Introductory Physics: Calculus-Based NAME:

Homework 1: Introduction to Physics, Units, Conversions: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

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001 qmult 00010 1 1 1 easy memory: name written down

1. Have you written your name in the name place at the upper right of this paper?

   a) Yes. b) No. c) Nope. d) Nein. e) Never.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   e) A nonsense answer—I hope.

   Redaction: Jeffery, 2008jan01

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001 qmult 00100 1 4 5 easy deducto-memory: physics defined

2. "Let's play *Jeopardy*! For $100, the answer is: In shorthand, it is the science of matter and motion, space and time."

   What is ____________, Alex?

   a) biology b) phrenology c) economics d) rocket science e) physics

   SUGGESTED ANSWER: (e)

   Wrong answers:
   a) As Lurch would say AAAARGH.
   b) This is head bump reading.

   Redaction: Jeffery, 2008jan01

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001 qmult 00110 1 1 3 easy memory: theory of everything

3. The ultimate goal of physics—or at least one of the ultimate goals—is a theory of:

   a) something (TOS). b) nothing (TON). c) everything (TOE). d) woe (TOW).
   e) Fred (TOF).

   SUGGESTED ANSWER: (c)

   Wrong answers:
   b) Now what good would this be.

   Fortran-95 Code

   Redaction: Jeffery, 2008jan01

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000 qmult 00120 1 4 1 easy deducto-memory: Presocratics

4. "Let's play *Jeopardy*! For $100, the answer is: The most obvious choice for the first natural philosophers or physicists. They attempted to explain the natural world from simple general principles: from these principles complex structures could arise. In mythology on the other hand, usually deities with human-like personalities are invoked to explain structures. But there is probably no hard line between natural philosophy and mythology: one shades into the other in the early days."

   Who are ____________, Alex?

   a) Presocratic philosophers of ancient Greece b) Babylonian diviners c) Roman augurs
   d) Mongolian shamans e) Irish druids

   SUGGESTED ANSWER: (a)

   Wrong answers:

b) These guys did do mathematical astronomy, but they didn't leave us much idea about how they thought things worked in terms of basic principles.
c) They were into haruspicy.
e) Tree worshippers.

Redaction: Jeffery, 2008jan01

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001 qmult 00140 1 1 2 easy memory: mathematical physics origin

5. Mathematical physics in a sense began with the ancient Babylonian astronomers. Later the ancient Greeks added a bit to mathematical physics, particularly Archimedes (circa 287–212 BCE) with his work on statics and hydrostatics. But idea of physics as a largely mathematical science is largely attributable to ____________, ____________, and their contemporaries in the 17th century.

a) William Shakespeare (1564–1616); John Milton (1608–1674)
b) Galileo (1564–1642); Isaac Newton (1643–1727)
c) Bernini (1598–1680); Borromini (1599–1667)
d) Caravaggio (1571–1610); Poussin (1594–1665)
e) Wallenstein (1583–1634); Turenne (1611-1675)

SUGGESTED ANSWER: (b)

Before the 17th century, much of physics in Europe and elsewhere was qualitative and descriptive. In the European/Islamic worlds, Aristotelian physics (which was all/mainly qualitative) held sway.

Wrong answers:
a) Poets.
c) Architects.
d) Painters.
e) Generals.

Redaction: Jeffery, 2008jan01

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001 qmult 00150 1 1 4 easy memory: true exact emergent theory

6. Because there are no doubts about its validity in the classical limit (scales larger than microscopic, but smaller than cosmic, speeds much less than that of light, gravity much less strong than near a black hole) and because it is the exact limit of more general theories, Newtonian physics can be regarded as a/an:

a) utterly false theory. b) negligible theory c) useless theory.
d) true exact emergent theory. e) phantasm

SUGGESTED ANSWER: (d)

Wrong answers:
e) A nonsense answer.

Redaction: Jeffery, 2008jan01

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001 qmult 00192 1 1 5 easy memory: clearly in physics, a joke question

7. In physics jargon, the word "clearly" means:

a) clearly. b) unclearly. c) after 4 pages of algebra. d) wrongly. e) all of the above.

SUGGESTED ANSWER: (e) It seems to me that this is the right answer. When someone says "clearly" in a physics argument, one needs to reflect on their character.

Wrong answers:
a) Only sometimes.
b) Frequently.
c) Pretty often.
d) Pretty often too.

Redaction: Jeffery, 2001jan01

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001 qmult 00194 1 1 5 easy memory: must be in physics, a joke question

Extra keywords: Not a serious question.

8. In physics jargon, the phrase “must be” means:

   a) is. b) just accept it that. c) not necessarily so. d) can’t be. e) all of the above.

   SUGGESTED ANSWER: (e)

   It seems to me that this is the right answer. When someone says “must be” in a physics argument, what they mean depends on their stage of desperation.

   In fact, it “must be” must generally be interpreted from the emphasis of expression. If someone says casually the “door must be closed”, it probably is closed and it probably doesn’t matter much either way. But if someone says “door must be closed” emphatically, you know darn well the door is open—and disaster is about to ensue.

   Wrong answers:
   a) Only sometimes.
   b) Frequently.
   c) Pretty often.
   d) Pretty often too.

   Redaction: Jeffery, 2001jan01

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001 qmult 00260 1 4 2 easy deducto-memory: scientific idealization

9. “Let’s play *Jeopardy*! For $100, the answer is: In scientific theorizing and model building, it is the elimination/neglect of complicating secondary factors in order to understand the main factors. In reality, the secondary factors may or may not be eliminated/neglected to one degree or another. If they cannot be eliminated/neglected, they have to be modeled in order to test the understanding of the main factors.”

   What is ____________, Alex?

   a) Platonic idealization b) scientific idealization c) the scientific method d) experiment e) observation

   SUGGESTED ANSWER: (b) In history, Galileo is cited as a pioneer of scientific idealization (Wikipedia: Idealization (philosophy of science: Early use).

   Wrong answers:
   a) Certainly a related concept and an ingredient the origin of scientific idealization.
   c) Scientific idealization is part of the usual practice of the scientific method.
   d) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2008jan01

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001 qmult 00300 1 4 2 easy deducto-memory: scientific notation defined

Extra keywords: physci

10. “Let’s play *Jeopardy*! For $100, the answer is: It is a notation in which one expresses a number by a coefficient decimal number multiplied explicitly by 10 to the appropriate power. If the coefficient is in the range 1 to 10, but not including 10, the notation is called normalized.”

    What is ____________, Alex?

    a) British notation b) scientific notation c) metric notation d) tensy notation e) Irish notation

    SUGGESTED ANSWER: (b)

    Wrong answers:
    e) Tis yourself that does not know that scientific notation was invented by St. Patrick who, alas, was actually British.

    Redaction: Jeffery, 2001jan01

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001 qmult 00310 1 3 4 easy math: hundred million billion in sci. not.

Extra keywords: physci

11. Write a hundred million billion miles in scientific notation.

    a) 10^2 mi. b) 10^6 mi. c) 10^9 mi. d) 10^{17} mi. e) 10^{-9} mi.

SUGGESTED ANSWER: (d)

As Andy Rooney says (or used to say), don't you just hate it when newspapers use expressions like hundred million billion miles—as if you were just going there to drop the kids off for soccer. We all know scientific notation or should nowadays.

Behold:

$$10^2 \times 10^6 \times 10^9 = 10^{17}\,\text{mi}\ .$$

Wrong answers:

e) Seems unlikely.

Redaction: Jeffery, 2001jan01

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001 qmult 00320 1 3 1 easy math: show in scientific notation

12. Express 4011 and 0.052 in normalized scientific notation form.

    a) 4.011×10^3 and 5.2×10^{-2}. b) 40.11×10^3 and $52. \times 10^{-2}$.
    c) 40.11×10^2 and $52. \times 10^{-3}$. d) 4.011×10^{-2} and 5.2×10^3. e) 4011 and 0.052.

SUGGESTED ANSWER: (a)

Wrong answers:

c) The numbers are, of course, equal to the numbers in the problem and are in scientific notation, but not the most conventional form as most people would say.
d) The same remark as for answer (c) applies.

Redaction: Jeffery, 2008jan01

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001 qmult 00410 1 4 5 easy deducto-memory: units needed

Extra keywords: physci

13. "Let's play *Jeopardy*! For $100, the answer is: In any measurements of quantities, they are conventionally agreed upon standard things."

    What are ____________, Alex?

    a) unities b) dualities c) duplicities d) quantons e) units

SUGGESTED ANSWER: (e)

Wrong answers:

d) I think this is a pretty good alternative to units.

Redaction: Jeffery, 2001jan01

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001 qmult 00420 1 1 3 easy memory: SI or metric units

Extra keywords: physci

14. The modern standard set of units for science, most engineering, and much of everyday life (except in the 2nd largest country in North America) is the International System of Units (Système International d'Unités or SI) which is often called the:

    a) English units. b) Mesopotamian system. c) metric system. d) Paraguayan system.
    e) United States customary units.

SUGGESTED ANSWER: (c)

Wrong answers:

a) More or less exactly wrong.
b) In timekeeping and angular measurement, we still use a lot of the sexagesimal system of the ancient Mesopotamians.
e) What the U.S. uses for many non-scientific purposes.

Redaction: Jeffery, 2001jan01

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001 qmult 00422 1 5 3 easy thinking: US Customary Unit System 1

Extra keywords: the horror, the horror

15. Why is the US Customary Units system (loosely called English Units) hard to use in calculations in comparison with the metric system?

    a) The US Customary Units system has no unit of mass. Consequently, when you need to use mass, you have to mentally work around the lack of a unit.
    b) The US Customary Units system is completely unknown by scientists and engineers, and so, of course, can scarcely be used.
    c) The US Customary Units system has units in a host of irregular sizes: 16 ounces to the pound, 8 pints in a gallon, 12 inches to the foot, and 45 inches to the ell (fact!). This makes calculations and conversions tortuous. In particular, it is awkward that cubic units of length (e.g., cubic inches, feet, etc.) are not simply related to standard volume units (e.g., a U.S. gallon is defined as 231 cubic inches). In everyday life the irregularities of the US Customary Units system don't cause much of a problem and maybe even have mnemonic value. But that hardly helps people who want to send probes to Mars.
    d) The complete regularity of the US Customary Units system means that you never know if a number is 10, 100, 1000, etc., except by context.
    e) It's the revenge for Bunker Hill.

SUGGESTED ANSWER: (c)

In late 1999, NASA lost the Mars Climate Orbiter before its mission could begin by crashing it on Mars due to confusion between English and metric units.

Wrong answers:

a) There is a unit of mass: the slug: a slug weighs about 32.2 pounds, and if acted upon by a one pound force will have an acceleration of 1 foot/s**2. But I don't think a slug has ever been acted by any force at all.
d) Actually the ancient Mesopotamians in math and astronomy used a sexagesimal system (base 60). But they didn't specify absolute values (no "decimal" point) leaving the absolute size to context which often hard to know 4000 years later.
e) Not an answer. However, still usings essentially English units is probably one of the disadvantages of the American Revolution.

Redaction: Jeffery, 2008jan01

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001 qmult 00430 1 4 3 easy deducto-memory: basic quantities

16. Three quantities usually adopted as basic (i.e., not reducible to other kinds of quantities by convention) are:

    a) length, area, volume. b) mass, weight, heft. c) length, mass, time.
    d) time, duration, age. e) length, mass, density.

SUGGESTED ANSWER: (c)

People should just know that (c) is the right answer. Of course, which quantities are taken as basci is to a degree arbitrary. One could take length, density, and time as basic and define mass from length and density. But the simple way we think of things suggests length, mass, time as fundamental.

Wrong answers:

a) Area and volume can be formed from length and so arn't usually considered fundamental.
b) heft can mean weight or heaviness or to lift or heave.
e) Density isn't independent of length and mass in the way we usually think of things, and a time-relating quantity is needed.

Redaction: Jeffery, 2001jan01

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001 qmult 00432 1 1 1 easy memory: the 7 base units

17. The 7 base units are meter, kilogram (gram would have been more logical), second, ampere,

a) kelvin, mole, and candela. b) calvin, mold, and candeleria.
c) kelvin, mouse, and candace. d) melvin, moose, and cantrip.
e) kludge, moor, and mountain.

SUGGESTED ANSWER: (a)

Wrong answers:
d) A cantrip is a magical spell.

Redaction: Jeffery, 2008jan01

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001 qmult 00450 1 4 3 easy deducto-memory: MKS acronym for what?

18. MKS stands for:

a) meters, kilometers, centimeters. b) meters, kilometers, seconds.
c) meters, kilograms, seconds. d) millimeters, kilometers, seconds.
e) millimeters, kilograms, seconds.

SUGGESTED ANSWER: (c)

Wrong answers:
a) Does this seem likely?

Redaction: Jeffery, 2008jan01

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001 qmult 00452 1 5 3 easy thinking: consistent MKS units in calculations

19. If one used **ONLY** MKS units (i.e., units of meters, kilograms, seconds, coulombs, amperes, kelvins, moles, etc., plus MKS units derived from these) in calculations, then one will get answers in:

a) CGS units only. b) MKS or CGS units. c) MKS units only. d) English units only.
e) any old units.

SUGGESTED ANSWER: (c)

An easy thinking question. All physical quantities can be expressed in the 7 fundamental MKS units plus the MKS units derived from them. Any calculation in pure MKS units can only generate results in pure MKS units.

Wrong answers:
a) Not possible.
b) Still not possible.
d) This is delusional.

Redaction: Jeffery, 2008jan01

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001 qmult 00454 1 5 5 easy thinking: consistent and convenient units, natural units

20. In scientific calculations, it is best to stick to one complete, consistent set of units (MKS or CGS):

a) always. b) half the time. c) never. d) whenever. e) except when its not convenient. In most specialized fields, there are natural units that are convenient in actual calculations, but overwhelmingly most often only in human readable outputs. Natural units are most useful for understanding not calculation. Astronomers, for example often use the solar mass (1.9891×10^{30} kg) as a unit for the masses of stars since the Sun for Earthlings is basic standard of reference for stars.

SUGGESTED ANSWER: (e)

Wrong answers:
d) A nonsense answer.

Redaction: Jeffery, 2008jan01

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001 qmult 00460 1 1 1 easy memory: metric kilo and centi
Extra keywords: physci

21. In SI, the prefixes kilo and centi indicate, respectively, multiplication by:

a) 1000 and 0.01. b) 0.01 and 1000. c) 1000 and 100. d) 60 and 0.01. e) π and e.

SUGGESTED ANSWER: (a)

Wrong answers:
e) As Lurch would say: “Aaaarh.”

Redaction: Jeffery, 2001jan01

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001 qmult 00462 1 4 2 easy deducto-memory: SI prefix symbols M- and m-

22. The metric or SI unit prefix symbols M- and m- stand for:

a) mega (factor of 10^6) and milli (factor of 10^{-6}). b) mega (factor of 10^6) and milli (factor of 10^{-3}). c) kilo (factor of 10^6) and milli (factor of 10^{-3}). d) kilo (factor of 10^6) and milli (factor of 10^{-6}). e) merger (factor of 10^9) and melba (factor of 10^{-6}).

SUGGESTED ANSWER: (b)

Wrong answers:
a) This is the tough one to rule out since reason suggests a symmetry between M and m.
c) kilo is for thousand.
d) kilo is for thousand.
e) Moore?

Redaction: Jeffery, 2001jan01

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001 qmult 00500 1 1 1 easy memory: conversion in general, factors of unity
Extra keywords: physci

23. In conversions, one can just treat units as variables whose values are never specified. One can do algebra with them and cancel them. One also knows a set of equalities relating units, and so can write down factors of unity or conversion factors. For example, $1000\,\text{m} = 1\,\text{km}$, and so a factor of unity (or conversion factor) is

$$1 = \frac{1000\,\text{m}}{1\,\text{km}} \ .$$

This factor would be used to convert an amount in kilometers to meters by:

a) multiplication. b) division. c) addition. d) subtraction. e) squaring.

SUGGESTED ANSWER: (a)

Wrong answers:
b) An amount in meters could be converted to kilometers by division.
e) As Lurch would say: “Aaaarh.”

Redaction: Jeffery, 2001jan01

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001 qmult 00510 1 4 1 easy deducto-memory: conversion factor=unity factor
Extra keywords: physci

24. A units conversion factor is sometimes called a factor of:

a) unity. b) 2. c) a few. d) 10. e) fact.

SUGGESTED ANSWER: (a) This isn’t a real common expression, but yours truly uses it along with the more common conversion factor. Note, since conversion doesn’t change physical size, factor of unity makes sense.

Wrong answers:
b) All too often the size of astronomical uncertainty or error.
c) Same as (b).
d) Same as (b).
e) A nonsense answer.

Redaction: Jeffery, 2001jan01

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001 qmult 00520 1 5 1 easy thinking: meter to centimeters

25. Using compact one-number conversion factors is often trickier than just using explicit factors of unity in doing calculations. But if one has to do repeated conversions, it is convenient to have them. Some

are straightforward to find. For example, what is the conversion factor for converting meters **TO** centimeters?

a) 100 cm/m. b) (1/100) cm/m. c) (1/10) cm/m. d) 10 cm/m. e) 10 m/cm.

SUGGESTED ANSWER: (a)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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001 qmult 00540 1 4 1 easy deducto-memory: kilogram to grams
Extra keywords: physci

26. A kilogram is:

a) 1000 grams. b) 1×10^{-3} grams. c) 3.1416 grams. d) 2 grams. e) 0 grams.

SUGGESTED ANSWER: (a)

Wrong answers:
b) kilograms just sound bigger than grams. Even before this course, everyone should know this is wrong.
c) Not π grams.
d) C'mon, metric is a decimal system.
e) A nonsense answer.

Redaction: Jeffery, 2001jan01

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001 qmult 00550 1 4 4 easy deducto-memory: conversion of an inch to cm
Extra keywords: physci

27. One inch is:

a) 1 m. b) 1 km. c) 1 cm. d) 2.54 cm. e) 1 ft.

SUGGESTED ANSWER: (d)

You can deduce the answer.
In modern times, the yard is defined to be exactly 0.9144 m. Thus,

$$1\text{ inch} = \frac{91.44\text{ cm}}{36} = \left(2.5 + \frac{1.44}{36}\right)\text{ cm} = 2.54\text{ cm} \qquad \text{exactly.}$$

Wrong answers:
a) A meter is about a yard=36 inches.
b) A kilometer is about 0.6215 miles and a mile is 1.609 kilometers.
c) Doesn't seem likely given generally offset between SI and British units.
e) Oh c'mon.

Redaction: Jeffery, 2001jan01

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001 qmult 00560 1 4 3 easy deducto-memory: conversion mile to km

28. 1 mile is nearly exactly:

a) 1 m. b) 1 km. c) 1.609 km. d) 2.54 cm. e) 1 ft.

SUGGESTED ANSWER: (c)

You can deduce the answer. How likely is it that 1 mi would nearly exactly 1 km?
Actually, the modern mile is defined to be exactly 1.609344 km (http://en.wikipedia.org/wiki/Mile).

Wrong answers:
a) A meter is about a yard=36 inches.
b) Doesn't seem likely given generally offset between SI and British units.
d) Not too likely

e) Oh c'mon.

Redaction: Jeffery, 2001jan01

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001 qmult 00570 2 4 4 moderate deducto-memory: 10 m/s to mph
Extra keywords: physci

29. A human (a very speedy human) can run 10 m/s. What is this speed in miles per hour (mi/h)? **HINT:** You do not need to do any explicit calculation (although that will work too). Just think about everyday reality. Can you outrun a car? Yes/no/maybe?

a) 100 mi/h. b) 1 mi/h. c) 11.2 km/s. d) 22.37 mi/h. e) 22.37 miles.

SUGGESTED ANSWER: (d)

One should be able to deduce the only reasonable answer. But if one wants to do the calculation, then behold:

$$10\,\mathrm{m/s} \times \left(\frac{1\mathrm{mi}}{1609\,\mathrm{m}}\right) \times \left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right) = 22.37\,\mathrm{mi/h}\ .$$

Fortran-95 Code

```
          print*
          speed=10.*(1./1609.)*(3600./1.)
          print*,'speed in mi/h'
          print*,speed
!                 22.374146
```

Wrong answers:

a) Not even Donovan Bailey. (A long ago Canadian sprint hero. Not the one who used steroids.)
b) Your kid sibling could do this.
c) Wrong units. Also this is the escape speed from the surface of the Earth (Fr-454). Humans arn't constantly just about launching themselves into orbit.
e) Wrong units.

Redaction: Jeffery, 2001jan01

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001 qmult 00580 1 3 4 easy math: seconds in a day
Extra keywords: physci

30. Exactly how many seconds are there in a day? How many seconds are there in a day to order of magnitude? **HINT:** "To order of magnitude" means that the number is rounded off to the nearest power of 10. There is no universal rule about the dividing line between rounding down and rounding up. However, using $\sqrt{10} \approx 3.162$ is a reasonable choice. Thus, the prefix number in normalized scientific notation is rounded down to 1 if it is less than $\sqrt{10}$, rounded up to 10 if it is larger than $\sqrt{10}$, and rounded to make an even power of 10 if it is exactly $\sqrt{10}$. For example, 2.2 rounds off to $10^0 = 1$, 991 rounds off to 10^3, and $\sqrt{10000} = \sqrt{10} \times 10^2$ rounds off to 10^2.

a) 86400 s and 10^4 s. b) 1440 s and 10^4 s. c) 1440 s and 10^5 s. d) 86400 s and 10^5 s.
e) $\pi \times 10^7$ s.

SUGGESTED ANSWER: (d)

Wrong answers:

e) This curiously enough is the number of seconds in a year (Julian or tropical) to better than 0.5 % accuracy. This is a simple coincidence, but its a useful mnemonic.

Redaction: Jeffery, 2001jan01

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001 qmult 00582 1 3 4 easy math: convert hundred million billion miles to km

31. **CONVERT** a hundred million billion miles into kilometers.

a) 1.6×10^2 km. b) 1.6×10^6 km. c) 1.6×10^9 km. d) 1.6×10^{17} km.
e) 1.6×10^{-9} km.

SUGGESTED ANSWER: (d) As Andy Rooney says (or used to say), don't you just hate it when newspapers quote numbers as hundred million billion miles. We all know scientific notation or should nowadays. $10^2 \times 10^6 \times 10^9 = 10^{17}$. A kilometer is about 0.6215 miles and a mile is about 1.609 kilometers.

Wrong answers:

e) Seems unlikely.

Redaction: Jeffery, 2001jan01

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001 qfull 00510 1 3 0 easy math: acre-foot conversion

Extra keywords: an exercise in conversion only: there is no useful purpose

32. American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot.

a) Say 3.00 in of rain fell on a plain of $30.0\,\mathrm{km}^2$. How many acre-feet of water fell? Note 1 square mile equals 640 acres and $1\,\mathrm{mi} = 1.609344\,\mathrm{km}$ exactly by the mile definition. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

b) Now do a perhaps more useful calculation. Find the conversion factor from acre-feet to cubic meters. Note that $1\,\mathrm{in} = 2.54\,\mathrm{cm}$ exactly by the modern inch definition.

SUGGESTED ANSWER:

a) Behold:

$$\begin{aligned} 3.0\,\mathrm{in} \times 30\,\mathrm{km}^2 &= 3.0\,\mathrm{in} \times 30\,\mathrm{km}^2 \times \left(\frac{1\,\mathrm{ft}}{12\,\mathrm{in}}\right) \times \left(\frac{1\,\mathrm{mi}}{1.609344\,\mathrm{km}}\right)^2 \times \left(\frac{640\,\mathrm{acres}}{1\,\mathrm{mi}^2}\right) \\ &= 1853\,\mathrm{acre\,ft}\ . \end{aligned}$$

b) Behold:

$$\begin{aligned} 1\,\mathrm{acre\,ft} &= 1\,\mathrm{acre\,ft} \times \left(\frac{1\,\mathrm{mi}^2}{640\,\mathrm{acre}}\right)\left(\frac{1.609344\,\mathrm{km}}{1\,\mathrm{mi}}\right)^2\left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right)^2\left(\frac{12\,\mathrm{in}}{1\,\mathrm{ft}}\right)\left(\frac{2.540\,\mathrm{cm}}{1\,\mathrm{in}}\right)\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right) \\ &= 1233.5\,\mathrm{m}^3\ . \end{aligned}$$

Thus the conversion factor is

$$1233.5\,\mathrm{m}^3/(\mathrm{acre\,ft})\ .$$

Wikipedia gives $1233.5\,\mathrm{m}^3/(\mathrm{acre\,ft})$, and so Wik and I agree at last.

Fortran-95 Code

```
          print*
          volume=3.d0*30.d0*(1.d0/12.d0)*(1.d0/1.609344d0)**2*(640.d0/1.d0)
          conv=(1.d0/640.d0)*(1.609344d0/1.d0)**2*(1000.d0/1.d0)**2
&
         &     *(12.d0/1.d0)*(2.54d0/1.d0)*(1.d0/100.d0)
          print*,'volume in acre-feet,conv'
          print*,volume,conv
    !  1853.29036100374 1233.4818375475203   ! new, improved way
    !  1853.4622079999996 1233.3674731176392  ! old way
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Redaction: Jeffery, 2001jan01

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001 qfull 00520 1 3 0 easy math: Fermi on lectures and the micro-century

Extra keywords: Fermi, micro-century, microcentury

33. Italian-American physicist Enrico Fermi once noted that a standard 50 minute university lecture was nearly a micro-century. **NOTE:** A Julian year, which is exactly 365.25 days, is exactly 3.15576×10^7 seconds. Actually, a convenient mnemonic is that a Julian year is $\pi \times 10^7\,\mathrm{s}$ which is too small by only 0.5 %. It just a coincidence that the Julian year is almost this number of seconds. More exactly a Julian year is $1.0045096\pi \times 10^7\,\mathrm{s}$.

a) How long is a micro-century in minutes actually?

b) What is the percentage difference between a standard lecture period and a micro-century.

SUGGESTED ANSWER: HRW-9 gives this factoid.

a) Behold:

$$
\begin{aligned}
1\,\text{micro-century} &= 1\,\text{micro-century} \times \left(\frac{10^{-6}\,\text{centuries}}{1\,\text{micro-century}}\right)\left(\frac{100\,\text{Jy}}{1\,\text{century}}\right)\left(\frac{3.15576\times10^{7}\,\text{s}}{1\,\text{Jy}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) \\
&= 52.5960\,\text{minutes}
\end{aligned}
$$

which is indeed nearly a lecture. As said Marilyn Monroe (1926–1962) on the flight of time in *Some Like it Hot*: "It makes a girl think."

b) Behold:

$$\left(\frac{52.596-50}{50}\right)\times 100 = 5.192\,\% = 5\,\% \ .$$

Fortran-95 Code

```
          print*
          xmucen=1.d0
          r1=1.d-6
          r2=100.d0
          r3=3.15576d7
          r4=1.d0/60.d0
          xmin=xmucen*r1*r2*r3*r4
          pi1=acos(-1.d0)
          yearmult=r3/(pi1*1.d7)
          diff=(xmin-50.d0)/50.d0
          print*,'yearmult,xmin,diff'
          print*,yearmult,xmin,diff
!     1.00450960642336        52.5960000000000        5.191999999999993E-002
```

Redaction: Jeffery, 2001jan01

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001 qfull 00930 1 3 0 easy math: light travel time to Sun, amount rate problem

34. The mean Sun-Earth distance, 1.496×10^{13} cm, is a convenient natural unit in astronomy. (All astronomy is CGS, not MKS by the by.) This unit is called the astronomical unit. Given the speed of light is approximately 3.00×10^{8} m/s, what is the light travel time from the Sun in minutes?

SUGGESTED ANSWER:

This problem belong to the general class of amount-rate problems. You are given an amount A to be accumulated and a constant rate of accumulation R. The time t to accumulate the amount satisfies

$$A = Rt \ ,$$

and so the time is given by

$$t = \frac{A}{R} \ .$$

In resource economics such accumulation times are called R/P-radios: R being the resource amount (and not rate) and P being the production rate.

In the present case, the mean Sun-Earth distance r is the amount and the speed of light c is the rate. Thus, we have

$$\frac{r}{c} = \frac{1.5\times10^{13}}{3\times10^{10}} = 0.5\times10^{3}\,\text{s} = 8.3\,\text{min}$$

to about 2-digit accuracy.

Redaction: Jeffery, 2001jan01

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001 qfull 00950 2 3 0 moderate math: leap seconds, amount rate problem

35. The mean solar day is known to increase secularly (i.e., in a long-term way) by about 1.70×10^{-3} seconds per century due mainly to the tidal interaction of the Moon and Sun (Wikipedia: Tidal acceleration; Wikipedia: Leap second: Rationale). Currently, the mean solar day is about 86400.002 seconds long (Wikipedia: Solar time). **HINT:** Both parts (a) and (b) are questions of how long at a given rate does a given amount take to accumulate.

   a) About how often does a leap second need to be introduced in standard time in order to keep standard and solar time synchronized?

   b) If the current rate of increase in the day continues, about how long in years will it take before a leap second is needed daily: i.e., how long until the mean solar day is about 86401.0 seconds?

   SUGGESTED ANSWER:

   a) The trouble is that the modern standard time clock runs fast in comparison to actual astronomical mean solar time. If they start synchronized, the standard clock completes a day in less time than solar time: about 0.002 s. So if the two clocks (standard and astronomical) are synchronized at the start of day 1, then as each day goes by there is an accumulating time difference between them. At some point the standard clock completes a day 1 s before the astronomical mean solar clock.

   The problem is essentially an amount-rate problem. You have an amount $A = 1\,\text{s}$ and a rate $R = 0.002\,\text{s/day}$. The time t to accumulate the amount satisfies $A = Rt$. Thus,

$$t = \frac{A}{R} = \frac{1\,\text{s}}{0.002\,\text{s/day}} = 500 \text{ days} .$$

   This simple calculation illustrates a vital, but simple, skill: calculating time needed from a rate and an amount. Learn it. Actually leap seconds are introduced at midnight on December 31 and June 30 as needed. For more on leap seconds see Wikpedia: Leap second.

   Actually, there is a proposal to abolish leap seconds. It's a bother to keep adjusting for them. They are added every 18 months or so. So we need 2 every 3 years, 20 every 30 years, and 200 every 300 years. But 200 s is just 3.3 minutes. This amount of desychronization is not noticeable for most human purposes. It wouldn't be that 12 pm would being occuring at solar dawn or anything like that. So maybe we should have leap minutes when needed: i.e., every 100 years or so. Much less bother—and remember "don't do today what you can put off till tomorrow since tomorrow may never come".

   b) This is another amount-rage problem—er amount-rate problem. In this case, $A = 1\,\text{s}$ again and $R = 1.7 \times 10^{-5}\,\text{s/year}$. So the time to accumulate the amount is

$$t = \frac{A}{R} = \frac{1\,\text{s}}{1.7 \times 10^{-5}\,\text{s/year}} \approx 6 \times 10^4 \text{ years}$$

   or about 60 millennia. And it was such an effort to get the 2nd millennium over with. The rate of increase in the day will likely not stay constant.

   FINAL NOTE: You may wonder, but probably not, how the discrepancy solar time and standard time grows if one accounts for the fact that that length of the day is growing too. It's turns out to be very tricky to do it exactly.

   Let's say t' is solar time measured in solar seconds and t is standard time measured in standard seconds. Both are clocks and the differential relationship is

$$dt' = R\,dt$$

   where R is a time-dependent scaling factor. So if the standard time clock passes time dt, then the solar time clock passes $R\,dt$. It's not that more or less time in a classical sense has passed for the solar clock: it's just recorded more or fewer ticks: it's running fast or slow. Say we model R by

$$R = R_0 + 2R_1 t ,$$

   where R_0 is the time zero scaling factor and $2R_1$ is the time zero derivative of the scaling factor. The explicit 2 is inserted for a reason. For a finite standard time, one obtains by integration

$$t' = R_0 t + R_1 t^2 .$$

One sees that the explicit 2 vanishes and that simplifies the appearance of the equation.

So one could evaluate t' for any t if one knew the parameters R_0 and R_1. One can obtain the them from two time measurements. These two time measurements give two equations

$$t'_1 = R_0 t_1 + R_1 t_1^2 \qquad \text{and} \qquad t'_2 = R_0 t_2 + R_1 t_2^2$$

which can be solved for the unknown parameters. The solution can be found exactly. But let's say we take t_1 to be so small that the term $R_1 t_1^2$ is negligible. Then the solution for R_0 is just

$$R_0 = \frac{t'_1}{t_1} \ .$$

If t_1 is one day, then

$$R_0 = \frac{t_1 - x}{t_1} \ ,$$

where x is the amount by which solar time lags from standard time which is about 0.002 s.

What of R_1? Well dividing by t_1 and t_2, we get

$$\frac{t'_1}{t_1} = R_0 + R_1 t_1 \qquad \text{and} \qquad \frac{t'_2}{t_1} = R_0 + R_1 t_2$$

and then substracting the latter from the former and dividing by $t_2 - t_1$, we get

$$R_1 = \frac{t'_2/t_2 - t'_1/t_1}{t_2 - t_1} \ .$$

If we could evaluate this from the results we know, it would be great. But what we know is that the increase in the length of the solar day 1.70×10^{-5} s/year. Well by considering four times, two day intervals far apart, we could evaluate the R_1 and verify that it has negligible effect for the length of day which confirms our approximation for evaluating R_0. Then we could use

$$t' = R_0 t + R_1 t^2$$

to evaluate any discrepancy assuming our model was right and find out how long before the second term has an effect on the discrepancy. But I've lost patience with this issue.

Redaction: Jeffery, 2001jan01

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001 qfull 01130 2 3 0 moderate math: Taylor's series

36. Some have seen Taylor's series, some have not. We won't prove it: we'll just write it down. Any sufficiently differentiable function in the neighborhood of a point x_0 can be approximated by a Taylor's series expansion about x_0 for a sufficiently small neighborhood. The general formula is

$$f(x) = \sum_{\ell=0}^{\infty} \frac{(x-x_0)^\ell}{\ell!} f^{(\ell)}(x_0) \ ,$$

where $f^{(\ell)}(x_0)$ is the ℓth derivative of f evaluated at x_0. The terms in the series are labeled by their power of $(x-x_0)$: $(x-x_0)^0$ is 0th (zeroth) order, $(x-x_0)^1$ is 1st order, $(x-x_0)^2$ is 2nd order, and so on. The Taylor's series is convergent (i.e., doesn't explode to infinity) for sufficiently small $(x-x_0)$. The smaller $(x-x_0)$ is, the less important the higher order terms. In physics (and other fields no doubt), one is often interested in the behavior near some important point x_0, and so truncates the Taylor's series to find an simple approximate expression for the neighborhood of x_0. For example

$$f(x) = \begin{cases} f(x_0) \ , & \text{0th order or constant approximation;} \\ f(x_0) + (x-x_0)f'(x_0) \ , & \text{1st order or linear approximation;} \\ f(x_0) + (x-x_0)f'(x_0) + \dfrac{(x-x_0)^2}{2} f''(x_0) \ , & \text{2nd order or quadratic approximation.} \end{cases}$$

The 0th order expansion, approximates the function as a constant which is the function value at x_0; the 1st order expansion approximates the function by a line that is tangent to the function at x_0; the 2nd order expansion approximates the function by a quadratic that is tangent to the function x_0.

a) Draw a general function on the Cartesian plane and at a general point x_0 schematically show how the 0th, 1st, and 2nd order approximations to the function behave.

b) Taylor expand

$$f(x) = \frac{1}{1+x}$$

to 2nd order about $x_0 = 0$. (Here $x - x_0$ is just x, of course.)

c) Taylor expand

$$f(x) = \sqrt{1+x}$$

to 2nd order about $x = 0$.

d) Taylor expand

$$f(x) = \frac{1}{\sqrt{1+x}}$$

to 2nd order about $x = 0$.

e) Taylor expand

$$f(x) = \sin(x)$$

to 3rd order about $x = 0$. Note that x must be in radians for the Taylor's expansion to work for the trigonometric functions. This is because the derivatives of these functions are derived using radians.

f) Taylor expand

$$f(x) = \cos(x)$$

to 2nd order about $x = 0$.

SUGGESTED ANSWER:

a) You'll have to use your imagination.

b) Behold:

$$f(x) = \frac{1}{1+x} = 1 - x + x^2 + \ldots \ .$$

Actually the complete expansion can be done in this case:

$$f(x) = \frac{1}{1+x} = \sum_{\ell=0}^{\infty} (-1)^\ell x^\ell \ .$$

This expansion is convergent for $|x| < 1$. Note the usual power series is a Taylors expansion of $1/(1-x)$: i.e.,

$$\frac{1}{1-x} = \sum_{\ell=0}^{\infty} x^\ell$$

(Ar-238).

c) Behold:

$$f(x) = \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots \ .$$

d) Behold:

$$f(x) = \frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \ldots \ .$$

e) Behold:

$$f(x) = \sin(x) = x - \frac{1}{6}x^3 + \ldots \ .$$

f) Behold:

$$f(x) = \cos(x) = 1 - \frac{1}{2}x^2 + \ldots \ .$$

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

---

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

---

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\ , \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \text{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\rm rel} = x_2 - x_1 \qquad v_{\rm rel} = v_2 - v_1 \qquad a_{\rm rel} = a_2 - a_1$$

$$x' = x - v_{\rm frame}t \qquad v' = v - v_{\rm frame} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\rm avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\rm avg} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{{\rm for}\ y\ {\rm max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\rm max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\rm for\ max} = \frac{\pi}{4} \qquad x_{\rm max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

---

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho Av^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

---

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1 m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times 10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\max}\sin[k(x \mp vt)] = y_{\max}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32°\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$