Intro Physics Semester I **Name:**

Homework 5: Classical Mechanics I: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

1. "Let's play *Jeopardy*. For $100, the answer is: The branch of physics that explains motion and acceleration in terms of forces and masses."

   What is ____________, Alex?

   a) kinematics b) dynamics c) statics d) economics e) cinematics

2. Dynamics is that branch of physics that:

   a) explains motion and acceleration in terms of the kinematic equations.
   b) explains motion and acceleration in terms of error analysis.
   c) treats dynamos.
   d) treats electricity and magnetism or electromagnetism.
   e) explains motion and acceleration in terms of forces and masses.

3. The area of physics dealing with **ONLY** cases of balanced forces (or equilibrium) is called:

   a) statics. b) dynamics. c) kinematics. d) kinesiology. e) cinema.

4. Forces can cause accelerations relative to inertial frames or cancel other forces. Another manifestation (which actually follows from their property of causing acceleration) is that they can cause:

   a) velocity (without causing acceleration).
   b) mass.
   c) bodies to distort: i.e., flex, compress, stretch, etc.
   d) bodies to live
   e) bodies to rule.

5. Accelerations with respect to ____________—which we will call natural frames for the nonce—though no one calls them that—and only they require forces as causes as prescribed by Newton's 2nd law ($\vec{F}_{\text{net}} = m\vec{a}$) in the classical limit. What are natural frames? They have been elusive historically. Newton hypothesized that a primary natural frame defined by the mean position of the fixed stars—absolute space as he called it. But the fixed stars move—as Newton knew himself—and revolve around the center of the Milky Way in complex orbits—as Newton did not know himself. The Milky Way and other galaxies are also in complex orbits in galaxy clusters or otherwise in complex relative motions. In modern cosmological theory, natural frames are frames of reference attached to points in space that participate in the mean expansion of the universe. Space is growing—just accept it. Not all space—not space within bound systems like you, me, and the Milky Way—but the space in between bound systems like galaxy clusters. To every point participating in the mean expansion of the universe attach the origin of a local pimary natural frame. It is called local because sufficiently close to the origin, the frame has the behavior given above. As you move away from the origin, there is a progressive departure from the behavior, but you have to move over distance scales larger than a galaxy cluster for that to become very noticeable. Now any frame in uniform motion (i.e., unaccelerated) with respect to a local primary natural frame is also a local natural frame. Say the pimary frame is unprimed and the non-primary is primed. Then we have

   $$\vec{r}\,' = \vec{r} - \vec{r}_{\text{prime}} \ ,$$

   where is $\vec{r}_{\text{prime}}$ is the position of the primed frame in the unprimed frame. Differentiate twice and you get $\vec{a}\,' = \vec{a}$. So accelerations in the unprimed frame are exactly those of the primed frame. So Newton's 2nd law ($\vec{F}_{\text{net}} = m\vec{a}$) must be obeyed for accelerations relative to non-primary local frames. Forces themselves are frame-independent in classical mechanics. If you need relativistic physics the story changes.

Actually, Newton's 2nd law can be generalized to non-natural frames by introducing what are called inertial forces which are not real forces, but force-like terms that account for using non-natural frames. In fact, using inertial forces is usually the best approach to non-natural frames.

By the by, we can actually identify natural frames in the universe and our own local one very precisely using astronomical measurements. However, for many purposes we can find non-natural frames that are sufficiently close to being natural frames that they can be used as natural frames to some degree of approximation. The local Earth surface (i.e., the ground) is natural enough for many purposes: not long-range gunnery or large-scale weather phenomena. If you need a more natural natural frame, you can use the fixed stars. For highest accuracy, we can use the local primary natural frame using cosmological knowledge.

a) rotating frames. b) accelerated frames. c) non-inertial frames. d) inertial frames.
e) picture frames.

6. How many laws of motion did Newton posit?

   a) 1. b) 2. c) 3. d) 4. e) 5.

7. Newton's 1st law is.

   a) **PHYSICALLY INDEPENDENT** of the other two laws of motion and **CANNOT** be dispensed with as an axiom of Newtonian physics.
   b) **PHYSICALLY INDEPENDENT** of the other two laws of motion, but nonetheless it **CAN** be dispensed with as an axiom of Newtonian physics.
   c) actually a **SPECIAL CASE** of the **2ND LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
   d) actually a **SPECIAL CASE** of the **3RD LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
   e) is **INCORRECT**, but is kept in the books for historical reasons.

8. Newton's 2nd law is:

   a) $m = \vec{F}_{\text{net}}\vec{a}$.
   b) $\vec{a} = m\vec{F}_{\text{net}}$.
   c) $\vec{F}_{\text{net}} = m\vec{a}$.
   d) For every force there is an equal and opposite force.
   e) For every acceleration there is an equal and opposite acceleration.

9. From here on in this course, a key thing to remember (to recite to yourself) when faced with any force problem is that Newton's 2nd law ($\vec{F}_{\text{net}} = m\vec{a}$) is:

   a) **ALWAYS VALID**. And it is a **VECTOR** equation, and so is always **VALID** component by component. And $\vec{F}_{\text{net}}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero vectorially, $\vec{F}_{\text{net}} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\text{net}} = m\vec{a}$ to solve for an unknown force.
   b) **ALWAYS VALID**. And it is a **SCALAR** equation. And $\vec{F}_{\text{net}}$ is the **SCALAR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero, $\vec{F}_{\text{net}} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\text{net}} = m\vec{a}$ to solve for an unknown force.
   c) **ONLY VALID** when there is a **NON-ZERO** net force. Because the 2nd law is a **VECTOR** equation, it is valid (when it is valid) component by component. And $\vec{F}_{\text{net}}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If you are given the acceleration, then you can often use $\vec{F}_{\text{net}} = m\vec{a}$ to solve for an unknown force.
   d) **ALWAYS INVALID**.
   d) **NEVER VALID**.

10. If Newton's 3rd law is true, why then does anything accelerate at all?

    a) The equal and opposite forces **DO NOT** have to be on the same body.
    b) The equal and opposite forces **DO** have to be on the same body.

c) Nothing moves at all as Parmenides argued in the 5th century BC. Motion is but seeming. Anyway Parmenides seems to have been a pretty smart guy since he's credited with the spherical Earth theory and the discovery that the Moon shines by reflected light.
d) Acceleration has nothing do with forces.
e) Forces have nothing do with acceleration.

11. "Let's play *Jeopardy*! For \$100, the answer is: Laws that prescribe forces for physical systems. They must exist independent of Newton's 3 laws of motion in order for Newtonian physics to be useful."

What are ____________, Alex?

a) Newton's 3 laws b) accelerations c) velocities d) force inequalities e) force laws

12. The base SI unit of force is the:

a) farad (F); $1\,\text{F} = 1\,\text{kg m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
b) henry (H); $1\,\text{H} = 1\,\text{kg m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
c) watt (W); $1\,\text{W} = 1\,\text{kg m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
d) joule (J); $1\,\text{J} = 1\,\text{kg m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
e) newton (N); $1\,\text{N} = 1\,\text{kg m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.

13. A bicycle-rider system has a mass of 80 kg. The bike is traveling on level and has initial velocity 6 m/s north. What is the constant force needed to stop the bike in 4 s?

a) 80 N south. b) 80 N north. c) 80 N east. d) 100 N south. e) 120 N south.

14. The magnitude of the gravitational force on an object of mass m for a uniform gravitational field (such as the gravitational field near the Earth's surface for human-size and somewhat larger objects) is given by the formula:

a) $F = mg$. b) $F = m/g$. c) $F = g/m$. d) $F = ma$. e) $F = m/a$.

15. Newton's 2nd law applied to the vertical direction with only the gravity force acting and down defined as positive leads to the scalar equation of motion:

a) $g = m/a$. b) $g = ma$. c) $mg = a$. d) $mg = ma$. e) $m/g = m/a$.

16. If you have a mass of 60 kg and $g = 9.8\,\text{m/s}^2$, you weigh about:

a) 10 N. b) 60 N. c) 600 N. d) 500 N. e) 20 N.

17. The force of gravity reaches out across space and pulls on each bit of your body independently of every other bit. We call a force like this a **FIELD FORCE** or a **BODY FORCE**. Why don't you accelerate downward, except when off the ground.

a) The **GROUND FORCE** reaches out across space and pushes upward on each bit of your body independently of every other bit. The ground force is also a **FIELD FORCE**.
b) The ground exerts a force on the soles of your feet and the soles of your feet on the next layer of your body and the next layer of your body on the next layer of your body and so on until the top of your head. Each layer pushes up with only enough force to balance the gravity force on the mass above. The ground force and the forces exerted by the layers of our bodies are **CONTACT FORCES**. A **CONTACT FORCE** acts over a very short range: so short that if the distance between the two objects exerting equal and opposite contact forces on each other is more than microscopic there is no contact force at all.
c) Since you are always off the ground, the question has no answer.
d) Since you are always off the ground, the question is hypothetical and the answer, speculative.
e) In orbit, you don't accelerate downward and you are certainly off the ground. So being on the ground may have nothing to do with why you don't accelerate downward.

18. "Let's play *Jeopardy*! For \$100, the answer is: they are, respectively, the resistance of a body to acceleration and the magnitude of the force of gravity on a body."

What are ____________ and ____________, Alex?

a) acceleration; normal force b) mass; normal force c) force; weight d) mass; weight
e) gravity; momentum

19. What is the approximate mass of a woman who weighs 500 N? What is gravitational force that Earth exerts on her. After she jumps **UPWARD** from a diving board, what is her acceleration in the absence of air drag?

    a) About 50 kg, 500 N, and $9.8\,\mathrm{m/s^2}$ downward once she starts moving downward, but **ZERO** before that.
    b) About 50 kg, 50 N, and $9.8\,\mathrm{m/s^2}$ downward once she starts moving downward, but **ZERO** before that.
    c) About 50 kg, 50 N, and $9.8\,\mathrm{m/s^2}$ downward at **ALL** times.
    d) About 50 kg, 500 N, and $9.8\,\mathrm{m/s^2}$ downward at **ALL** times.
    e) None of these questions can be answered with the given information.

20. The normal force is:

    a) a repulsive contact force exerted by a surface that points perpendicularly outward from that surface. The force turns out to resist compression. In principle, the force can be calculated from the compressional displacement of the surface from equilibrium, but in elementary problems one usually calculates it from Newton's 2nd or 3rd law assuming the surface to be completely rigid.
    b) $\vec{F}_{\mathrm{net}}$ in $\vec{F}_{\mathrm{net}} = m\vec{a}$.
    c) the tension force in a rope.
    d) the tension force in a rope that allows you to push on a rope.
    e) an ordinary, run-of-the-mill force, a pedestrian force, a force without pretensions or airs, a downright force, a regular-guy force, just a plain salt-of-the-earth force.

21. An object of mass m is on a rigid, frictionless slope of angle θ from the horizontal. What is the magnitude of normal force on the object? What is the component of the gravitational force in the positive x direction which is down the slope? What is the expression for the position x of the object as a function of time t when it starts from rest at $t = 0$ and $x = 0$?

    a) $mg\cos\theta$; $mg\cos\theta$; $x = (1/2)(g\cos\theta)t^2$. b) $mg\cos\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$.
    c) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$. d) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\cos\theta)t^2$.
    e) $mg\sin\theta$; $mg\sin\theta$; $x = (g\cos\theta)t$.

22. A book sits at rest on a table. The reaction force that follows from Newton's 3rd law to the gravitational force of the Earth on the book is the:

    a) gravitational force of the book on the Earth.
    b) normal (i.e., perpendicular upward) force of the table on the book.
    c) table friction force on the book.
    d) book friction force on the table.
    e) book normal force on the table.

23. A woman who has a mass of 50 kg is in an elevator that is accelerating downward at $2\,\mathrm{m/s^2}$. What is the force the floor exerts on her? What is the force she exerts on the floor?

    a) 390 N upward; 390 N downward. b) 390 N downward; 390 N upward.
    c) 490 N downward; 490 N upward. d) 490 N upward; 490 N downward.
    e) 100 N upward; 100 N downward.

24. Tension is the magnitude of the force in an object that resists:

    a) extension. b) compression.
    c) shearing (i.e., the deformation of the object without change in volume).
    d) creaking (i.e., the deformation of the object with noise). e) concession.

25. "Let's play *Jeopardy*! For \$100, the answer is: It has zero thickness and only resists extension along its length. In fact, resists extension completely. Usually, but not always, it is assumed to have zero mass and be unbreakable"

    What is a/an ____________, Alex?

    a) ideal monkey b) ideal rigid rod c) ideal surface d) ideal rope e) real rope

26. The normal force magnitude per unit length exerted by the curved surface on an ____________ at any

general point s is

$$f_{\rm nor} = \frac{T}{r} ,$$

where s is measured from the start of the ____________, T is the tension at point s r is the radius of curvature at s, and the normal force per unit length points radially outward from the center of curvature. The center of curvature is the center of a circle that approximates the curve at s to first order. The normal force per unit length exerted by the rope on the curved surface is equal in magnitude to $f_{\rm nor}$, but points radially inward by the 3rd law.

a) ideal rigid rod b) ideal rope c) unreal rigid rod d) uaenrl riigd rod e) ideal door

27. A taut ideal, massless rope should have ________________ tension (i.e., constant magnitude of tension force) between two endpoints provided no external forces parallel to the rope act on it **BETWEEN** the endpoints: there will in general be external applied forces to hold it taut at the endpoints. The rope does not have to be straight. It can be wrapped around constraints as long as their surfaces exert no parallel forces on it.

    a) wildly varying b) constant c) complexly varying d) zero e) 9.8 N/kg

28. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the tension force that the rope exerts on the mass?

    a) 100 N downward. b) 200 N downward. c) 200 N upward. d) 100 N upward.
    e) 200 N horizontally.

29. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the **NET** force on the mass? It is:

    a) about 100 N downward. b) 0 N. c) about 200 N upward. d) about 100 N upward.
    e) about 200 N horizontally.

30. An elevator just starts moving upward.

    a) You feel slightly heavy for a moment.
    b) You feel slightly light for a moment.
    c) You feel slightly light and carefree for a moment.
    d) You feel totally carefree and ethereal.
    e) You come to understand that there are no forces in foxholes.

31. As this is (or was within living memory) 2001, let's say you are David Bowman and you've just arrived at Jupiter. Before going off to investigate that monolith (and go beyond humankind), you decide on a little excursion to Callisto, one of Jupiter's 4 major moons. Assume you are so close to Callisto's surface throughout the maneuvers of this question the gravitational field $g_{\rm Cal}$ can be approximated as a constant.

    a) As your landing pod descends straight down to the Callisto surface and when your are relatively close to touchdown, your rocket thrust is 3260 N and your descent velocity is **CONSTANT**. What is the gravitational force on your pod? Take the upward direction as the positive direction.

    b) Say you reduce thrust to 2200 N and find that the pod has a downward acceleration of $0.39\,{\rm m/s^2}$. What is the mass of your pod including yourself?

    c) What's the free-fall acceleration magnitude due to gravity near the Callisto surface (i.e., $g_{\rm Cal}$, the analog to g for gravity near Earth's surface)? The free-fall acceleration acceleration magnitude is also the gravitational field magnitude.

    d) Say you have a mass of 70 kg. What's your **WEIGHT** on Callisto and what is your Callisto weight divided by your Earth weight (i.e., what is the weight **RATIO**)?

    e) Now the hard part. After finishing your excursion on the icy surface, you launch and go into uniform circular motion, low-Callisto orbit. The gravitational acceleration is approximately the same as the surface gravitational acceleration and the radius of the orbit is approximately just Callisto's radius of 2400 km. Calculate the **ORBITAL SPEED**. Then find the **ORBITAL PERIOD** P (i.e., the time to orbit once) in seconds and in hours. **HINT:** Remember centripetal acceleration and $\vec F_{\rm net} = m\vec a$.

32. There is a 2 kg block on a frictionless incline that is at $\theta = 30^\circ$ from the horizontal.

    a) What is the normal force on the block? **HINTS:** Draw a free body diagram and remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

    b) What is the net force down the slope?

    c) What is the acceleration down the slope?

    d) Starting from rest how far does the block slide in 10 s?

33. Physics students frequently flub analyzing forces and accelerations on an inclined plane. Let's get it straight.

    a) A Cairn terrier named Bit has dog-rolled to a point on an inclined plane. The plane has an angle from the horizontal of θ. Bit's mass is m. What is the component of gravitational force on Bit parallel to the inclined plane? What is the normal force on Bit? **HINT:** Draw a diagram.

    b) Fun-loving pig Waldo Pepper (mass m) is sliding down a frictionless incline (with angle θ form the horizontal). What is his acceleration? What is the normal force on Waldo? **HINT:** Draw a diagram.

    c) Underdog has just alighted on an inclined plane from which the Wonder Woofer surveys the world with a flint-hard gaze. The inclined plane has an angle of θ from the horizontal. What is the gravitational force component parallel to the inclined plane on the Caring Canine (mass m)? What is the normal force on the Magnificent Mutt? **HINT:** Draw a diagram.

    d) A 1992 GM Geo Metro (mass m) is sliding down a frictionless incline (with angle θ form the horizontal) which is sort of like a hill in Moscow, Idaho in January. What is Baby's acceleration? What is the normal force on Baby? **HINT:** Draw a diagram.

34. You have block just sitting on horizontal flat ground. **HINT:** This is a problem for rumination—or perhaps ruminants.

    a) Draw a free body diagram for the block. Indicate all the forces acting on the block. What is the cause of these forces and are they contact or field forces?

    b) Now by the 3rd law, what forces does the block exert and on what and where exactly does it exert them?

35. You are in a car accelerating at a constant $10\,\mathrm{m/s^2}$ in a constant direction. The car is on level ground. A pair of fuzzy dice is hanging by a cord from the mirror at an angle θ from the vertical. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is this angle? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

36. You have a **FRICTIONLESS** triangular block which gives you two inclines: i.e., double incline. Incline 1 is at θ_1 to the horizontal and incline 2 at θ_2. You have an ideal massless pulley (or altneratively and equivalently, a friction-free bend) at the apex and a taut ideal rope connecting two blocks, one on each slope. The rope is parallel to both inclines. The incline 1 block has mass m_1 and the incline 2, mass m_2. The masses of the blocks and the incline angles are the formal knowns of the problem.

    a) Write down Newton's 2nd law for each block for the direction along the inclines. Take up as positive for incline 1 and down as positive for incline 2. **HINT:** Draw two free body diagrams on a diagram of the double incline.

    b) Derive the explicit formula for acceleration of the two blocks along their respective inclines and the explicit formula for the tension in the rope? **NOTE:** By "explicit" in this context, we mean formulae with $a = \ldots$ and $T = \ldots$ with no a's or T's on the right-hand sides.

    c) Specialize the formulae from the part (b) answer for the case of $\theta_1 = \theta_2 = \pi/2$. This case is Atwood's machine.

    d) Specialize the formulae from the part (b) answer for the case of $\theta_1 = 0$ and $\theta_2 = \pi/2$.

    e) Specialize the formulae from the part (b) answer for the case of $\theta_2 = \pi/2$.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \quad 0 = ax^2 + bx + c\,, \quad \text{then} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

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14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

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15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total}\ f} = KE_{\text{total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 **Static Equilibrium**

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\mathrm{max}}\sin[k(x \mp vt)] = y_{\mathrm{max}}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\mathrm{max}}^2$$

$$y = 2y_{\mathrm{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$