Intro Physics Semester I Name:

Homework 2: Vectors and Trigonometry: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

1. "Let's play *Jeopardy*! For \$100, the answer is: An incomplete definition is that it is a quantity with both a magnitude and a direction. The direction is in ordinary physical space (i.e., space space). The extent is in ordinary physical space for displacement only and in abstract spaces otherwise (e.g., velocity space)."

   What is a/an ____________, Alex?

   a) director b) aviator c) bisector d) scalar e) vector

2. "Let's play *Jeopardy*! For \$100, the answer is: It is a physical quantity expressible by a single real number whose value is independent of the coordinate system (of ordinary physical space). A ____________ can have a direction in a 1-dimensional abstract space. For example, temperature on the Celsius scale: it can have positive or negative value. So ____________ can be regarded as a 1-dimensional vector with the 1 direction being in an abstract space, but in the usual way of speaking one would say it is **NOT** a vector."

   What is a/an ____________, Alex?

   a) director b) aviator c) bisector d) scalar e) vector

3. "Let's play *Jeopardy*! For \$100, the answer is: It is quantity with a value at every point in space or at least some region of space. The quantity can be a scaler, vector, or tensor."

   What is a/an ____________, Alex?

   a) area b) electric area c) field d) electric space e) zone

4. A vector (i.e., a true vector, not just something called a vector since it has some vector-like aspects) has the properties that the components are ____________ and the magnitude of the vector (in its space abstract or otherwise) is determined using the ____________ with components acting as "sides". Also vectors have certain coordinate transformation properties. Pseudovectors (e.g., angular momentum and torque) act like vectors in most respects, but do not have the exactly the same coordinate tranformation properties.

   a) scalars; Pythagorean theorem b) scalars; absolute value operation
   c) vectors; Pythagorean theorem d) vectors; absolute value operation
   e) integers; Pythagorean theorem

5. "Let's play *Jeopardy*! For \$100, the answer is: It is a vector that is specified by giving the direction and a straight-line distance in that direction."

   What is ____________, Alex?

   a) velocity b) acceleration c) displacement d) force e) distance

6. The prototype vector in physics (i.e., the one that is usually thought of setting what the standard physical vector properties are) is ____________. One main reason for this is that of the physical vectors, only ____________ has extension in ordinary physical space.

   a) force b) mass c) displacement d) speed e) the electric field vector

7. Velocity is:

   a) speed. b) a scalar. c) displacement. d) acceleration. e) a vector.

8. Acceleration is:

   a) speed.
   b) velocity.

c) the rate of change of velocity with time. It is a **SCALAR**.
d) the rate of change of velocity with time. It is a **VECTOR**.
e) the rate of change of displacement with time. It is a **VECTOR**.

9. You are going in a circle at a uniform speed (i.e., a constant speed). Is your **VELOCITY** ever changing?

   a) No, the speed is constant.
   b) Yes, it is constantly changing since the motion is in a **STRAIGHT LINE**.
   c) Yes, it is constantly changing since the motion is in a **CIRCLE** and direction is constantly changing.
   d) No, the velocity is constant.
   e) Yes, on **EVERY OTHER LEFT** bend.

10. Which of the following quantities is vector?

    a) mass. b) force. c) energy. d) speed. e) temperature.

11. Which of the following can be the unit of a vector?

    a) kilogram. b) second. c) meter/second. d) liter. e) acceleration.

12. By the ancient traditional meaning trignonometry (abbreviated to trig) is the branch of mathematics dealing with triangles. The word trignonometry is derived from trigonon (Greek for triangle) and metron (Greek for measure). Actually trignonometry is generalized beyond triangles to deal with the components of a radius of a circle in a Cartesian coordinate system. This generalization gives the definitions of the ____________ functions.

    a) polynomial b) trignonometric c) transcendental d) quadratic e) cubic

13. Trigonometry is often abbreviated to

    a) triggy. b) trig. c) gono. d) metro. e) monstro.

14. For reasons they never bothered to record for posterity, the ancient Babylonians of circ 500 BCE divided the circle into 360 units (i.e., 360 degrees). One likely reason is that they didn't like dealing with decimal fractions (actually sexagesimal fractions in their system), and so chose a number of divisions to be one with a lot of whole number factors. How many whole number factors does 360 have?

    a) 6. b) 8. c) 12. d) 16. e) 24.

15. Nothing requires us to divide the circle into a whole number or even a rational number of divisions. In fact, it gives a natural angular unit to divide the circle into 2π divisions. The division or angular unit obtained is $1/(2\pi)$ of a circle and is called a radian (rad): thus

    $$\frac{1\,\mathrm{rev}}{2\pi} = 1\,\mathrm{rad}\ .$$

    The radian is the natural unit of angular measure since arc length s is given by ____________, where r is the circle's radius and θ is angle in radians. Also all of calculus with trigonometric functions is simplified by using radians.

    a) $s = \theta/r$ b) $s = [\theta/(2\pi)]r$ c) $s = \theta r$ d) $s = \theta r^2$ e) $s = r/\theta$

16. "Let's play *Jeopardy*! For \$100, the answer is: They are functions that cannot be exactly evaluated by a finite sequence of the algebraic operations of addition, multiplication, and root taking. Examples are the trigonometric, logarithm, and exponential functions."

    What are the ____________ functions, Alex?

    a) algebraic b) polynomial c) real d) emergent e) transcendental

17. "Let's play *Jeopardy*! For \$100, the answer is: They are functions whose argument is an angle and which yield the ratios of sides of a right triangle (or right-angled triangle). The functions are extended so that the argument angle can be any value in the range $[0^\circ, 360^\circ]$. The "sides" of the triangle in the extension are the magnitude of a radius vector (which is the hypotenuse) and the components of the radius vector along the x and y axes of the Cartesian plane. The component "sides" can be positive or negative depending on quadrant where the radius lies."

    What are the ____________ functions, Alex?

a) vector b) eigen c) venial d) trigonometric e) bodily

18. The three basic trigonometric functions have abbreviated names:

a) sin, cos, tan. b) sly, crow, tawn. c) slip, crape, toon. d) slop, crip, troop.
e) snood, croon, troon.

19. The 3 basic trigonometric functions are defined by

$$\sin\theta = \frac{y}{r} , \qquad \cos\theta = \frac{x}{r} , \qquad \tan\theta = \frac{y}{x} ,$$

where r is the magnitude of a radius vector $\vec{r}$, θ is the angle of the radius vector measured counterclockwise from the positive x axis, and (x, y) are the ordered pair that locate the head of the radius vector. Just to be clear, the argument of the functions is θ and the trigonometric functions are the ratios. Immediately, one sees that:

a) $\tan\theta = \cos(\theta)\sin(\theta)$. b) $\tan\theta = \dfrac{\cos\theta}{\sin\theta}$. c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. d) $\tan\theta = \dfrac{1}{\cos\theta}$.
e) $\tan\theta = \dfrac{1}{\sin\theta}$.

20. Given a right triangle with sides of length x and y being adjacent to the right angle and a side of length r being opposite to the right angle (this side is the hypotenuse), one has the exact relationship

$$r^2 = x^2 + y^2 .$$

This relationship is called the:

a) Anaxagorean theorem. b) Euclidean theorem. c) Sumerian theorem.
d) Taurean theorem. e) Pythagorean theorem.

21. Sin, cos, and tan of 0° are, respectively:

a) 0, 1, 0. b) $1/2$, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, $1/2$, $\sqrt{3}$. e) 1, 0, ∞.

22. Sin, cos, and tan of 60° are, respectively:

a) 0, 1, 0. b) $1/2$, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, $1/2$, $\sqrt{3}$. e) 1, 0, ∞.

23. An object is undergoing uniform circular motion (i.e., revolving in a circular in a circle at a uniform speed). If the object's motion is projected on a line in the plane of the circle and passing through the center of the circle, the motion along the line is ____________ as a function of time.

a) a sawtooth wave b) time-like c) sinusoidal d) a square wave e) a lissajous curve

24. Vector components are computed by multiplying the magnitude (or length) of a vector by the cosines of the angles the vector makes with the positive coordinate directions of a:

a) circle. b) coordinate system. c) rotation. d) square. e) wheel.

25. In Cartesian coordinates, a two-dimensional vector $\vec{a}$ is given by

$$\vec{a} = (a_x, a_y) ,$$

where (a_x, a_y) given by

$$a_x = a\cos\theta \qquad \text{and} \qquad a_y = a\cos\theta_y ,$$

where θ is the standard angle measured from the positive x axis and θ_y is the angle measured from the y axis. From trigonometry, we know that

$$\frac{a_y}{a} = \cos\theta_y$$

and that

a) $\dfrac{a_y}{a} = \tan\theta$. b) $\dfrac{a_y}{a} = \cos\theta$. c) $\dfrac{a_y}{a} = \sin\theta$. d) $\dfrac{a_y}{a} = \cot\theta$. e) $\dfrac{a_y}{a} = \csc\theta$.

26. The components of multi-dimensional physical vectors:

a) are unique.
b) can be chosen only two ways: the two ways will lead to different physical behavior.
c) can be chosen in infinitely many ways: each way leads to a different physical behavior.
d) can be chosen in infinitely many ways. However, the physics of the vector remains the same and in any problem the choice of components (i.e., the choice of a coordinate system) is arbitrary. But some choices make the problem a lot easier.
e) cannot be determined at all in principle.

27. Vector addition is defined to be done by adding the vector components by the:

a) ordinary real number addition rule. b) ordinary real number multiplication rule.
c) extraordinary real number multiplication rule.
d) super-unusual real number multiplication rule. e) law of cosines.

28. You can add vectors:

a) geometrically or adding their magnitudes. b) geometrically or by components. c) adding their magnitudes or by components. d) adding their magnitudes or by division. e) adding their magnitudes or by integration.

29. Vector addition is:

a) independent of the order of addition: i.e., it is commutative.
b) depends on the order of addition.
c) not possible.
d) only possible for displacement vectors.
e) only possible for velocity vectors.

30. Say you had two vectors of equal magnitude A, but with opposite directions. What can you say about the vector sum of these vectors? **HINT:** A diagram might help.

a) $2A$ is the magnitude, but the direction cannot be determined without more information.
b) $2A$ is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
c) $2A$ is the magnitude. The direction is the direction of the **SECOND** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **SECOND** stage clearly dominates where you would be.
d) **ZERO** is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
e) **ZERO** is the magnitude. The direction of a zero magnitude vector is **UNDEFINED AND UNNEEDED**. Say you added a zero magnitude vector to a non-zero magnitude vector. In any reasonable mathematical system, the vector sum should just be the original non-zero vector and any direction defined for the zero magnitude vector would have no use.

31. The formula for the component form of dot product of general vectors $\vec{A}$ and $\vec{B}$ is:

a) $\vec{A}\cdot\vec{B} = A_xB_xA_yB_yA_zB_z$. b) $\vec{A}\cdot\vec{B} = \dfrac{A_xA_yA_z}{B_xB_yB_z}$. c) $\vec{A}\cdot\vec{B} = \dfrac{B_xB_yB_z}{A_xA_yA_z}$.
d) $\vec{A}\cdot\vec{B} = \dfrac{A_xB_yA_z}{B_xA_yB_z}$. e) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

32. The coordinate-system-independent or non-component formula for the dot product of general vectors $\vec{A}$ and $\vec{B}$ with angle θ between is:

a) $\vec{A}\cdot\vec{B} = AB\sin\theta\hat{n}$. b) $\vec{A}\cdot\vec{B} = AB\cos\theta$. c) $\vec{A}\cdot\vec{B} = \dfrac{A}{B}\cos\theta$. d) $\vec{A}\cdot\vec{B} = \dfrac{A}{B}\sin\theta\hat{n}$.
e) $\vec{A}\cdot\vec{B} = -AB\sin\theta\hat{n}$.

33. For general vectors $\vec{A}$ and $\vec{B}$, the dot product $\vec{A}\cdot\vec{B}$ equals:

a) $\vec{B}\cdot\vec{B}$. b) $\vec{A}\cdot\vec{A}$. c) $\vec{B}\cdot\vec{A}$. d) $\vec{A}\cdot\vec{B}^{\,2}$. e) $\vec{A}^{\,2}\cdot\vec{B}$.

34. For vectors $\vec{A}$ and $\vec{B}$ with the angle between them θ equal to 0°, 90°, 180°, one has, respectively, the dot products:

 a) AB, 0, $-AB$. b) $-AB$, 0, AB. c) AB, $-AB$, 0. d) 0, AB, $-AB$.
 e) 0, $-AB$, AB.

35. A man walks 40 m on level ground to an elevator and then rises 70 m. What approximately is the magnitude of his displacement from the starting point?

 a) 8 m. b) 110 m. c) 30 m. d) 6500 m. e) 80 m.

36. You are in Las Vegas at the intersection of the Strip and Tropicana (where the MGM Grand, New York, New York, Excalibur, and Tropicana are). You go about **1 mile north** on the east side of the Strip to the Harley-Davidson Cafe, cross the Strip to the west side, and go about **half a mile south** to the Monte Carlo and there lose **most** of your of \$100 stake at the roulette table.

 a) Your total travel distance is about **1.5 miles**, total displacement about **1 mile north**, and you have **more** than \$50 left.
 b) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **more** than \$50 left.
 c) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **less** than \$50 left.
 d) Your total travel distance is about **1.5 miles**, total displacement about **1.5 miles north**, and you have **more** than \$50 left.
 e) Your total travel distance is about **0.5 miles**, total displacement about **1.5 miles north**, and you have havn't got **bus fare** left.

37. You are in Vegas again. You start from 965 E. Cottage Grove and drive 0.50 mi east to S. Maryland Parkway and then drive 1.00 mi south to Tropicana. What is your displacement?

 a) 1.1 mi, 63° south of east. b) 0.9 mi, 63° north of east. c) 1.3 mi, 63° south of east.
 d) 1.1 mi, 27° north of east. e) 1.3 mi, 27° south of east.

38. An airplane flying horizontally has an air speed of 250 km/h and is flying in a horizontal wind of 70 km/h. The ground speed of the airplane must be somewhere in the range:

 a) 250–320 km/h. b) 180–320 km/h. c) 180–250 km/h. d) 240–260 km/h.
 e) 70–250 km/h.

39. The law of sines is

$$\frac{\sin\theta_a}{a}=\frac{\sin\theta_b}{b}=\frac{\sin\theta_c}{c}\ ,$$

where a, b and c are the sides of a general triangle and θ_a, θ_b, θ_c are the angles opposite those sides. Prove the law of sines. **HINT:** Use trigonometry and draw an illustrative diagram.

40. You are given two vectors in component form:

$$\vec{A}=(3.2,4.2) \qquad \text{and} \qquad \vec{B}=(-10.5,3.0)\ .$$

 a) Give the vector sum $\vec{A}+\vec{B}$ and vector difference $\vec{A}-\vec{B}$ in component form.
 b) What is the magnitude of $\vec{A}+\vec{B}$?
 c) What is the angle of $\vec{A}+\vec{B}$ relative to the positive x-axis? The positive x-axis is the normal reference direction on the Cartesian plane.

41. If the sum of two vectors is perpendicular to their difference, prove that the vectors have equal magnitude. **HINT:** Use the dot or scalar product of sum and difference. A diagram might make the result look plausible.

42. Say one has a general triangle with sides a, b, and r with angle θ opposite side r. The law of cosines relates the sides and angle:

$$r^2=a^2+b^2-2ab\cos\theta\ .$$

The law of cosines can be viewed as a generalization of the Pythagorean theorem.

One can prove the law of cosines using elementary geometrical means, but one can also easily prove it using the dot product. The trick is the seeing where to start. And the trick is to see side r as a vector $\vec{r}$ that is the sum of vectors $\vec{a}$ and $\vec{b}$ made out of sides a and b. Take the dot product of $\vec{r}$ with itself and carry on with the proof. **HINT:** Draw a diagram.

43. The eye of a hurricane passes over Bermuda moving 20.0° north of west at $40\,\mathrm{km/h}$ for 2 hours and then turns due north moving at $20\,\mathrm{km/h}$. What is its displacement relative to Bermuda after 4 hours being at Bermuda: give distance from Bermuda and angle relative to north? Neglect the curvature of the Earth.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

5 Quadratic Formula

If $\quad 0=ax^2+bx+c\ ,\quad$ then $\quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=-\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\mathrm{for}\ y\ \mathrm{max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\mathrm{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\mathrm{for\ max}} = \frac{\pi}{4} \qquad x_{\mathrm{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

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14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

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15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total }f} = KE_{\text{total }i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\mathrm{max}}\sin[k(x \mp vt)] = y_{\mathrm{max}}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\mathrm{max}}^2$$

$$y = 2y_{\mathrm{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L}\ :\ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32°\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$