There is no friction, no drag, and no dissipation forces in general. Thus the moition is perpetual. The rope sweeps out a cone and the opening angle of the cone from the vertical is θ.

    a) Write down Newon's 2nd law for the horizontal direction (taking inward to the axis of rotation as positive) and vertical direction (taking upward as positive). Make use of the centripetal acceleration formula. Remember, the rope sweeps out a cone and the opening angle of the cone from the vertical is θ. **HINT:** Draw diagram.

    b) Solve for θ as a function of g, r and v and for T as a functions of m, g and θ.

    c) For a person just swinging a conical pendulum by hand, one can set m and ℓ directly and with some skill period $p = 2\pi r/v$. So those variables are the obvious control parameters of the system. Determine in order formulae for θ, T, r, and v in terms of m, ℓ, and p.

    d) Determine p_{max}, the maximum period for the circular motion allowed by the formulae. For what angle θ does this occur?

    e) You swing a crude pendulum with $\ell = 0.5\,\text{m}$ such that $p \approx 1\,\text{s}$. What is the angle θ?

18. Consider an ordinary road corner that is level ground. Such corners are usually not banked very much, in fact.

    a) What force supplies the centripetal force that allows a vehichle to make the corner?

    b) For a corner turn with radius of curvature r and a vehicle of mass m, derive the formula for the maximum turn speed v before slipping occurs.

    c) Say $r = 10\,\text{m}$ (which seems reasonable for ordinary corners) and $\mu_{\text{st}} = 1.0$ (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Now say $\mu_{\text{st}} = 0.30$ (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Convert the answers to miles per hour.

19. The drag equation drag law is

    $$F_{\text{DE}} = \frac{1}{2}\rho v^2 C_{\text{D}} A = bv^2$$

    where v is flow speed, ρ is fluid density, $(1/2)\rho v^2$ is the relative kinetic energy density of the fluid, C_{D} is the drag coefficient (which depends on many factors in general, but the always depends on the shape of the object and probably is often determined empirically), and A is the reference area (for many objects just projected frontal area of the object), and b is a combined coefficient introduced for simplicity.

    a) Taking downward as positive, write down Newton's 2nd law for an object of mass m falling under gravity with drag acting. Determine the formula for acceleration in the form $a = g[1 - \ldots]$ with v/v_{ter} as one for the terms with v_{ter} defined appropriately.

    b) Now define $z = v/v_{\text{ter}}$ and reformulate the formula for acceleration a as a formula with differential $dt = (v_{\text{ter}}/g)\,dz/(\ldots)$.

    c) Integrate the dt expression from part (b) to obtain $t = t(v)$ assuming initial time and velocity are zero. You will need the table integral

    $$\int \frac{dz}{1 - z^2} = \text{artanh}(z) \qquad \text{for } z < 1\ .$$

    where *artanh* is the inverse hyperbolic tangent function.

d) Invert the expression from part (c) to obtain the function $v(t)$. Note the hyperbolic tangent function is defined

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \ .$$

It is an odd function (i.e., $\tanh(-x) = -\tanh(x)$ and $\lim_{x\to\pm\infty} \tanh(x) = \pm 1$. What is the terminal velocity (i.e., the velocity as $t \to \infty$)? What is the scale time (i.e, the time when hyperbolic tangent argument is 1)? What is v at the scale time? What is the meaning of the scale time?

20. There is a block of mass m on an incline with inclination angle θ. The incline has static friction coefficient $\mu_{\rm s}$ and kinetic friction coefficient $\mu_{\rm k}$.

a) The block is not sliding. Determine the gravity component along the incline taking downward as positive. Determine the normal force taking outward from the incline as postive. Determine the friction force. **HINT:** Draw a diagram

b) Determine for the angle formula at which the block is reaches the upper limit of the static friction force. What is the angle for $\mu_{\rm s} = 0.5$?

c) Determine for the acceleration formula of the block when sliding. What is the accleration for $\mu_{\rm k} = 0.2$ and $\theta = 60°$?

21. Newton's 2nd law in ordinary form is

$$\vec{F}_{\rm net} = m\vec{a}_{\rm cm} \ ,$$

where $\vec{F}_{\rm net}$ is the net force on a system (AKA object, AKA body), the system mass is m, and the sytem acceleration is $\vec{a}_{\rm cm}$. Newton's 2nd law is the basis for determining the motion of the system center of mass: i.e., as a functions of time acceleration $\vec{a}_{\rm cm}$, velocity $\vec{v}_{\rm cm}$, and position $\vec{r}_{\rm cm}$. Like all general physical laws, Newton's 2nd law is what is eternally true everywhere: i.e, eternally true everywhere in the classical limit. Other features of the system and its environment are contingent on the physical history.

In fact, Newton's 2nd law has to be generalized to allow for inflow (which is outflow if negative) of mass which can change the (system) acceleration $\vec{a}_{\rm cm}$ without a net external force because it changes the (system) momentum. Note, we drop the center-of-mass subscript cm hereafter for formula simplicity. Now momentum (i.e., linear momentum, not angular momentum) for a system is defined

$$\vec{p} = m\vec{v} \ .$$

The generalized Newton's 2nd law is

$$\vec{F}_{\rm net} + \vec{v}_{\rm flow}\frac{dm}{dt} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} + \vec{v}\frac{dm}{dt} \ ,$$

where dm/dt is the mass inflow rate which can be positive (i.e., actual inflow) or negative (i.e., actual outflow) and $\vec{v}_{\rm flow}$ is the velocity of inflowing mass. Note, we have assumed there is a single $\vec{v}_{\rm flow}$ for simplicity in our discussion. The inflow can change the momentum, and therefore accelerate the system without exerting any net external force. The inflow when not part of the system can exert an external force, but it cannot by definition when part of the system since then it can only exert internal forces. In fact, it may be an analysis choice when to consider the inflow as part of the system.

In this problem, we consider the generalized Newton's 2nd law.

a) If $\vec{F}_{\rm net}$ and dm/dt are zero, what can one say about the system momentum and acceleration?

b) If $\vec{F}_{\rm net}$ is zero, $dm/dt \neq 0$, and $\vec{v}_{\rm flow} = \vec{v}$, what can one say about the system momentum and acceleration?

c) What is the explicit formula for acceleration?

d) An interesting special case for the generalized Newton's 2nd law is the (Tsiolkovsky) rocket problem (Wikipedia: Tsiolkovsky rocket equation). Say you have a rocket in empty space with $\vec{F}_{\rm net} = 0$. It can still be accelerated by thrust: ejecting burnt exhaust fuel opposite the direction of motion. The speed of the exhausted fuel relative to the rocket is called the exhaust speed $v_{\rm ex}$ which is a parameter of the rocket. The rate of mass flow (i.e., the rate of ejection of exhausted fuel) $dm/dt < 0$ is also a parameter of the rocket. For our analysis of the rocket problem, we will consider only 1-dimensional motion. Specialize the generalized Newton's second law for the 1-dimentional case of the rocket problem.

e) What is the velocity of the ejected exhausted fuel in terms of the generalized Newton's 2nd law. What is this velocity in the outside inertial frame the rocket is traveling in, not relative to the rocket? What does it mean if the velocity is positive?

f) Making use of the part (e) result, write down the formula for the acceleration of the rocket simplified as much as possible.

g) Solve the part (f) equation (which is a differential equation) for velocity as a function of mass by integration recalling $a = dv/dt$. Assume the initial mass m_0 and the intial velocity is v_0. What happens to velocity as $m \to 0$? Why must the velocity formula actually fail to be physically real if the velocity gets too large? Why is this failure unlikely in practice.

g) Invert formula found in part (g) to find the amount of fuel m_{fuel} needed to achieve a given change in velocity $\Delta v = v - v_0$.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

If $\quad 0 = ax^2 + bx + c\,,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\mathrm{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\mathrm{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

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14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2\rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_\mathrm{T}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_\mathrm{max}\sin[k(x \mp vt)] = y_\mathrm{max}\sin(kx \mp \omega t)$$

$$\mathrm{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_\mathrm{max}^2$$

$$y = 2y_\mathrm{max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_\mathrm{medium} = \frac{f_0}{1 - v_0/v_\mathrm{medium}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_\mathrm{K} = T_\mathrm{C} + 273.15\,\mathrm{K} \qquad T_\mathrm{F} = 1.8 \times T_\mathrm{C} + 32°\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_\mathrm{avg} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_\mathrm{RMS}^2\right)$$

$$v_\mathrm{RMS} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$