Intro Physics Semester I **Name:**

Homework 3: One-Dimensional Kinematics: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

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002 qmult 00050 1 4 5 easy deducto-memory: calculus limited definition

1. "Let's play *Jeopardy*! For $100, the answer is: It is the branch of mathematics that deals with limits, derivatives, differentiation, integrals, and integration."

   What is ____________, Alex?

   a) geometry b) algebra c) number theory d) set theory e) calculus

   SUGGESTED ANSWER: (e)

   Wrong answers:
   c) I've no idea really.
   d) I've no idea really.

   Redaction: Jeffery, 2008jan01

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002 qmult 00051 1 1 2 easy memory: Delta use defined

2. It is common to specify a change in a quantity (specified by a symbol) by a prefixed capital Greek letter:

   a) Alpha A. b) Delta Δ. c) Lambda Λ. d) Psi Ψ. e) Omega Ω.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   a) Now would this look much like a change in sign.

   Redaction: Jeffery, 2008jan01

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002 qmult 00052 1 1 3 easy memory: derivative definition

3. The formula

   $$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \ ,$$

   where $f(x)$ is a general function of x is the not-explicit definition of the:

   a) integration. b) integral. c) derivative. d) differentiation. e) differential.

   SUGGESTED ANSWER: (c) The explicit definition is

   $$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \ .$$

   Wrong answers:
   b) Exactly wrong.

   Redaction: Jeffery, 2008jan01

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002 qmult 00054 1 1 2 easy memory: differentials

4. The quantity dx in calculus is the:

   a) derivative of x. b) differential of x. c) integral of x. d) integratial of x.
   e) product of d and x.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   d) It's begging to exist, but it doesn't.

e) In a calculus context, no one would buy this.

Redaction: Jeffery, 2008jan01

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002 qmult 00056 1 1 2 easy memory: approximate derivative

5. We can approximate derivative df/dx to some degree of accuracy by:

a) $\Delta f \Delta x$. b) $\dfrac{\Delta f}{\Delta x}$. c) $\dfrac{\Delta x}{\Delta f}$. d) $\dfrac{d^2 f}{dx^2}$. e) $\dfrac{df^2}{\Delta x}$.

SUGGESTED ANSWER: (b)

Wrong answers:

a) Oh, c'mon.

Redaction: Jeffery, 2008jan01

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002 qmult 00058 1 1 4 easy memory: power-law derivative

6. The derivative of

$$Ax^p ,$$

where A is a general constant and p is a general power, is:

a) $\dfrac{Ax^{p+2}}{p+2}$. b) $\dfrac{Ax^p}{p}$. c) $\dfrac{Ax^{p+1}}{p+1}$ or $A\ln(x)$ if $p = -1$.
d) pAx^{p-1} or 0 if p and x are both zero. e) Ax^p .

SUGGESTED ANSWER: (d)

Wrong answers:

c) This is the indefinite integral or antiderivative.

Redaction: Jeffery, 2008jan01

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002 qmult 00060 1 1 1 easy memory: definite and indefinite integrals

7. The expression

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

is a/an ____________ and the expression

$$\int f(x)\,dx = F(x) + C$$

is a/an ____________.

a) definite integral; indefinite integral b) indefinite integral; definite integral
c) antiderivative; derivative d) differential equation; antidifferential equation
e) differential; antidifferential

SUGGESTED ANSWER: (a) An indefinite integral can also be called an antiderivative and the constant of integration C can be omitted since it is understood.

Wrong answers:

b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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002 qmult 00100 1 2 1 moderate memory: kinematics definition 1

Extra keywords: physci

8. Kinematics is:

a) the description of motion. b) the techniques of the cinema. c) dynamics by another name. d) the study of the causes of motion in terms of physical quantities, most prominently force and mass. e) the rate of change of acceleration with time.

SUGGESTED ANSWER: (a)

Wrong answers:

b) Cinematics? Both kinematics and cinema derive from the Greek word kinema meaning motion. I think the trouble is that the Romans used c for the k sound, but in medieval times, the c became soft in medieval Latin. A great improvement in most cases. Circe is sounds better than Kirke to me.
c) Nah, nah. Dynamics is (d)
d) This is dynamics.
e) The rate of change of acceleration doesn't have a common name although I vaguely recall from my first year physics course that it may have been called jerk. Yes/no?

Redaction: Jeffery, 2001jan01

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002 qmult 00110 1 1 1 easy memory: quantities of kinematics

9. The three quantities that are of most obvious interest in kinematics are displacement, velocity, and:

a) acceleration. b) deceleration. c) mass. d) force. e) inertia.

SUGGESTED ANSWER: (a)

Wrong answers:

d) Force doesn't formally come into kinematics since it is part of dynamics.

Redaction: Jeffery, 2008jan01

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002 qmult 00112 1 1 3 easy memory: magnitudes of x,v,a

10. The magnitudes of displacement, velocity, and acceleration are usually called distance, speed, and:

a) acceleration speed. b) deceleration. c) acceleration. d) accelmag.
e) the unnameable.

SUGGESTED ANSWER: (c) The terminology of kinematics is not entirely regular. But that's true of most fields in life.

Wrong answers:

d) Maybe this should be the name.

Redaction: Jeffery, 2008jan01

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002 qmult 00120 1 1 3 easy memory: one-dimension vectors

11. Displacement, velocity, and acceleration are quantities that have both magnitude and direction, and so are:

a) scalars. b) unities. c) vectors. d) multiplicities. e) Templars.

SUGGESTED ANSWER: (c)

Wrong answers:

a) Exactly wrong.
e) Ah, the Templars—and who could ever forget Simon Templar.

Redaction: Jeffery, 2008jan01

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002 qmult 00122 1 4 5 easy deducto-memory: direction in 1-d

12. "Let's play *Jeopardy*! For $100, the answer is: All that is used to indicate direction in 1-dimensional kinematics."

What is ____________, Alex?

a) symbol b) image c) vision d) vision quest e) sign

SUGGESTED ANSWER: (e)

Wrong answers:

d) Probably wouldn't hurt.

Redaction: Jeffery, 2008jan01

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002 qmult 00140 1 1 3 easy memory: definitions of v(avg), etc.

13. The following equations

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} , \qquad v = \frac{dx}{dt} , \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} , \qquad a = \frac{dv}{dt}$$

are, respectively, the 1-dimensional definitions of average velocity, velocity, ____________, acceleration.

a) average deceleration b) deceleration c) average acceleration d) speed
e) average speed

SUGGESTED ANSWER: (c)

Wrong answers:
a) Oh, c'mon.

Redaction: Jeffery, 2008jan01

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002 qmult 00142 1 1 2 easy memory: average velocity sensible

14. There are different ways of defining average quantities. But one wants definitions that are useful. For example, the conventional definition for average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} ,$$

where Δx is the displacement that occurred in time Δt. This definition is **USEFUL** since $v_{\text{avg}}\Delta t$ is:

a) a random number. b) the displacement that takes place in time Δt.
c) a quantity with the dimension of length. d) a quantity with the dimension of velocity.
e) a time.

SUGGESTED ANSWER: (b)

Wrong answers:
a) Now is this useful?
c) Yes, it has that dimensions, but that's not why it is useful.

Redaction: Jeffery, 2008jan01

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002 qmult 00150 1 1 2 easy memory: motion diagrams

15. Graphs of kinematic variables (i.e., displacement, velocity, acceleration, jerk) versus time can all be called:

a) emotion diagrams. b) motion diagrams. c) velocity diagrams.
d) acceleration diagrams. e) graphoi

SUGGESTED ANSWER: (b) Google AI (the ultimate authority) tells me motion diagram is, indeed, a conventional term.

Wrong answers:
a) Oh, c'mon.
e) Graphoi in ancient Greek meant sketches or a book. Did Aristarchos of Samos (c. 310—c. 230 BCE) write a book about his heliocentric model or just some sketches for friends to admire. We'll never know: Archimedes (c. 287–c. 212 BCE) or whoever ghostwrote his book *The Sand Reckoner* failed to clarify this point.

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002 qmult 00152 1 1 3 easy memory: constant acceleration gives quadratic displacement

16. If acceleration is constant, then velocity is linear with time and displacement is ____________ with time.

a) constant b) linear c) quadratic d) cubic e) quartic

SUGGESTED ANSWER: (c)

Wrong answers:
a) Not likely.

Redaction: Jeffery, 2008jan01

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002 qmult 00154 1 1 4 easy memory: singularity

17. Cusps, discontinuities, and infinities of functions are all:

    a) anomies. b) autonomies. c) anomalies. d) singularities. e) multiplicities.

    SUGGESTED ANSWER: (d)

    Wrong answers:
    a) Apparently, anomie has no plural: its uncountable.

    Redaction: Jeffery, 2008jan01

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002 qmult 00210 1 1 4 easy memory: kinematic equations by antidifferentiation

18. Given that one-dimensional acceleration a is a constant, one obtains the equations

$$v = at + v_0$$

and

$$x = \frac{1}{2}at^2 + v_0 t + x_0 \ ,$$

where v_0 is an initial velocity and x_0 is an initial displacement. The equations are obtained by:

a) algebra. b) distinction. c) differentiation. d) antidifferentiation. e) antialgebra.

SUGGESTED ANSWER: (d) Antidifferentiation or integration

Wrong answers:
a) Exactly wrong.

Redaction: Jeffery, 2008jan01

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002 qmult 00220 1 1 4 easy memory: constant accel. kinematic equations

19. For 1-dimensional, constant acceleration cases there are ____ **INDEPENDENT** equations. Using these equations, 1-dimensional, constant acceleration problems can be solved. Only ____ unknowns can be solved in general from the ____ **INDEPENDENT** equations. One can derive by **ALGEBRA** extra kinematic equations that speed the solution of problems in some cases. But one can still only solve for ____ unknowns in general.

    a) 5 b) 4 c) 3 d) 2 e) 1

    SUGGESTED ANSWER: (d)

    The 2 independent equations are derived by taking the antiderivative of a and then of v. The other 3 independent equations are obtained by algebra from the first 2. The 5 equations are given in the table below.

One-Dimensional, Constant Acceleration Kinematic Equations

Number	Equation	Missing Variable
1	$v = at + v_0$	Δx
2	$\Delta x = (1/2)at^2 + v_0 t$	v
3	$v^2 = v_0^2 + 2a\Delta x$	t
4	$\Delta x = (1/2)(v_0 + v)t$	a
5	$\Delta x = -(1/2)at^2 + vt$	v_0

The usual procedure for using the equations is to identify the 3 knowns and the 2 unknowns. One identifies the equation not containing the unknown you are not interested in at least at first. Thus you have a one unknown in one equation, and the solution is straightforward.

Many textbooks omit the 5th equation probably on the grounds that it is little used. But to complete the set of equations it should be included. Actually, both the 4th and 5th equations are little used in problems. There is no physical reason for this. It's just that the folks who set problems don't bother with them much. The first 3 equations do get a lot of use. I call the 3rd equation the timeless equation since it has no time in it. It's the equation to use when you don't know time and don't need to know it.

Wrong answers:
a) 5 is the total number of standard equations in some formulations including mine.
b) 4 is the total number of standard equations in most textbook formulations.

Redaction: Jeffery, 2008jan01

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002 qmult 00250 1 1 5 easy memory: timeless equation

20. The constant-acceleration kinematic equation without the time variable (which can be called the timeless equation for mnemonic reasons) is:

a) $v^2 = v_0^2 - 2a\Delta x$. b) $v = v_0 + 2a\Delta x$. c) $v^2 = v_0^2 + 2a/\Delta x$. d) $v^2 = v_0^2 + a\Delta x$
e) $v^2 = v_0^2 + 2a\Delta x$.

SUGGESTED ANSWER: (e)

Wrong answers:
b) Not dimensionally correct.
c) Not dimensionally correct.
d) Dimensionally correct, but still wrong.

Redaction: Jeffery, 2008jan01

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002 qmult 00320 1 1 3 easy math: travel time from distance/speed: Knoxville 1
Extra keywords: physci

21. You have just traveled the back roads from Knoxville to Nashville. Your average speed was 60 mi/h, but you occasionally hit an instantaneous speed of 130 mi/h. (Could be you're hauling white lightning.) Your odometer travel distance is 250 miles. How long have you been on the road?

a) 1/4 hours. b) 10 hours. c) 4.17 hours. d) 6 hours. e) about 2 hours.

SUGGESTED ANSWER: (c)

The students have to be clear on how you get a time from a distance and an average speed: distance/speed. The calculation is

$$\frac{\Delta s}{\dot{s}_{\rm avg}} = \frac{250}{60} = 4.17\,\mathrm{h} \;,$$

where $\dot{s}_{\rm avg}$ stands for average speed.

The question is a remnant of my hillbilly days in Tennessee. Actually, the only time I drank white lightning in Tennessee it was imported by friends from Romania. Alas, the great days of *Thunder Road* are mostly over.

Wrong answers:

Redaction: Jeffery, 2001jan01

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002 qmult 00324 1 3 1 easy memory: average velocity: Knoxville 3

22. You have just traveled 400 km on a trip to Knoxville and back. Knoxville is due east of your starting point. It took 8 hours. Your average **VELOCITY** (with velocity definitely meaning a vector here) was:

a) 0 km/h with an indeterminate direction. b) 50 km/h west. c) 100 km/h east.
d) 200 km/h west. e) 400 km/h north.

SUGGESTED ANSWER: (a)

Note that it doesn't matter what the actual path to Knoxville and back was. All the little displacement vectors in a loop add up to zero and zero divided by a non-zero time is still zero.

Wrong answers:
e) North? C'mon.

Redaction: Jeffery, 2008jan01

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002 qmult 00330 1 3 5 easy math: average speed over two phases
Extra keywords: physci

23. You move 3 meters due west and then, **WITHOUT** a discontinuous change in direction, go onto a circular path (circle **RADIUS** 2 meters) bending to the left until you are headed due east. This has taken you 10 seconds. Your average speed is approximately ____________. **HINT:** Draw a diagram.

    a) 3 m/s. b) 1.55 m/s. c) 0.3 m/s. d) 15.5 m/s. e) 0.9 m/s.

    SUGGESTED ANSWER: (e)

    Remember circumference is $2\pi r \approx 2 \times 3 \times 2 = 12\,\text{m}$, but you don't go a whole circle circumference only half of one. Thus

$$v_{\text{ave}} = \frac{\ell + (1/2)(2\pi r)}{t} = \frac{3 + \pi \times 2}{10} \approx 0.9\,\text{m/s} \ .$$

    Wrong answers:

    Redaction: Jeffery, 2001jan01

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002 qmult 00340 1 1 3 easy memory: lightning and thunder
Extra keywords: physci KB-60-5

24. The speed of sound in air (at 1 atm pressure and 20°C) is 343 m/s. A lightning flash occurs 1.5 km away. How long until you hear thunder?

    a) 228.7 s. b) 0.0044 s. c) 4.4 s. d) 515 s. e) 1.5 s.

    SUGGESTED ANSWER: (c)

    Remember this is time-to-exhaustion question where the solution is always amount/rate assuming a constant rate. Behold:

$$t = \frac{\text{distance}}{\text{sound speed}} = \frac{1500}{343} = 4.37318 \approx 4.4\,\text{s} \ ,$$

    where 4.4 s is to the correct number of significant figures actually. HRW-400 gives the sound speed.

    Fortran Code

```
* code
      print*
      vsound=343.  ! in m/s at 1 atm and 20 C
      dd=1.5e+3    ! distance in meters
      tt=dd/vsound
      print*,'Time until you hear thunder is
     &       ',tt,' seconds.' ! 4.37318
```

    Wrong answers:

    Redaction: Jeffery, 2001jan01

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002 qmult 00400 1 1 2 easy memory: acceleration and speed
Extra keywords: physci

25. If an object's speed changes, the object:

    a) stops. b) accelerates. c) starts. d) goes forward. e) hesitates.

    SUGGESTED ANSWER: (b) An acceleration is a change in velocity, and therefore can be a change in speed and/or direction.

    Wrong answers:
    a) Not necessarily.
    e) As Lurch would say: "Aaaarh."

    Redaction: Jeffery, 2001jan01

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002 qmult 00410 2 5 2 moderate thinking: acceleration opposite velocity
Extra keywords: physci

26. Say you are moving in the positive x-direction, but your speed is decreasing.

a) Your acceleration points in the **POSITIVE** x-direction.
b) Your acceleration points in the **NEGATIVE** x-direction.
c) You are **NOT** accelerating at all. You are decelerating.
d) Your acceleration points **PERPENDICULAR** to the x-axis.
e) Your acceleration points in the **POSITIVE** x-direction. Acceleration always points in the same direction as velocity.

SUGGESTED ANSWER: (b)

At least on a dull afternoon, this question seems moderately hard. If your velocity is decreasing while you are traveling in a certain direction, the changes in velocity point in the opposite direction. Thus the acceleration points in the opposite direction.

Wrong answers:
c) Deceleration is usually just means speed decreasing I think. Acceleration can always be used instead.
d) No way.
e) No acceleration can point in any direction relative to velocity.

Redaction: Jeffery, 2001jan01

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002 qmult 00420 2 5 5 moderate thinking: acceleration and instantaneous acc.
Extra keywords: physci

27. At time zero you are moving at $10\,\mathrm{m/s}$ in the positive y-direction. At time $10\,\mathrm{s}$, you are moving at $15\,\mathrm{m/s}$ in the positive y-direction. What is your average acceleration over the $10\,\mathrm{s}$ of travel? What is your instantaneous acceleration at time $5\,\mathrm{s}$?

a) The average acceleration and the $5\,\mathrm{s}$ instantaneous acceleration are both $0.5\,\mathrm{m/s^2}$ in the **POSITIVE** y-direction.
b) The average acceleration and the $5\,\mathrm{s}$ instantaneous acceleration are both $0.5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction.
c) The average acceleration is $0.5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction. There is **NOT** enough information to determine the $5\,\mathrm{s}$ instantaneous acceleration.
d) The average acceleration and the $5\,\mathrm{s}$ instantaneous accelerations are both $5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction.
e) The average acceleration is $0.5\,\mathrm{m/s^2}$ in the **POSITIVE** y-direction. There is **NOT** enough information to determine the $5\,\mathrm{s}$ instantaneous acceleration.

SUGGESTED ANSWER: (e)

The average acceleration can be found, but the velocity may have changed in any fashion in between the two time limits. So the acceleration at the $5\,\mathrm{s}$ point cannot be determined.

Wrong answers:
a) You havn't answered the second question.

Redaction: Jeffery, 2001jan01

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002 qmult 00500 1 1 2 easy memory: surface gravity points down
Extra keywords: physci

28. The acceleration due to gravity near the Earth's surface (whose magnitude is denoted by g and whose value is nearly constant and is near fiducial value $9.8\,\mathrm{m/s^2}$):

a) points east. b) points down toward the Earth's center more or less. c) points straight up more or less. d) has no direction at all. e) points toward the Moon.

SUGGESTED ANSWER: (b) An easy observation question.

Wrong answers:

d) Acceleration is a vector (when the term is being used to mean magnitude of acceleration), and so points somewhere.

Redaction: Jeffery, 2001jan01

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002 qmult 00510 1 1 1 easy memory: free-fall speed and acceleration
Extra keywords: physci KB-58-4

29. As an object falls freely under gravity. Neglecting air drag, its speed ____________ and its acceleration is ____________.

a) increases; constant. b) decreases; increases. c) increases; increases.
d) decreases; decreases. e) is constant; constant.

SUGGESTED ANSWER: (a)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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002 qmult 00530 1 1 3 easy memory: free fall independent of mass: Galileo
Extra keywords: physci

30. It is reported that Galileo (circa 1590) dropped balls of different mass at the same time from the top of the Leaning Tower of Pisa in order to demonstrate that:

a) the heavier ball hit the ground first by a large margin.
b) the lighter ball hit the ground first by a large margin.
c) both balls hit the ground at more or less the same time.
d) the balls would levitate toward the Moon.
e) the Leaning Tower was leaning.

SUGGESTED ANSWER: (c)

This is an easy observation question.

The tower story was ahistorically embellished, but probably something of the sort did occur. Viviani, who knew Galileo, told us that it happened without much elaboration. It was likely a public demonstration, not a real experiment. Of course, the balls wouldn't hit at exactly the same time and the hardcore Aristotelians likely pointed to this as the significant fact verifying Aristotle. The point of the ideal Galileo (if not exactly the Galileo of history though maybe him too) was that this slight discrepancy didn't matter. You imagined going to the ideal limit in which air drag and release-time errors vanished and in that limit the balls would fall in exactly the same time. No physical theory can be verified exactly. It can only be verified to within experimental error. If that error can be made small, then the theory is very adequate and could become widely accepted. This reasoning was an important conceptual leap in the development of science—of course, it didn't happen all at once—millennia were involved—but Galileo's career exemplified it.

I don't think Galileo himself ever made explicit error estimates.

Wrong answers:
a) Air drag does tend to cause denser bodies to fall faster, but the effect is small for reasonably dense bodies over not so large distances

Redaction: Jeffery, 2001jan01

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002 qmult 00540 1 3 2 easy math: ball thrown down
Extra keywords: physci KB-60-17

31. A ball is thrown downward at 12 m/s. About what is its **SPEED** 2.0 s later assuming no air drag?

a) 20 m/s. b) 32 m/s. c) 22 m/s. d) -2 m/s. e) 12 m/s.

SUGGESTED ANSWER: (b)

Behold

$$v = v_0 + gt = 12 + 9.8 \times 2 = 31.6 \approx 32\,\mathrm{m/s}\ ,$$

where 32 m/s is to the correct number of significant figures.

Wrong answers:

e) All things are wrong.

Redaction: Jeffery, 2001jan01

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002 qmult 00550 2 3 4 moderate math: kinematic equations: arrow flight 1

Extra keywords: physci

32. A tall archer with her longbow shoots an arrow straight up at 100 m/s. The arrow rises, slows, holds for an instant, and then descends picking up speed. The rise time, neglecting air drag, is:

a) 100 s. b) 100.2 s. c) 9.8 s. d) 10.2 s. e) 980 s.

SUGGESTED ANSWER: (d)

Use the kinematic equation

$$v = v_0^2 + at$$

with $v = 0$ for the time when the arrow reaches, the highest point, $v_0 = 100\,\mathrm{m/s}$ for the initial speed, and $a = -g = -9.8\,\mathrm{m/s^2}$ for the acceleration. We then have

$$0 = 100 - 9.8 \times t$$

which has solution $t \approx 10.2\,\mathrm{s}$.

I was thinking of Geena Davis for the tall archer apropos of nothing at all: she was an Olympic hopeful for 2004.

Wrong answers:

a) Bad guess.

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002 qmult 00560 1 3 1 easy math: free-fall speed in 3 seconds

33. How fast is a person falling after 3 s starting from rest? Recall the acceleration due to gravity is $g = 9.8\,\mathrm{m/s^2}$ (which is the fiducial value). Neglect air drag.

a) 29.4 m/s. b) 44.1 m/s. c) 9.8 m/s. d) 88.2 m/s. e) At the speed of light.

SUGGESTED ANSWER: (a)

Behold:

$$y = gt = 9.8 \times 3 = 29.4\,\mathrm{m/s}\ .$$

Wrong answers:

e) Wow.

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002 qmult 00562 1 3 1 easy math: kinematic equations: free fall distance

Extra keywords: in 3 seconds

34. A human falls off some high scaffolding. About how far do they fall in 3 seconds? (Neglect air drag.)

a) 44 m. b) 88 m. c) 22 m. d) 9.8 m. e) 4.9 m.

SUGGESTED ANSWER: (a)

Use the kinematic equation

$$y = \frac{1}{2}at^2 + v_0 t + y_0$$

to find

$$y = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 9 = 4.9 \times 9 = 44.1\,\mathrm{m}\ .$$

Wrong answers:

Redaction: Jeffery, 2001jan01

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002 qmult 00610 1 4 3 easy deducto-memory: terminal velocity defined
Extra keywords: physci

35. "Let's play *Jeopardy*! For $100, the answer is: It occurs to a dense falling object falling near the Earth's surface when the force of gravity and the force of air drag (AKA air resistance) cancel to give no net force on an object."

What is ____________, Alex?

a) acceleration upward b) acceleration downward c) terminal velocity
d) initial velocity e) parabolic motion

SUGGESTED ANSWER: (c)

Actually buoyancy for must be considered too. So one should say when gravity, drag, and buoyancy force cancel. But for dense objects, the buoyancy force is usually negligible.

Wrong answers:
a) Bad guess.

Redaction: Jeffery, 2001jan01

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002 qmult 00620 3 1 2 easy math: travel time, human terminal velocity 1
Extra keywords: physci

36. The terminal velocity of a human in air is about 120 mi/h. At this speed how long does it take to fall 2 miles.

a) 2 minutes. b) 1 minute. c) 1 hour. d) 2 hours. e) 1 second.

SUGGESTED ANSWER: (b)

The students have to be clear on how you get a time from a distance and speed: distance/speed. In this case

$$\frac{2\,\mathrm{mi}}{120\,\mathrm{mi/h}} = \frac{1}{60}\,\mathrm{h} \times \left(\frac{60\,\mathrm{minutes}}{1\,\mathrm{h}}\right) = 1\,\mathrm{minute}\ .$$

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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002 qmult 00640 1 4 5 easy deducto-memory: cat fall
Extra keywords: mathematical physics

37. "Let's play *Jeopardy*! For $100, the answer is: These features allow cat victims of the high-rise syndrome (i.e., the propensity to taking flying leaps into oblivion—cats being so darn smart you know) to survive falls of $\gtrsim 20$ m without major injuries—sometimes that is."

What are ____________, Alex?

a) feline insouciance, savoir-faire, panache, et je-ne-sais-quoi.
b) the cat **WRONGING** reflex and relatively **LOW** terminal velocity when spread-eagled
c) the cat **WRONGING** reflex and relatively **HIGH** terminal velocity when spread-eagled
d) the cat **RIGHTING** reflex and relatively **HIGH** terminal velocity when spread-eagled
e) the cat **RIGHTING** reflex and relatively **LOW** terminal velocity when spread-eagled

SUGGESTED ANSWER: (e)

See

http://en.wikipedia.org/wiki/Cat_righting_reflex

and

http://en.wikipedia.org/wiki/High-rise_syndrome .

Wrong answers:
a) Or so cats would have you believe.

Redaction: Jeffery, 2008jan01

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002 qfull 00052 2 3 0 moderate math: differentiation power law

38. Differentiation is an extremely important mathematical operation in physics.

    a) Differentiate the power-law formula

    $$y = Ax^p \ ,$$

    where A is a constant and p is a general power.

    b) Find the derivatives of

    $$y = A \ , \qquad y = Ax \ , \qquad y = Ax^2 \ .$$

SUGGESTED ANSWER:

a) Behold:

$$\frac{dy}{dx} = Apx^{p-1} \ .$$

Note that $p = 0$ is a special case in that derivative is zero and not a power-law function with power -1. The derivative of $\ln(x)$ is a power-law function with power -1.

b) Behold:

$$\frac{dy}{dx} = 0 \ , \qquad \frac{dy}{dx} = A \ , \qquad \frac{dy}{dx} = 2Ax \ .$$

Redaction: Jeffery, 2008jan01

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002 qfull 00210 2 3 0 moderate math: linear acceleration with time, constant jerk

Extra keywords: calculus-based question

39. Jerk is derivative of acceleration. The term jerk is not used much by a lot of people. Say jerk for a one-dimensional case is a constant b. This means that $b = da/dt$, where a is acceleration. Using antidifferentiation (i.e., integration), write down the expressions for acceleration, velocity, and position remembering that a constant of integration arises at every integration.

SUGGESTED ANSWER:

Given $b = da/dt$ constant,

$$\begin{aligned} a &= bt + a_0 \ , \\ v &= \frac{1}{2}bt^2 + a_0 t + v_0 \ , \\ x &= \frac{1}{6}bt^3 + \frac{1}{2}a_0 t^2 + v_0 t + x_0 \ , \end{aligned}$$

where a_0, v_0, and x_0 are constants of integration and also initial conditions.

Redaction: Jeffery, 2001jan01

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002 qfull 00410 1 3 0 easy math: easy car motion

40. A car starts from **REST** with constant acceleration $1\,\mathrm{m/s^2}$.

    a) What is its velocity after 10 s from its **START POSITION**?

    b) What is its displacement traveled after 10 s from its **START POSITION**?

    c) What is its velocity when it has traveled 100 m from its **START POSITION**?

SUGGESTED ANSWER:

a) Behold:

$$v = at = 10\,\mathrm{m/s} \ .$$

b) Behold:

$$x = \frac{1}{2}at^2 = 50\,\mathrm{m} \ .$$

c) Behold:

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{200} = 14.1\,\mathrm{m/s}\ .$$

Redaction: Jeffery, 2001jan01

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002 qfull 00440 2 3 0 moderate math: two-object car passing problem

41. At time zero, there is a car stopped at a light and a truck moving at a constant velocity of $30.0\,\mathrm{m/s}$ is $200\,\mathrm{m}$ behind the car. Also at time zero, the car starts accelerating forward at $1.00\,\mathrm{m/s^2}$.

a) Draw a **QUALITATIVE** plot of position x versus time t showing the trajectory curves of both vehicles. What are the three qualitatively different ways the curves can intersect?

b) At what time or times do the car and truck pass? Is it possible for there to be only one pass with the given conditions?

SUGGESTED ANSWER:

a) You will have to imagine the plot. The car curve is a parabola that rises from the origin. The truck curve is a straight line that intersects the x-axis at a negative point. The curves can intersect in only three ways: (i) the truck curve can always be below the car curve (i.e., no passes or catch-up) and this is the no-intersection case of intersection; (ii) the truck curve can just be tangent to the car curve at one point (i.e., no passes, but one catch-up); (iii) the truck curve can intersect the car curve twice (truck passes car, then car repasses truck).

Note that since the car is accelerating, it must eventually have a higher velocity than the truck and therefore must in a finite time go ahead of the truck forever no matter what else happens at early times.

b) There are two bodies moving with constant acceleration, and so two sets of kinematic equations are needed in general to describe the motion. However, we immediately see that we need only the two position equations

$$x_{\mathrm{tr}} = vt + x_0$$

for the truck and

$$x_{\mathrm{ca}} = \frac{1}{2}at^2$$

for the car. Passing gives us a condition that allows us to solve for the time of passing: i.e., passing occurs when $x_{\mathrm{tr}} = x_{\mathrm{ca}}$. If we equate our equations, we obtain the quadratic equation

$$0 = \frac{1}{2}at^2 - vt - x_0\ .$$

We can reflect for a moment on the quadratic equation. It gives the zeros of the quadratic function $f(t) = (1/2)at^2 - vt - x_0$ which is a parabola opening upward since $a > 0$. The zeros occur where the function crosses the t axis. It can do this in two places if the minimum of the function is less than zero, once if the minimum is on the t axis, or never if the minimum is above the t axis. What decides which case applies?

One solves the quadratic equation using the standard quadratic equation solution formula. The discriminant of the quadratic equation solution decides. If the discriminant is positive, then there are solutions and two passes: the truck passes the car, then the car passes the truck. If the discriminant is zero, then there are no solutions and no passes, but the truck does catch up to the car for a moment. If the discriminant is negative, there are no solutions and the truck never even catches up to the car.

The solution of the quadratic equation is

$$t = \frac{v \pm \sqrt{v^2 + 2ax_0}}{a} = \frac{30 \pm \sqrt{500}}{1} \approx 7.6\,\mathrm{s} \qquad \text{or} \qquad 52\,\mathrm{s}\ .$$

The discriminant is positive in fact, and thus we have the two-pass case. More exact time values are $7.64\,\mathrm{s}$ and $52.4\,\mathrm{s}$.

Note that by making x_0 smaller (i.e., $|x_0|$ bigger) or a bigger, we make the discriminant approach zero and then go negative. This just corresponds to what one would think: the

larger the car's initial lead or the car's acceleration, the time interval between the passes and eventually the time interval drops to zero and then becomes complex which in this context means the truck never catches up to the car.
Actually, the numerically robust solutions for a quadratic equation

$$0 = ax^2 + bx + c$$

are given by

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] , \qquad x_1 = \frac{q}{a} \qquad \text{and} \qquad x_2 = \frac{c}{q}$$

(Press-178). The one of conventional solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

will have have terrible roundoff error as $4ac$ becomes small.

Fortran Code

```
      print*
      v=30.
      a=1.
      x0=-200.
      t1=(v-sqrt(v**2+2.*a*x0) )/a
      t2=(v+sqrt(v**2+2.*a*x0) )/a
      print*,'t1,t2'
      print*,t1,t2
*  7.63932037  52.3606796
```

Redaction: Jeffery, 2001jan01

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002 qfull 00510 1 3 0 easy math: raindrops sans air drag

42. Say that raindrops fall 1800 m to the ground starting from rest.

a) Assuming there is no air drag (AKA air resistance) what is touch-down velocity?

b) Would it be safe to walk in the rain without air drag? Compare the drop speed to bullet speed? You will find some useful information about raindrop speeds at

http://hypertextbook.com/facts/2007/EvanKaplan.shtml

and about bullet speeds at

http://hypertextbook.com/facts/1999/MariaPereyra.shtml .

Of course, this is no help on a test.

SUGGESTED ANSWER:

a) The appropriate kinematic equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} \approx \sqrt{2 \times 10 \times 1800} \approx 190\,\mathrm{m/s} .$$

b) These raindrops have speeds comparable to those of low-velocity bullets. However, their density is probably only about 1/8 of a steal bullet and their size is smaller by a factor of 10 or so I'm guessing. So the raindrops are of order a hundred times less massive than a bullet. I'd say that individually they'd be much less devastating than a bullet, but if you were hit by one it could be painful and a rain of them would be pretty bad—a super rain.

Actually mass times velocity gives momentum which is a good measure of impact effect. If raindrops have a hundredth of a bullet's momentum, their impact effect would be of order a hundredth of a bullet.

Redaction: Jeffery, 2001jan01

002 qfull 00610 2 5 0 moderate thinking: relative kinematic equations

43. The 5 standard 1-dimensional, constant-acceleration kinematic equations were derived assuming motion with respect to some fixed reference frame. But what if you had say a two-object 1-dimensional, constant-acceleration that you wished to reduce to a one-object problem by changing reference frames: i.e., changing to the rest frame of one of the objects or in other words to a relative frame.

Note the relative position (or displacement), velocity, and acceleration for object 2 in the frame of object 1 are given by, respectively,

$$x_{\rm rel} = x_2 - x_1 \ ,$$
$$v_{\rm rel} = v_2 - v_1 \ ,$$
$$a_{\rm rel} = a_2 - a_1 \ .$$

The first formula is a definition—a very reasonable one—and the others follow from differentiation of the first. The formulae just express what one means by relative position, velocity, and acceleration in one dimension.

a) Show that the first two kinematic equations apply to relative motion if the objects involved both have constant acceleration (relative to some standard frame of reference like the ground). Recall these equations are:

$$v = at + v_0 \qquad \text{and} \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \ ,$$

where the subscript "0" indicates initial value.

b) Show that the other 3 kinematic equations apply to relative motion. **HINT:** This isn't very hard if you know the trick.

SUGGESTED ANSWER:

a) We note that positions, velocities, and accelerations all occur linearly in the first two kinematic equations: i.e., there are no squares or square roots, etc., of them. There is a square of time t, but time is the same in both frames since we have implicitly assumed non-relativistic behavior as this is an intro physics course problem after all. Thus, when we subtract these kinematic motion equations for one object from those of another, we get a set of relative equations.

Say we have objects 1 and 2 and we subtract the equations for object 1 from those for object 2. This means that object 2's motion is being referenced to the frame defined by object 1 or that object 2's motion is being analyzed relative to object 1. We obtain

$$v_2 - v_1 = (a_2 - a_1)t + (v_{2,0} - v_{1,0})$$

and

$$x_2 - x_1 = \frac{1}{2}(a_2 - a_1)t^2 + (v_{2,0} - v_{1,0})t + (x_{2,0} - x_{1,0})$$

or writing these expression in terms of $x_{\rm rel} = x_2 - x_1$, etc, we obtain

$$v_{\rm rel} = a_{\rm rel}t + v_{0,\rm rel} \qquad \text{and} \qquad x_{\rm rel} = \frac{1}{2}a_{\rm rel}t^2 + v_{0,\rm rel}t + x_{0,\rm rel} \ .$$

These relative equations are exactly the first 2 kinematic equations, but in terms of the relative quantities. So yes, the first two equations apply to relative motion.

Actually, there is a simpler solution for those who know calculus. If the relative acceleration $a_{\rm rel}$ is constant (as it would be for a_1 and a_2 constant), then

$$v_{\rm rel} = a_{\rm rel}t + v_{0,\rm rel} \qquad \text{and} \qquad x_{\rm rel} = \frac{1}{2}a_{\rm rel}t^2 + v_{0,\rm rel}t + x_{0,\rm rel}$$

are obtained by antidifferentiation (or integration) where $v_{0,\rm rel}$ and $x_{0\,\rm rel}$ are constants of integration. One integrates once to get $v_{\rm rel}$ and then again to get $x_{\rm rel}$.

b) The trick is reduction to an already solved problem. The first 2 kinematic equations are the fundamental ones. The other 2 (or 3) were derived from the first 2 equations by algebra: the algebra is the same no matter what quantities the first 2 (or 3) equations apply to. Thus if the first 2 equations apply to relative motion, then so do the other 2 (or 3).

Reduction of a problem to an already solved problem is a standard procedure in science. This kind of reduction is also useful in everyday life. Here is an example:

> If you get into a mess, ask yourself this one question: "What would the Lone Ranger do?"

It's a waste of time, but we might as well repeat the algebra to satisfy ourselves about the other kinematic equations. Using the 1st kinematic equation

$$v_{\rm rel} = a_{\rm rel}t + v_{0,\rm rel} \ ,$$

we find

$$t = \frac{v_{\rm rel} - v_{0,\rm rel}}{a_{\rm rel}} \ ,$$

and then we find by clever substitutions into the 2nd kinematic equations

$$\begin{aligned} x_{\rm rel} &= \frac{1}{2}a_{\rm rel}t^2 + v_{0,\rm rel}t + x_{0,\rm rel} = \frac{1}{2}(v_{0,\rm rel} + a_{\rm rel}t)t + \frac{1}{2}v_{0,\rm rel}t + x_{0,\rm rel} \\ &= \frac{1}{2}(v_{0,\rm rel} + v_{\rm rel})t + x_{0,\rm rel} \end{aligned}$$

which is the 3rd kinematic equation and

$$x_{\rm rel} = \frac{1}{2}(v_{0,\rm rel} + v_{\rm rel})\left(\frac{v_{\rm rel} - v_{0,\rm rel}}{a}\right) + x_{0,\rm rel} = \left(\frac{v_{\rm rel}^2 - v_{0,\rm rel}^2}{2a}\right) + x_{0,\rm rel} \ .$$

The last of these can be rearrange to get

$$v_{\rm rel}^2 = v_{0,\rm rel}^2 + 2a(x_{\rm rel} - x_{0,\rm rel})$$

which is the 4th or timeless kinematic equation. Lastly, by substituting

$$v_{0,\rm rel} = v_{\rm rel} - a_{\rm rel}t$$

(obtained by rearranging the 1st kinematic equation) into the 2nd kinematic equation to get

$$x_{\rm rel} = -\frac{1}{2}a_{\rm rel}t^2 + v_{\rm rel}t + x_{0,\rm rel}$$

which the 5th (and unappreciated) kinematic equation. Thus, very concretely we know that the 4 (or 5) kinematic equations also apply to relative motion.

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta)\qquad \cos(\theta+90^\circ)=-\sin(\theta)\qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta)\qquad \cos(180^\circ-\theta)=-\cos(\theta)\qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta)\qquad \cos(90^\circ-\theta)=\sin(\theta)\qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)\qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a)\qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]\qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)]\qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)]\qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx}\qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta\qquad \tan\theta\approx\theta\qquad \cos\theta\approx 1-\frac{1}{2}\theta^2\qquad \text{all for } \theta<<1$$

5 Quadratic Formula

$$\text{If}\qquad 0=ax^2+bx+c\ ,\qquad \text{then}\qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\frac{b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}} t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

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15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total }f} = KE_{\text{total }i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

19 **Static Equilibrium**

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 **Gravity**

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 **Oscillation**

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\max}\sin[k(x \mp vt)] = y_{\max}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L}\ :\ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32°\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$