Introductory Physics: Calculus-Based NAME:

Homework 1: Introduction to Physics, Units, Conversions: Homeworks are due as posted on the course web site. Multiple-choice questions will **NOT** be marked, but some of them will appear on exams. One or more full-answer questions may be marked as time allows for the grader. Hand-in the full-answer questions on other sheets of paper: i.e., not crammed onto the downloaded question sheets. Make the full-answer solutions sufficiently detailed that the grader can follow your reasoning, but you do **NOT** be verbose. Solutions will be posted eventually after the due date. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

1. Have you written your name in the name place at the upper right of this paper?

   a) Yes. b) No. c) Nope. d) Nein. e) Never.

2. "Let's play *Jeopardy*! For $100, the answer is: In shorthand, it is the science of matter and motion, space and time."

   What is ____________, Alex?

   a) biology b) phrenology c) economics d) rocket science e) physics

3. The ultimate goal of physics—or at least one of the ultimate goals—is a theory of:

   a) something (TOS). b) nothing (TON). c) everything (TOE). d) woe (TOW).
   e) Fred (TOF).

4. "Let's play *Jeopardy*! For $100, the answer is: The most obvious choice for the first natural philosophers or physicists. They attempted to explain the natural world from simple general principles: from these principles complex structures could arise. In mythology on the other hand, usually deities with human-like personalities are invoked to explain structures. But there is probably no hard line between natural philosophy and mythology: one shades into the other in the early days."

   Who are ____________, Alex?

   a) Presocratic philosophers of ancient Greece b) Babylonian diviners c) Roman augurs
   d) Mongolian shamans e) Irish druids

5. Mathematical physics in a sense began with the ancient Babylonian astronomers. Later the ancient Greeks added a bit to mathematical physics, particularly Archimedes (circa 287–212 BCE) with his work on statics and hydrostatics. But idea of physics as a largely mathematical science is largely attributable to ____________, ____________, and their contemporaries in the 17th century.

   a) William Shakespeare (1564–1616); John Milton (1608–1674)
   b) Galileo (1564–1642); Isaac Newton (1643–1727)
   c) Bernini (1598–1680); Borromini (1599–1667)
   d) Caravaggio (1571–1610); Poussin (1594–1665)
   e) Wallenstein (1583–1634); Turenne (1611-1675)

6. Because there are no doubts about its validity in the classical limit (scales larger than microscopic, but smaller than cosmic, speeds much less than that of light, gravity much less strong than near a black hole) and because it is the exact limit of more general theories, Newtonian physics can be regarded as a/an:

   a) utterly false theory. b) negligible theory c) useless theory.
   d) true exact emergent theory. e) phantasm

7. In physics jargon, the word "clearly" means:

   a) clearly. b) unclearly. c) after 4 pages of algebra. d) wrongly. e) all of the above.

8. In physics jargon, the phrase "must be" means:

   a) is. b) just accept it that. c) not necessarily so. d) can't be. e) all of the above.

9. "Let's play *Jeopardy*! For $100, the answer is: In scientific theorizing and model building, it is the elimination/neglect of complicating secondary factors in order to understand the main factors. In reality, the secondary factors may or may not be eliminated/neglected to one degree or another. If they

cannot be eliminated/neglected, they have to be modeled in order to test the understanding of the main factors."

What is ____________, Alex?

a) Platonic idealization b) scientific idealization c) the scientific method d) experiment e) observation

10. "Let's play *Jeopardy*! For \$100, the answer is: It is a notation in which one expresses a number by a coefficient decimal number multiplied explicitly by 10 to the appropriate power. If the coefficient is in the range 1 to 10, but not including 10, the notation is called normalized."

    What is ____________, Alex?

    a) British notation b) scientific notation c) metric notation d) tensy notation e) Irish notation

11. Write a hundred million billion miles in scientific notation.

    a) 10^2 mi. b) 10^6 mi. c) 10^9 mi. d) 10^{17} mi. e) 10^{-9} mi.

12. Express 4011 and 0.052 in normalized scientific notation form.

    a) 4.011×10^3 and 5.2×10^{-2}. b) 40.11×10^3 and $52. \times 10^{-2}$. c) 40.11×10^2 and $52. \times 10^{-3}$. d) 4.011×10^{-2} and 5.2×10^3. e) 4011 and 0.052.

13. "Let's play *Jeopardy*! For \$100, the answer is: In any measurements of quantities, they are conventionally agreed upon standard things."

    What are ____________, Alex?

    a) unities b) dualities c) duplicities d) quantons e) units

14. The modern standard set of units for science, most engineering, and much of everyday life (except in the 2nd largest country in North America) is the International System of Units (Système International d'Unités or SI) which is often called the:

    a) English units. b) Mesopotamian system. c) metric system. d) Paraguayan system. e) United States customary units.

15. Why is the US Customary Units system (loosely called English Units) hard to use in calculations in comparison with the metric system?

    a) The US Customary Units system has no unit of mass. Consequently, when you need to use mass, you have to mentally work around the lack of a unit.
    b) The US Customary Units system is completely unknown by scientists and engineers, and so, of course, can scarcely be used.
    c) The US Customary Units system has units in a host of irregular sizes: 16 ounces to the pound, 8 pints in a gallon, 12 inches to the foot, and 45 inches to the ell (fact!). This makes calculations and conversions tortuous. In particular, it is awkward that cubic units of length (e.g., cubic inches, feet, etc.) are not simply related to standard volume units (e.g., a U.S. gallon is defined as 231 cubic inches). In everyday life the irregularities of the US Customary Units system don't cause much of a problem and maybe even have mnemonic value. But that hardly helps people who want to send probes to Mars.
    d) The complete regularity of the US Customary Units system means that you never know if a number is 10, 100, 1000, etc., except by context.
    e) It's the revenge for Bunker Hill.

16. Three quantities usually adopted as basic (i.e., not reducible to other kinds of quantities by convention) are:

    a) length, area, volume. b) mass, weight, heft. c) length, mass, time. d) time, duration, age. e) length, mass, density.

17. The 7 base units are meter, kilogram (gram would have been more logical), second, ampere,

    a) kelvin, mole, and candela. b) calvin, mold, and candeleria. c) kelvin, mouse, and candace. d) melvin, moose, and cantrip.

e) kludge, moor, and mountain.

18. MKS stands for:

    a) meters, kilometers, centimeters. b) meters, kilometers, seconds.
    c) meters, kilograms, seconds. d) millimeters, kilometers, seconds.
    e) millimeters, kilograms, seconds.

19. If one used **ONLY** MKS units (i.e., units of meters, kilograms, seconds, coulombs, amperes, kelvins, moles, etc., plus MKS units derived from these) in calculations, then one will get answers in:

    a) CGS units only. b) MKS or CGS units. c) MKS units only. d) English units only.
    e) any old units.

20. In scientific calculations, it is best to stick to one complete, consistent set of units (MKS or CGS):

    a) always. b) half the time. c) never. d) whenever. e) except when its not convenient. In most specialized fields, there are natural units that are convenient in actual calculations, but overwhelmingly most often only in human readable outputs. Natural units are most useful for understanding not calculation. Astronomers, for example often use the solar mass (1.9891×10^{30} kg) as a unit for the masses of stars since the Sun for Earthlings is basic standard of reference for stars.

21. In SI, the prefixes kilo and centi indicate, respectively, multiplication by:

    a) 1000 and 0.01. b) 0.01 and 1000. c) 1000 and 100. d) 60 and 0.01. e) π and e.

22. The metric or SI unit prefix symbols M- and m- stand for:

    a) mega (factor of 10^6) and milli (factor of 10^{-6}). b) mega (factor of 10^6) and milli (factor of 10^{-3}). c) kilo (factor of 10^6) and milli (factor of 10^{-3}). d) kilo (factor of 10^6) and milli (factor of 10^{-6}). e) merger (factor of 10^9) and melba (factor of 10^{-6}).

23. In conversions, one can just treat units as variables whose values are never specified. One can do algebra with them and cancel them. One also knows a set of equalities relating units, and so can write down factors of unity or conversion factors. For example, $1000\,\mathrm{m} = 1\,\mathrm{km}$, and so a factor of unity (or conversion factor) is

    $$1 = \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} \ .$$

    This factor would be used to convert an amount in kilometers to meters by:

    a) multiplication. b) division. c) addition. d) subtraction. e) squaring.

24. A units conversion factor is sometimes called a factor of:

    a) unity. b) 2. c) a few. d) 10. e) fact.

25. Using compact one-number conversion factors is often trickier than just using explicit factors of unity in doing calculations. But if one has to do repeated conversions, it is convenient to have them. Some are straightforward to find. For example, what is the conversion factor for converting meters **TO** centimeters?

    a) 100 cm/m. b) (1/100) cm/m. c) (1/10) cm/m. d) 10 cm/m. e) 10 m/cm.

26. A kilogram is:

    a) 1000 grams. b) 1×10^{-3} grams. c) 3.1416 grams. d) 2 grams. e) 0 grams.

27. One inch is:

    a) 1 m. b) 1 km. c) 1 cm. d) 2.54 cm. e) 1 ft.

28. 1 mile is nearly exactly:

    a) 1 m. b) 1 km. c) 1.609 km. d) 2.54 cm. e) 1 ft.

29. A human (a very speedy human) can run 10 m/s. What is this speed in miles per hour (mi/h)? **HINT:** You do not need to do any explicit calculation (although that will work too). Just think about everyday reality. Can you outrun a car? Yes/no/maybe?

a) 100 mi/h. b) 1 mi/h. c) 11.2 km/s. d) 22.37 mi/h. e) 22.37 miles.

30. Exactly how many seconds are there in a day? How many seconds are there in a day to order of magnitude? **HINT:** "To order of magnitude" means that the number is rounded off to the nearest power of 10. There is no universal rule about the dividing line between rounding down and rounding up. However, using $\sqrt{10} \approx 3.162$ is a reasonable choice. Thus, the prefix number in normalized scientific notation is rounded down to 1 if it is less than $\sqrt{10}$, rounded up to 10 if it is larger than $\sqrt{10}$, and rounded to make an even power of 10 if it is exactly $\sqrt{10}$. For example, 2.2 rounds off to $10^0 = 1$, 991 rounds off to 10^3, and $\sqrt{10000} = \sqrt{10} \times 10^2$ rounds off to 10^2.

    a) 86400 s and 10^4 s. b) 1440 s and 10^4 s. c) 1440 s and 10^5 s. d) 86400 s and 10^5 s.
    e) $\pi \times 10^7$ s.

31. **CONVERT** a hundred million billion miles into kilometers.

    a) 1.6×10^2 km. b) 1.6×10^6 km. c) 1.6×10^9 km. d) 1.6×10^{17} km.
    e) 1.6×10^{-9} km.

32. American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot.

    a) Say 3.00 in of rain fell on a plain of 30.0 km^2. How many acre-feet of water fell? Note 1 square mile equals 640 acres and 1 mi = 1.609344 km exactly by the mile definition. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

    b) Now do a perhaps more useful calculation. Find the conversion factor from acre-feet to cubic meters. Note that 1 in = 2.54 cm exactly by the modern inch definition.

33. Italian-American physicist Enrico Fermi once noted that a standard 50 minute university lecture was nearly a micro-century. **NOTE:** A Julian year, which is exactly 365.25 days, is exactly 3.15576×10^7 seconds. Actually, a convenient mnemonic is that a Julian year is $\pi \times 10^7$ s which is too small by only 0.5 %. It just a coincidence that the Julian year is almost this number of seconds. More exactly a Julian year is $1.0045096\pi \times 10^7$ s.

    a) How long is a micro-century in minutes actually?

    b) What is the percentage difference between a standard lecture period and a micro-century.

34. The mean Sun-Earth distance, 1.496×10^{13} cm, is a convenient natural unit in astronomy. (All astronomy is CGS, not MKS by the by.) This unit is called the astronomical unit. Given the speed of light is approximately 3.00×10^8 m/s, what is the light travel time from the Sun in minutes?

35. The mean solar day is known to increase secularly (i.e., in a long-term way) by about 1.70×10^{-3} seconds per century due mainly to the tidal interaction of the Moon and Sun (Wikipedia: Tidal acceleration; Wikipedia: Leap second: Rationale). Currently, the mean solar day is about 86400.002 seconds long (Wikipedia: Solar time). **HINT:** Both parts (a) and (b) are questions of how long at a given rate does a given amount take to accumulate.

    a) About how often does a leap second need to be introduced in standard time in order to keep standard and solar time synchronized?

    b) If the current rate of increase in the day continues, about how long in years will it take before a leap second is needed daily: i.e., how long until the mean solar day is about 86401.0 seconds?

36. Some have seen Taylor's series, some have not. We won't prove it: we'll just write it down. Any sufficiently differentiable function in the neighborhood of a point x_0 can be approximated by a Taylor's series expansion about x_0 for a sufficiently small neighborhood. The general formula is

$$f(x) = \sum_{\ell=0}^{\infty} \frac{(x-x_0)^\ell}{\ell!} f^{(\ell)}(x_0) ,$$

where $f^{(\ell)}(x_0)$ is the ℓth derivative of f evaluated at x_0. The terms in the series are labeled by their power of $(x-x_0)$: $(x-x_0)^0$ is 0th (zeroth) order, $(x-x_0)^1$ is 1st order, $(x-x_0)^2$ is 2nd order, and so on. The Taylor's series is convergent (i.e., doesn't explode to infinity) for sufficiently small $(x-x_0)$. The smaller $(x-x_0)$ is, the less important the higher order terms. In physics (and other fields no doubt), one is often interested in the behavior near some important point x_0, and so truncates the Taylor's series to find an simple approximate expression for the neighborhood of x_0. For example

$$f(x) = \begin{cases} f(x_0)\ , & \text{0th order or constant approximation;} \\ f(x_0) + (x - x_0)f'(x_0)\ , & \text{1st order or linear approximation;} \\ f(x_0) + (x - x_0)f'(x_0) + \dfrac{(x - x_0)^2}{2} f''(x_0)\ , & \text{2nd order or quadratic approximation.} \end{cases}$$

The 0th order expansion, approximates the function as a constant which is the function value at x_0; the 1st order expansion approximates the function by a line that is tangent to the function at x_0; the 2nd order expansion approximates the function by a quadratic that is tangent to the function x_0.

a) Draw a general function on the Cartesian plane and at a general point x_0 schematically show how the 0th, 1st, and 2nd order approximations to the function behave.

b) Taylor expand

$$f(x) = \frac{1}{1+x}$$

to 2nd order about $x_0 = 0$. (Here $x - x_0$ is just x, of course.)

c) Taylor expand

$$f(x) = \sqrt{1+x}$$

to 2nd order about $x = 0$.

d) Taylor expand

$$f(x) = \frac{1}{\sqrt{1+x}}$$

to 2nd order about $x = 0$.

e) Taylor expand

$$f(x) = \sin(x)$$

to 3rd order about $x = 0$. Note that x must be in radians for the Taylor's expansion to work for the trigonometric functions. This is because the derivatives of these functions are derived using radians.

f) Taylor expand

$$f(x) = \cos(x)$$

to 2nd order about $x = 0$.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta)\qquad \cos(\theta+90^\circ)=-\sin(\theta)\qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta)\qquad \cos(180^\circ-\theta)=-\cos(\theta)\qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta)\qquad \cos(90^\circ-\theta)=\sin(\theta)\qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)\qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a)\qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]\qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)]\qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)]\qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

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4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx}\qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta\qquad \tan\theta\approx\theta\qquad \cos\theta\approx 1-\frac{1}{2}\theta^2\qquad \text{all for } \theta<<1$$

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5 Quadratic Formula

$$\text{If}\qquad 0=ax^2+bx+c\ ,\qquad \text{then}\qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\frac{b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = p x^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\rm cm} = \frac{\sum_i m_i \vec{r}_i}{m_{\rm total}} = \frac{\sum_{\rm sub} m_{\rm sub} \vec{r}_{\rm cm\ sub}}{m_{\rm total}} \qquad \vec{v}_{\rm cm} = \frac{\sum_i m_i \vec{v}_i}{m_{\rm total}} \qquad \vec{a}_{\rm cm} = \frac{\sum_i m_i \vec{a}_i}{m_{\rm total}}$$

$$\vec{r}_{\rm cm} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\rm total}}$$

$$\vec{F}_{\rm net} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,{\rm m/s^2}$$

$$\vec{F}_{\rm normal} = -\vec{F}_{\rm applied} \qquad F_{\rm linear} = -kx$$

$$f_{\rm normal} = \frac{T}{r} \qquad T = T_0 - F_{\rm parallel}(s) \qquad T = T_0$$

$$F_{\rm f\ static} = \min(F_{\rm applied}, F_{\rm f\ static\ max}) \qquad F_{\rm f\ static\ max} = \mu_{\rm static} F_{\rm N} \qquad F_{\rm f\ kinetic} = \mu_{\rm kinetic} F_{\rm N}$$

$$v_{\rm tangential} = r\omega = r\frac{d\theta}{dt} \qquad a_{\rm tangential} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\rm centripetal} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\rm centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\rm drag,lin} = bv \qquad v_{\rm T} = \frac{mg}{b} \qquad \tau = \frac{v_{\rm T}}{g} = \frac{m}{b} \qquad v = v_{\rm T}(1 - e^{-t/\tau})$$

$$F_{\rm drag,quad} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\rm T} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\rm mechanical} = KE + PE$$

$$P_{\rm avg} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\rm net} \quad \Delta PE_{\rm of\ a\ conservative\ force} = -W_{\rm by\ a\ conservative\ force} \quad \Delta E = W_{\rm nonconservative}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \qquad \tau_z = RF_{xy} \sin \gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

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20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\text{max}}\sin[k(x \mp vt)] = y_{\text{max}}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

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24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K} \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^\circ\mathrm{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}KE_{\mathrm{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\mathrm{RMS}}^2\right)$$

$$v_{\mathrm{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\mathrm{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\mathrm{H}}} = \frac{Q_{\mathrm{H}} - Q_{\mathrm{C}}}{W} = 1 - \frac{Q_{\mathrm{C}}}{Q_{\mathrm{H}}} \qquad \eta_{\mathrm{heating}} = \frac{Q_{\mathrm{H}}}{W} = \frac{Q_{\mathrm{H}}}{Q_{\mathrm{H}} - Q_{\mathrm{C}}} = \frac{1}{1 - Q_{\mathrm{C}}/Q_{\mathrm{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\mathrm{cooling}} = \frac{Q_{\mathrm{C}}}{W} = \frac{Q_{\mathrm{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\mathrm{heating}} - 1$$

$$\varepsilon_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}} \qquad \eta_{\mathrm{heating,Carnot}} = \frac{1}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}} \qquad \eta_{\mathrm{cooling,Carnot}} = \frac{T_{\mathrm{C}}/T_{\mathrm{H}}}{1 - T_{\mathrm{C}}/T_{\mathrm{H}}}$$