Intro Physics Semester I **Name:**

Homework 2: Vectors and Trigonometry: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

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003 qmult 00100 1 4 5 easy deducto-memory: vector defined

Extra keywords: physci

1. "Let's play *Jeopardy*! For $100, the answer is: An incomplete definition is that it is a quantity with both a magnitude and a direction. The direction is in ordinary physical space (i.e., space space). The extent is in ordinary physical space for displacement only and in abstract spaces otherwise (e.g., velocity space)."

   What is a/an ____________, Alex?

   a) director b) aviator c) bisector d) scalar e) vector

   SUGGESTED ANSWER: (e)

   Wrong answers:
   d) Exactly wrong.

   Redaction: Jeffery, 2001jan01

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003 qmult 00120 1 4 4 easy deducto-memory: physics scalar defined

Extra keywords: physci

2. "Let's play *Jeopardy*! For $100, the answer is: It is a physical quantity expressible by a single real number whose value is independent of the coordinate system (of ordinary physical space). A ____________ can have a direction in a 1-dimensional abstract space. For example, temperature on the Celsius scale: it can have positive or negative value. So ____________ can be regarded as a 1-dimensional vector with the 1 direction being in an abstract space, but in the usual way of speaking one would say it is **NOT** a vector."

   What is a/an ____________, Alex?

   a) director b) aviator c) bisector d) scalar e) vector

   SUGGESTED ANSWER: (d)

   Wrong answers:
   e) Exactly wrong.

   Redaction: Jeffery, 2001jan01

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003 qmult 00150 1 4 3 easy deducto-memory: field in physics defined

3. "Let's play *Jeopardy*! For $100, the answer is: It is quantity with a value at every point in space or at least some region of space. The quantity can be a scaler, vector, or tensor."

   What is a/an ____________, Alex?

   a) area b) electric area c) field d) electric space e) zone

   SUGGESTED ANSWER: (c)

   Wrong answers:
   a) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2008jan01

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003 qmult 00160 1 4 1 easy deducto-memory: true vector

4. A vector (i.e., a true vector, not just something called a vector since it has some vector-like aspects) has the properties that the components are ____________ and the magnitude of the vector (in its space abstract or otherwise) is determined using the ____________ with components acting as "sides". Also

vectors have certain coordinate transformation properties. Pseudovectors (e.g., angular momentum and torque) act like vectors in most respects, but do not have the exactly the same coordinate tranformation properties.

a) scalars; Pythagorean theorem b) scalars; absolute value operation
c) vectors; Pythagorean theorem d) vectors; absolute value operation
e) integers; Pythagorean theorem

SUGGESTED ANSWER: (a)

Wrong answers:
e) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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003 qmult 00200 1 4 3 easy deducto-memory: displacement defined
Extra keywords: physci

5. "Let's play *Jeopardy*! For $100, the answer is: It is a vector that is specified by giving the direction and a straight-line distance in that direction."

What is ____________, Alex?

a) velocity b) acceleration c) displacement d) force e) distance

SUGGESTED ANSWER: (c)

Wrong answers:
e) This is not a vector.

Redaction: Jeffery, 2001jan01

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003 qmult 00210 1 1 3 easy memory: displacement prototype vector

6. The prototype vector in physics (i.e., the one that is usually thought of setting what the standard physical vector properties are) is ____________. One main reason for this is that of the physical vectors, only ____________ has extension in ordinary physical space.

a) force b) mass c) displacement d) speed e) the electric field vector

SUGGESTED ANSWER: (c)

Wrong answers:
b) Not a vector.

Redaction: Jeffery, 2008jan01

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003 qmult 00220 1 1 5 easy memory: velocity is a vector

7. Velocity is:

a) speed. b) a scalar. c) displacement. d) acceleration. e) a vector.

SUGGESTED ANSWER: (e)

Wrong answers:
a) Exactly wrong.
b) Exactly wrong too.

Redaction: Jeffery, 2008jan01

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003 qmult 00230 1 1 4 easy memory: definition of acceleration
Extra keywords: physci

8. Acceleration is:

a) speed.
b) velocity.
c) the rate of change of velocity with time. It is a **SCALAR**.
d) the rate of change of velocity with time. It is a **VECTOR**.
e) the rate of change of displacement with time. It is a **VECTOR**.

SUGGESTED ANSWER: (d)

Wrong answers:
c) There is a scalar meaning of acceleration, but that isn't rate of change of velocity and it isn't the first meaning.

Redaction: Jeffery, 2001jan01

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003 qmult 00232 1 1 3 easy memory: vector uniform circular motion
Extra keywords: physci

9. You are going in a circle at a uniform speed (i.e., a constant speed). Is your **VELOCITY** ever changing?

a) No, the speed is constant.
b) Yes, it is constantly changing since the motion is in a **STRAIGHT LINE**.
c) Yes, it is constantly changing since the motion is in a **CIRCLE** and direction is constantly changing.
d) No, the velocity is constant.
e) Yes, on **EVERY OTHER LEFT** bend.

SUGGESTED ANSWER: (c)

Going in a circle at a constant speed is usually called uniform circular motion in physics-astronomy jargon. Although your speed is constant, you are accelerating at every moment. Actually, the acceleration vector points toward the center, and so your acceleration is constantly changing too although not in magnitude.

Wrong answers:
a) Speed is constant, but velocity is still changing.
b) Circles are not straight lines.
d) Direction is changing constantly: velocity is not constant.
e) A nonsense answer. You may be continuously bending left or right, but there is no separate bends and even if you argue that there, you would have to argue that you accelerated on every "left" bend.

Redaction: Jeffery, 2001jan01

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003 qmult 00280 1 1 2 easy memory: identify a vector
Extra keywords: physci

10. Which of the following quantities is vector?

a) mass. b) force. c) energy. d) speed. e) temperature.

SUGGESTED ANSWER: (b)

You know force is a vector and since there is only one right answer, it is the only vector.

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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003 qmult 00284 1 1 3 easy memory: identify a unit that can be for a vector
Extra keywords: physci

11. Which of the following can be the unit of a vector?

a) kilogram. b) second. c) meter/second. d) liter. e) acceleration.

SUGGESTED ANSWER: (c)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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003 qmult 00300 1 1 2 easy memory: trig and trig functions

12. By the ancient traditional meaning trignonometry (abbreviated to trig) is the branch of mathematics dealing with triangles. The word trignonometry is derived from trigonon (Greek for triangle) and

metron (Greek for measure). Actually trignonometry is generalized beyond triangles to deal with the components of a radius of a circle in a Cartesian coordinate system. This generalization gives the definitions of the ____________ functions.

a) polynomial b) trignonometric c) transcendental d) quadratic e) cubic

SUGGESTED ANSWER: (b)

Wrong answers:
c) The trig functions are transcendental functions, but there are other kinds of transcental functions.

Redaction: Jeffery, 2008jan01

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003 qmult 00302 1 1 2 easy memory: trig abbreviation
Extra keywords: too simple-minded even for tests
13. Trigonometry is often abbreviated to

a) triggy. b) trig. c) gono. d) metro. e) monstro.

SUGGESTED ANSWER: (b)

Wrong answers:
a) Does anyone remember Twiggy. The ultra-slim British supermodel and sometime actress of the 1960s?

Redaction: Jeffery, 2008jan01

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003 qmult 00304 1 1 5 easy memory: division of circle into 360 degrees
14. For reasons they never bothered to record for posterity, the ancient Babylonians of circ 500 BCE divided the circle into 360 units (i.e., 360 degrees). One likely reason is that they didn't like dealing with decimal fractions (actually sexagesimal fractions in their system), and so chose a number of divisions to be one with a lot of whole number factors. How many whole number factors does 360 have?

a) 6. b) 8. c) 12. d) 16. e) 24.

SUGGESTED ANSWER: (e)

Behold the following table:

Whole Number Factors of 360

No. of factor pair	Smaller factor	Larger factor
1	1	360
2	2	180
3	3	120
4	4	90
5	5	72
6	6	60
7	8	45
8	9	40
9	10	36
10	12	30
11	15	24
12	18	20

So there are 24 whole number factors in 360.

Actually, the main reason why the Babylonians chose 360 divisions is probably because the Sun orbits the Earth (from the Earth's rest frame) relative to the fixed stars in about 365 days: the modern, current sidereal year is 365.256363004 days (value as of 2000jan01 noon or J2000.0 value). For astronomical computations, it would be convenient if the circle divisions were chosen such that the Sun moved one division per day. Unfortunately, that leads to an awkward non-whole number of divisions. So the Babylonians compromised by choosing 360 divisions which gave a approximately one division per day and had lots of whole number factors.

A third reason is that for astronomical purposes, the Babylonians use 60 as their base rather than 10, and 360 is conveniently 6 times their base. They probably chose 60 since also has a lot of divisors (12 in fact) making simple divisions easy. The divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

Wrong answers:
a) Bad guess.

Redaction: Jeffery, 2008jan01

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003 qmult 00306 1 1 3 easy memory: radians

15. Nothing requires us to divide the circle into a whole number or even a rational number of divisions. In fact, it gives a natural angular unit to divide the circle into 2π divisions. The division or angular unit obtained is $1/(2\pi)$ of a circle and is called a radian (rad): thus

$$\frac{1\,\text{rev}}{2\pi} = 1\,\text{rad} .$$

The radian is the natural unit of angular measure since arc length s is given by ____________, where r is the circle's radius and θ is angle in radians. Also all of calculus with trigonometric functions is simplified by using radians.

a) $s = \theta/r$ b) $s = [\theta/(2\pi)]r$ c) $s = \theta r$ d) $s = \theta r^2$ e) $s = r/\theta$

SUGGESTED ANSWER: (c)

In general, one has

$$s = \frac{\theta_{\text{unit}}}{\theta_{\text{max}}}(2\pi r) .$$

If angle is in radians, this reduces to the simple formula $s = \theta r$.

Wrong answers:
a) Oh, c'mon.

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003 qmult 00310 1 4 5 easy deducto-memory: transcendental functions

16. "Let's play *Jeopardy*! For $100, the answer is: They are functions that cannot be exactly evaluated by a finite sequence of the algebraic operations of addition, multiplication, and root taking. Examples are the trigonometric, logarithm, and exponential functions."

What are the ____________ functions, Alex?

a) algebraic b) polynomial c) real d) emergent e) transcendental

SUGGESTED ANSWER: (e)

Wrong answers:
a) Sort of exactly wrong.

Redaction: Jeffery, 2008jan01

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003 qmult 00320 1 4 4 easy deducto-memory: trigonometric functions
Extra keywords: mathematical physics

17. "Let's play *Jeopardy*! For $100, the answer is: They are functions whose argument is an angle and which yield the ratios of sides of a right triangle (or right-angled triangle). The functions are extended so that the argument angle can be any value in the range $[0°, 360°]$. The "sides" of the triangle in the extension are the magnitude of a radius vector (which is the hypotenuse) and the components of the radius vector along the x and y axes of the Cartesian plane. The component "sides" can be positive or negative depending on quadrant where the radius lies."

What are the ____________ functions, Alex?

a) vector b) eigen c) venial d) trigonometric e) bodily

SUGGESTED ANSWER: (d)

Wrong answers:

a) There are vector functions, but they don't fit the answer.
b) There are eigenfunctions, but they don't fit the answer.
e) Well no.

Redaction: Jeffery, 2008jan01

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003 qmult 00330 1 1 1 easy memory: basic trig function names

18. The three basic trigonometric functions have abbreviated names:

a) sin, cos, tan. b) sly, crow, tawn. c) slip, crape, toon. d) slop, crip, troop.
e) snood, croon, troon.

SUGGESTED ANSWER: (a)

Wrong answers:
e) It's hard to believe in snood, but it is in the dictionary.

Redaction: Jeffery, 2008jan01

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003 qmult 00332 1 1 3 easy memory: trig function definitions

19. The 3 basic trigonometric functions are defined by

$$\sin\theta = \frac{y}{r} , \qquad \cos\theta = \frac{x}{r} , \qquad \tan\theta = \frac{y}{x} ,$$

where r is the magnitude of a radius vector $\vec{r}$, θ is the angle of the radius vector measured counterclockwise from the positive x axis, and (x, y) are the ordered pair that locate the head of the radius vector. Just to be clear, the argument of the functions is θ and the trigonometric functions are the ratios. Immediately, one sees that:

a) $\tan\theta = \cos(\theta)\sin(\theta)$. b) $\tan\theta = \dfrac{\cos\theta}{\sin\theta}$. c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. d) $\tan\theta = \dfrac{1}{\cos\theta}$.
e) $\tan\theta = \dfrac{1}{\sin\theta}$.

SUGGESTED ANSWER: (c)

Behold:

$$\tan\theta = \frac{y}{x} = \frac{y/r}{x/r} = \frac{\sin\theta}{\cos\theta} .$$

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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003 qmult 00340 1 1 5 easy memory: Pythagorean theorem

20. Given a right triangle with sides of length x and y being adjacent to the right angle and a side of length r being opposite to the right angle (this side is the hypotenuse), one has the exact relationship

$$r^2 = x^2 + y^2 .$$

This relationship is called the:

a) Anaxagorean theorem. b) Euclidean theorem. c) Sumerian theorem.
d) Taurean theorem. e) Pythagorean theorem.

SUGGESTED ANSWER: (e)

Without a diagram, discussing a proof of the Pythagorean theorem is awkard. But imagine a 1st square of side length $a + b$. Put 2nd square inside the 1st square whose vertices just touch the 1st square at the point a length a along each side. The 2nd square has side length c. One can see there are four right triangles that fill the 1st square not filled by the 2nd square. These triangles are similar with sides of length a, b, and c, where c is the hypotenuse. The area of the triangle is

$$A = (a + b)^2 = a^2 + b^2 + 2ab .$$

But it also is

$$A = c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab \ ,$$

where we have used the area formula for a triangle of one half base times height. We then have

$$a^2 + b^2 + 2ab = c^2 + 2ab \ ,$$

from which it follows that

$$c^2 = a^2 + b^2$$

which is just the Pythagorean theorem.

The proof above really lacks some mathematical rigor. For example, we've relied on the area concept without explicating why that is fair to do. But "sufficient unto the day is the rigor thereof." Roger Penrose (*Road to Reality*, 2004, p. 25–33) gives a discussion of what is needed for a required for a rigorous proof in Euclidean geometry.

Other simple proofs exist, but this is seems simplest to me. See Wikipedia: Pythagorean theorem.

Wrong answers:

c) The Sumerians may have had a working knowledge of the theorem, but they may never have considered a formal proof necessary or any abstract statement of it.

Redaction: Jeffery, 2008jan01

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003 qmult 00350 2 1 1 moderate memory: trig functions with 0 degrees

21. Sin, cos, and tan of 0° are, respectively:

a) 0, 1, 0. b) $1/2$, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, $1/2$, $\sqrt{3}$. e) 1, 0, ∞.

SUGGESTED ANSWER: (a)

I thought of using 3.14159..., 2.71828..., and 0.57721566... as a wrong answer, but it turned out more plausible ones did cover the five possible answers. These numbers are π, e, and the Euler-Mascheroni constant (WA-266–267). The Euler-Mascheroni constant is a more obscure irrational number, but dear to some folks.

Wrong answers:

e) These are for 90°.

Redaction: Jeffery, 2008jan01

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003 qmult 00356 2 1 4 moderate memory: trig functions with 60 degrees

22. Sin, cos, and tan of 60° are, respectively:

a) 0, 1, 0. b) $1/2$, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, $1/2$, $\sqrt{3}$. e) 1, 0, ∞.

SUGGESTED ANSWER: (d)

Alternate set of standard results are those for 30° which is the complementary angle to 60°. For 30°, sin, cos, and tan are, respectively, $1/2$, $\sqrt{3}/2$, and $1/\sqrt{3}$.

You may be curious (but probably not) on how we know these results to be exact. Imagine a regular hexagon divided into 6 triangles by lines connecting opposing vertices. The lines must all intersect at the geometric center of the hexagon. By symmetry, the triangles must all be similar and all be isosceles triangles. They must all have angles 60° at the geometric center of the hexagon since these angles must add up to 360°.

Consider one triangle. Since it is isosceles, the non-center angles must be equal and since all the angles must add up to 180°, all the angles of the triangle are 60°. Thus, all the sides of the triangle are equal since the triangle is isosceles relative to any vertex. Let the side length be c. Bisect angle of the triangle that is at the center of the hexagon by a perpendicular to the opposing side.

Consider one of the triangles formed by the bisection. It is a right triangle with angles 30°, 60°, and, of course, 90°. The side opposing the 30° angle has length $c/2$. Using the Pythagorean theorem, it follows that the side opposing the 60° angle and adjacent to the 30° angle has length

$$\sqrt{c^2 - \left(\frac{c}{2}\right)^2} = c\frac{\sqrt{3}}{2} \ .$$

It now follows immediately that

$$\cos(30°) = \frac{\sqrt{3}}{2} \;, \qquad \sin(30°) = \frac{1}{2} \;, \qquad \tan(30°) = \frac{1}{\sqrt{3}} \;,$$

and

$$\cos(60°) = \frac{1}{2} \;, \qquad \sin(60°) = \frac{\sqrt{3}}{2} \;, \qquad \tan(60°) = \sqrt{3} \;.$$

I don't like to admit how long it took me to figure out this proof.

Wrong answers:
e) These are for 90°.

Redaction: Jeffery, 2008jan01

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003 qmult 00380 1 1 3 easy memory: sinusoidal motion
Extra keywords: sinusoidal motion from circular motion.

23. An object is undergoing uniform circular motion (i.e., revolving in a circular in a circle at a uniform speed). If the object's motion is projected on a line in the plane of the circle and passing through the center of the circle, the motion along the line is ____________ as a function of time.

a) a sawtooth wave b) time-like c) sinusoidal d) a square wave e) a lissajous curve

SUGGESTED ANSWER: (c)

Wrong answers:
a) The cusps in this pattern make it unlikely to arise from a smooth rotational motion.
b) In special relatively, events that can happen at the same place in some inertial reference frame (which may be a moving frame for most of the universe's in, cos, and tan of 60° are since the two events can never be made to be simultaneous by a frame transformation and one event always precedes the other. There can be a causal relationship between the events.
d) The cusps in this pattern make it unlikely to arise from a smooth rotational motion.
e) This is kind of wrapt-around sinusoidal motion frequently seen on oscilloscopes.

Redaction: Jeffery, 2008jan01

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003 qmult 00400 1 1 2 easy memory: vector components

24. Vector components are computed by multiplying the magnitude (or length) of a vector by the cosines of the angles the vector makes with the positive coordinate directions of a:

a) circle. b) coordinate system. c) rotation. d) square. e) wheel.

SUGGESTED ANSWER: (b)

For Cartesian coordinates, one can say coordinate axes. But for curvilinear coordinate systems (e.g., polar coordinates and spherical polar coordinates), it's a bit awkward to talk of their axes since the "axes" are position dependent. It better to say that the components are the dot products of the vector with the coordinate system unit vectors. It shorter to say this and clearer—but only after the unit vectors and the dot product have been introduced.

Wrong answers:
c) A nonsense answer.

Redaction: Jeffery, 2008jan01

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003 qmult 00402 1 1 3 easy memory: 2-d vectors

25. In Cartesian coordinates, a two-dimensional vector $\vec{a}$ is given by

$$\vec{a} = (a_x, a_y) \;,$$

where (a_x, a_y) given by

$$a_x = a\cos\theta \qquad \text{and} \qquad a_y = a\cos\theta_y \;,$$

where θ is the standard angle measured from the positive x axis and θ_y is the angle measured from the y axis. From trigonometry, we know that

$$\frac{a_y}{a} = \cos\theta_y$$

and that

a) $\dfrac{a_y}{a} = \tan\theta$. b) $\dfrac{a_y}{a} = \cos\theta$. c) $\dfrac{a_y}{a} = \sin\theta$. d) $\dfrac{a_y}{a} = \cot\theta$. e) $\dfrac{a_y}{a} = \csc\theta$.

SUGGESTED ANSWER: (c)

Wrong answers:

a) Does this seem likely.

Redaction: Jeffery, 2008jan01

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003 qmult 00410 1 1 4 easy memory: vector component non-uniqueness

26. The components of multi-dimensional physical vectors:

a) are unique.
b) can be chosen only two ways: the two ways will lead to different physical behavior.
c) can be chosen in infinitely many ways: each way leads to a different physical behavior.
d) can be chosen in infinitely many ways. However, the physics of the vector remains the same and in any problem the choice of components (i.e., the choice of a coordinate system) is arbitrary. But some choices make the problem a lot easier.
e) cannot be determined at all in principle.

SUGGESTED ANSWER: (d) This question obeys the longest answer is right rule.

Wrong answers:

Redaction: Jeffery, 2001jan01

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003 qmult 00510 1 1 1 easy memory: vector addition with components

27. Vector addition is defined to be done by adding the vector components by the:

a) ordinary real number addition rule. b) ordinary real number multiplication rule.
c) extraordinary real number multiplication rule.
d) super-unusual real number multiplication rule. e) law of cosines.

SUGGESTED ANSWER: (a)

Wrong answers:

e) A nonsense answer.

Redaction: Jeffery, 2008jan01

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003 qmult 00530 1 1 2 easy memory: vector addition

28. You can add vectors:

a) geometrically or adding their magnitudes. b) geometrically or by components. c) adding their magnitudes or by components. d) adding their magnitudes or by division. e) adding their magnitudes or by integration.

SUGGESTED ANSWER: (b)

Wrong answers:

d) A nonsense answer.
e) A nonsense answer.

Redaction: Jeffery, 2001jan01

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003 qmult 00540 1 1 1 easy memory: vector addition commutative

29. Vector addition is:

a) independent of the order of addition: i.e., it is commutative.

b) depends on the order of addition.
c) not possible.
d) only possible for displacement vectors.
e) only possible for velocity vectors.

SUGGESTED ANSWER: (a)

Wrong answers:
b) This is exactly wrong.
c) This is exactly wrong too.
d) Well no, a vector qua vector has all vector properties.
e) Well no, a vector qua vector has all vector properties.

Redaction: Jeffery, 2008jan01

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003 qmult 00580 1 5 5 easy thinking: vector addition to zero

30. Say you had two vectors of equal magnitude A, but with opposite directions. What can you say about the vector sum of these vectors? **HINT:** A diagram might help.

a) $2A$ is the magnitude, but the direction cannot be determined without more information.
b) $2A$ is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
c) $2A$ is the magnitude. The direction is the direction of the **SECOND** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **SECOND** stage clearly dominates where you would be.
d) **ZERO** is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
e) **ZERO** is the magnitude. The direction of a zero magnitude vector is **UNDEFINED AND UNNEEDED**. Say you added a zero magnitude vector to a non-zero magnitude vector. In any reasonable mathematical system, the vector sum should just be the original non-zero vector and any direction defined for the zero magnitude vector would have no use.

SUGGESTED ANSWER: (e)

An easy thinking question. Students have to remember about head-to-toe vector addition. They are given clues to wrong answers. Maybe I've obscured the answer. But the hope is that students think there way through the ramification to the right answer and the right way of thinking about things.

Wrong answers:
a) They really should know the magnitude must be zero.
d) They really should know that the two paths will lead to the same place (except for some offset for the curvature and roughness of the Earth, and therefore deduce that order is unimportant.

Redaction: Jeffery, 2008jan01

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003 qmult 00600 1 1 5 easy memory: dot product formula for components, dot product defined 1

31. The formula for the component form of dot product of general vectors $\vec{A}$ and $\vec{B}$ is:

a) $\vec{A}\cdot\vec{B} = A_xB_xA_yB_yA_zB_z$. b) $\vec{A}\cdot\vec{B} = \dfrac{A_xA_yA_z}{B_xB_yB_z}$. c) $\vec{A}\cdot\vec{B} = \dfrac{B_xB_yB_z}{A_xA_yA_z}$.
d) $\vec{A}\cdot\vec{B} = \dfrac{A_xB_yA_z}{B_xA_yB_z}$. e) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

SUGGESTED ANSWER: (e)

Wrong answers:
a) Well no.

Redaction: Jeffery, 2008jan01

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003 qmult 00612 1 1 2 easy memory: primary dot product formula

32. The coordinate-system-independent or non-component formula for the dot product of general vectors $\vec{A}$ and $\vec{B}$ with angle θ between is:

    a) $\vec{A}\cdot\vec{B} = AB\sin\theta\hat{n}$. b) $\vec{A}\cdot\vec{B} = AB\cos\theta$. c) $\vec{A}\cdot\vec{B} = \dfrac{A}{B}\cos\theta$. d) $\vec{A}\cdot\vec{B} = \dfrac{A}{B}\sin\theta\hat{n}$.
    e) $\vec{A}\cdot\vec{B} = -AB\sin\theta\hat{n}$.

    SUGGESTED ANSWER: (b)

    Wrong answers:
    a) This is the cross product formula where $\hat{n}$ is a normal to the two vectors with its sense determined by a right-hand rule where you sweep your right-hand fingers from $\vec{A}$ to $\vec{B}$ and your thumb gives the sense.

    Redaction: Jeffery, 2008jan01

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003 qmult 00620 1 1 3 easy memory: dot product commutes

33. For general vectors $\vec{A}$ and $\vec{B}$, the dot product $\vec{A}\cdot\vec{B}$ equals:

    a) $\vec{B}\cdot\vec{B}$. b) $\vec{A}\cdot\vec{A}$. c) $\vec{B}\cdot\vec{A}$. d) $\vec{A}\cdot\vec{B}^{\,2}$. e) $\vec{A}^{\,2}\cdot\vec{B}$.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    e) A nonsense answer.

    Redaction: Jeffery, 2008jan01

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003 qmult 00630 1 3 1 easy math: standard dot products

34. For vectors $\vec{A}$ and $\vec{B}$ with the angle between them θ equal to 0°, 90°, 180°, one has, respectively, the dot products:

    a) AB, 0, $-AB$. b) $-AB$, 0, AB. c) AB, $-AB$, 0. d) 0, AB, $-AB$.
    e) 0, $-AB$, AB.

    SUGGESTED ANSWER: (a)

    Behold:

$$\vec{A}\cdot\vec{B} = \begin{cases} AB\cos\theta & \text{in general;} \\ AB & \text{for } \theta = 0^\circ; \\ 0 & \text{for } \theta = 90^\circ; \\ -AB & \text{for } \theta = 180^\circ. \end{cases}$$

    Wrong answers:
    b) Well no.

    Redaction: Jeffery, 2008jan01

---

003 qmult 00810 1 3 5 easy math: walk and elevator displacement
Extra keywords: physci KB-60-9

35. A man walks 40 m on level ground to an elevator and then rises 70 m. What approximately is the magnitude of his displacement from the starting point?

    a) 8 m. b) 110 m. c) 30 m. d) 6500 m. e) 80 m.

    SUGGESTED ANSWER: (e)

    In this case, the two displacement vectors are perpendicular to each other. So one can find the magnitude of the vector sum just using the Pythagorean theorem. Behold:

$$d = \sqrt{70^2 + 40^2} = \sqrt{4900 + 1600} = \sqrt{6500} \approx 80\,\text{m} .$$

    Wrong answers:

b) This is the distance he has traveled.
d) Forgot to take the square root.

Redaction: Jeffery, 2001jan01

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003 qmult 00820 1 3 3 easy math: displacements in Vegas 1
Extra keywords: physci

36. You are in Las Vegas at the intersection of the Strip and Tropicana (where the MGM Grand, New York, New York, Excalibur, and Tropicana are). You go about **1 mile north** on the east side of the Strip to the Harley-Davidson Cafe, cross the Strip to the west side, and go about **half a mile south** to the Monte Carlo and there lose **most** of your of \$100 stake at the roulette table.

a) Your total travel distance is about **1.5 miles**, total displacement about **1 mile north**, and you have **more** than \$50 left.
b) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **more** than \$50 left.
c) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **less** than \$50 left.
d) Your total travel distance is about **1.5 miles**, total displacement about **1.5 miles north**, and you have **more** than \$50 left.
e) Your total travel distance is about **0.5 miles**, total displacement about **1.5 miles north**, and you have havn't got **bus fare** left.

SUGGESTED ANSWER: (c)

Ah, I remember the corner well from my days in Vegas working at UNLV (1998–1999, 2003–2004).

Wrong answers:
a) The displacement is only 0.5 miles north.
b) You have less than \$50 left, not more.
d) The displacement is only 0.5 miles north.
e) Distance and displacement numbers are interchanged and may or may not have bus fare: there's not enough information to tell; the question doesn't suggest you're broke though.

Redaction: Jeffery, 2001jan01

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003 qmult 00824 1 1 1 easy memory: displacements in Vegas 3

37. You are in Vegas again. You start from 965 E. Cottage Grove and drive 0.50 mi east to S. Maryland Parkway and then drive 1.00 mi south to Tropicana. What is your displacement?

a) 1.1 mi, 63° south of east. b) 0.9 mi, 63° north of east. c) 1.3 mi, 63° south of east.
d) 1.1 mi, 27° north of east. e) 1.3 mi, 27° south of east.

SUGGESTED ANSWER: (a)

From the Pythagorean theorem,

$$r = \sqrt{x^2 + y^2} = 1.1\,\text{mi}$$

and using the inverse tangent function

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = -63^\circ \ .$$

In compass direction terms, the direction is 63° south of east.
I used to live at 965 E. Cottage Grove—in the good old days.

Wrong answers:
b) How could you be north of east?

Fortran-95 Code

```
x=.5d0
y=-1.d0
r=sqrt(x**2+y**2)
```

```
        theta=atan(y/x)
        raddeg2=180.d0/pi
        print*,'r,theta'
        print*,r,theta*raddeg2
!   1.118033988749895 -63.43494882292201
```

Redaction: Jeffery, 2008jan01

---

003 qmult 00830 1 3 2 easy math: ground speed of an airplane

Extra keywords: physci KB-57-23

38. An airplane flying horizontally has an air speed of 250 km/h and is flying in a horizontal wind of 70 km/h. The ground speed of the airplane must be somewhere in the range:

a) 250–320 km/h. b) 180–320 km/h. c) 180–250 km/h. d) 240–260 km/h. e) 70–250 km/h.

SUGGESTED ANSWER: (b)

The two velocities must add vectorially. The actual value depends on the relative orientation of the velocities. But the minimum value is if the wind is a headwind and the maximum is if it is a tailwind.

Wrong answers:

d) I don't know why anyone would choose this answer.

Redaction: Jeffery, 2001jan01

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003 qfull 00350 1 3 0 easy math: prove law of sines

39. The law of sines is

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c} ,$$

where a, b and c are the sides of a general triangle and θ_a, θ_b, θ_c are the angles opposite those sides. Prove the law of sines. **HINT:** Use trigonometry and draw an illustrative diagram.

SUGGESTED ANSWER:

You will have to imagine the diagram.

Let triangle side c be the triangle base. The line collinear with c is the base line.

Let side a be the left up-angled side and side b be the right up-angled side. Let y be the height of the triangle: i.e., the length of a perpendicular from the vertex formed by intersection of sides a and b.

Let θ_1 be the angle measured counterclockwise from the horizontal to a. Let θ_2 be the angle measured counterclockwise from the horizontal to b. From trigonometry, we find

$$y = a\sin\theta_1 \qquad \text{and} \qquad y = b\sin\theta_2 .$$

Since y equals y, we find that

$$a\sin\theta_1 = b\sin\theta_2 .$$

Now θ_1 is actually θ_b and $\theta_2 = 180^\circ - \theta_a$. From a standard trigonometric identity, we find

$$\sin\theta_2 = \sin(180^\circ - \theta_a) = \sin\theta_a .$$

If you don't remember the trig identity $\sin(180^\circ - \theta) = \sin\theta$ (which I seldom to), then one should obtain it from the memorable trig identity

$$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) .$$

Now behold:

$$\begin{aligned}\sin(\theta_2) &= \sin(180^\circ - \theta_a) \\ &= \sin(180^\circ)\cos(\theta_a) + \cos(180^\circ)\sin(-\theta_a) = 0 - \sin(-\theta_a) \\ &= \sin\theta_a .\end{aligned}$$

We now have that

$$a \sin\theta_b = b \sin\theta_a \ ,$$

and thus

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} \ .$$

Sides a and b are general, and so this equality must hold for any sides a and b. To be concrete, if we'd flipped the triangle laterally so that side a was right up-angled side and side b the left up-angled side, we'd have arrived at the same result following the argument above.

If we chose side a as the base, we'd get

$$\frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

by the same argument as above.

So finally, we get

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

which is the law of sines QED.

Redaction: Jeffery, 2008jan01

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003 qfull 00510 1 3 0 easy math: adding two vectors

40. You are given two vectors in component form:

$$\vec{A} = (3.2, 4.2) \qquad \text{and} \qquad \vec{B} = (-10.5, 3.0) \ .$$

a) Give the vector sum $\vec{A} + \vec{B}$ and vector difference $\vec{A} - \vec{B}$ in component form.

b) What is the magnitude of $\vec{A} + \vec{B}$?

c) What is the angle of $\vec{A} + \vec{B}$ relative to the positive x-axis? The positive x-axis is the normal reference direction on the Cartesian plane.

SUGGESTED ANSWER:

a) Behold:

$$\vec{A} + \vec{B} = (-7.3, 7.2) \qquad \text{and} \qquad \vec{A} - \vec{B} = (13.7, 1.2) \ .$$

b) From the Pythagorean theorem we have

$$\sqrt{x^2 + y^2} = \sqrt{(-7.3)^2 + 7.2^2} = 10.25 \ .$$

c) Using the inverse tan function we find

$$\tan^{-1}\left(\frac{y}{x}\right) + 180^\circ = 135.4^\circ \ ,$$

where we have to add 180° to put the angle in the correct quadrant. Recall the calculator or computer inverse tan function is limited to the angular range -90° to 90°. The actual inverse tan function computer inverse tan function plus either 0° or 180° and the plus any multiple of 360° degrees since all those angles give the same trigonometric function values: the trigonometric functions are periodic over 360°. The 0° case is when $x > 0$: i.e., point is on the right-hand side of the Cartesian plane. The 180° case is when $x < 0$: i.e., point is on the left-hand side of the Cartesian plane. The ambiguity arises because the computer inverse tan function cannot tell the individual signs of the x and y values. It can only tell the sign of the quotient. Thus, the designers of the computer inverse tan functions as a matter of convention made the function assume that $x > 0$ always. One always has to check which quadrant the angle actually lies in. Note if both x and y are negative the angle lies in the 3rd quadrant, but the computer inverse tan function will give an angle in the 1st quadrant.

Fortran-95 Code

```
      print*
      x=-7.3
      y=7.2
      xmag=sqrt(x**2+y**2)
      angle=atand(y/x)+180
      print*,'xmag,angle'
      print*,xmag,angle
!      10.253292 135.39514
```

Redaction: Jeffery, 2008jan01

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003 qfull 00640 2 3 0 moderate math: vector identity: sum and difference

41. If the sum of two vectors is perpendicular to their difference, prove that the vectors have equal magnitude. **HINT:** Use the dot or scalar product of sum and difference. A diagram might make the result look plausible.

SUGGESTED ANSWER:

Let $\vec{A}$ and $\vec{B}$ be two vectors such that their sum $\vec{C}$ and difference $\vec{D}$ are perpendicular. Given that $\vec{C}$ and $\vec{D}$ are perpendicular, it follows that

$$0 = \vec{C} \cdot \vec{D} = (\vec{A} + \vec{B}\,) \cdot (\vec{A} - \vec{B}\,) = A^2 - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - B^2 = A^2 - B^2 \;:$$

i.e., that the magnitudes of A and B are the same. A rectangle diagram with a diagonals drawn through it suggests result.

Redaction: Jeffery, 2001jan01

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003 qfull 00650 2 3 0 moderate math: prove law of cosines

42. Say one has a general triangle with sides a, b, and r with angle θ opposite side r. The law of cosines relates the sides and angle:

$$r^2 = a^2 + b^2 - 2ab\cos\theta \;.$$

The law of cosines can be viewed as a generalization of the Pythagorean theorem.

One can prove the law of cosines using elementary geometrical means, but one can also easily prove it using the dot product. The trick is the seeing where to start. And the trick is to see side r as a vector $\vec{r}$ that is the sum of vectors $\vec{a}$ and $\vec{b}$ made out of sides a and b. Take the dot product of $\vec{r}$ with itself and carry on with the proof. **HINT:** Draw a diagram.

SUGGESTED ANSWER:

The diagram you will have to imagine.
Now

$$\begin{aligned} r^2 &= \vec{r} \cdot \vec{r} = (\vec{a} + \vec{b}\,) \cdot (\vec{a} + \vec{b}\,) \\ &= a^2 + b^2 + 2\vec{a} \cdot \vec{b} \\ &= a^2 + b^2 + 2ab\cos(180^\circ - \theta) \\ &= a^2 + b^2 - 2ab\cos\theta \;, \end{aligned}$$

where we have used the distributive and commutative properties of the dot product and the fact that the angle between $\vec{a}$ and $\vec{b}$ is the supplement angle of angle θ. From the above chain of calculations, one arrives at the law of cosines

$$r^2 = a^2 + b^2 - 2ab\cos\theta \;.$$

Redaction: Jeffery, 2001jan01

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003 qfull 00810 1 3 0 easy math: hurricane

43. The eye of a hurricane passes over Bermuda moving 20.0° north of west at 40 km/h for 2 hours and then turns due north moving at 20 km/h. What is its displacement relative to Bermuda after 4 hours

being at Bermuda: give distance from Bermuda and angle relative to north? Neglect the curvature of the Earth.

SUGGESTED ANSWER:

Well, the displacements are 80 km at 20.0° north of west and 40 km due north. There are various ways of the doing vector addition. The simplest is probably to start by decomposing the first displacement into west and north components. These are 75.175 km west and 27.362 km north. Then add 27.362 km and 40 km to get 67.362 km which is the total northward component. The magnitude of the displacement from Bermuda is then

$$\sqrt{75.175^2 + 67.362^2} = 101\,\mathrm{km}$$

and the angle west from the north is

$$\tan^{-1}\left(\frac{75.175}{67.362}\right) = 48^\circ\ .$$

Fortran-95 Code

```
          print*
          pi_50=3.14159265358979323846264338327950288419716939937510e0_np !
http://en.wikipedia.org/wiki/Pi#Approximate_value
          raddeg=180.e0_np/pi_50
          v1=40.d0
          t1=2.d0
          theta1=20.d0
          r1=v1*t1
          x1=r1*cos(theta1/raddeg)
          y1=r1*sin(theta1/raddeg)
          v2=20.d0
          t2=4.d0
          y2=v2*(t2-t1)
          x=x1
          y=y1+y2
          r=sqrt(x**2+y**2)
          theta=atan(x/y)*raddeg  ! This is west of north.
          print*,'r1,x1,y1,y2,x,y,r,theta'
          print*,r1,x1,y1,y2,x,y,r,theta
    ! 80.0 75.17540966287268 27.361611466053496 40.0 75.17540966287268
    ! 67.36161146605349 100.94022447609417 48.1377812784058
```

Redaction: Jeffery, 2008jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

---

2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

---

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

---

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1 - x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

---

5 Quadratic Formula

If $\quad 0 = ax^2 + bx + c\ ,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \text{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $g = 9.8\,\mathrm{m/s^2}$

$$x_{\rm rel} = x_2 - x_1 \qquad v_{\rm rel} = v_2 - v_1 \qquad a_{\rm rel} = a_2 - a_1$$

$$x' = x - v_{\rm frame}t \qquad v' = v - v_{\rm frame} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\rm avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\rm avg} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{{\rm for}\ y\ {\rm max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\rm max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\rm for\ max} = \frac{\pi}{4} \qquad x_{\rm max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

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14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total}\ f} = KE_{\text{total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

19 **Static Equilibrium**

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 **Gravity**

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 **Oscillation**

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\max}\sin[k(x \mp vt)] = y_{\max}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^{\gamma} = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$