Introductory Physics II: Calculus-Based: Final Exam Name:

Instructions: There are 20 multiple-choice problems worth 2 marks each for a total of 40 marks altogether. Choose the **BEST** answer, completion, etc. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized. Harder/longer multiple-choice problems (which are sometimes based on full-answer problems) are marked by asterisks: $*$ for easy, $**$ for moderate, $***$ for hard: all in the judgment of the instructor.

There are **SIX** full-answer problems worth 10 marks each for a total of 60 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may extend over multiple-pages: make sure you have answered everything. And **BOX-IN** your final answers. There are 2 scratch pages for additional calculations or to rewrite answers if needed. In the latter case, make clear which answer is to be marked. You can ask for more paper if needed.

This is a **CLOSED-BOOK: CLOSED-NOTES: NO-ONLINE-ACCESS: NO-AI** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 100 marks altogether and is a 100-minute exam.

Multiple Choice Questions:

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018 qmult 00100 1 4 2 easy deducto-memory: sound waves defined

1. "Let's play *Jeopardy*! For \$100, the answer is: They are pressure waves in a medium that can be any of gas, liquid, and solid. In gases and fluids, the waves can only be longitudinal. In solids, tranverse waves can occur."

   What are ____________, Alex?

   a) gravitational waves b) sound waves c) water waves
   d) seismic waves e) quantum mechanical waves

   SUGGESTED ANSWER: (b)

   Wrong answers:
   a) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2008jan01

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018 qmult 00210 1 4 1 moderate deducto-memory: sound speed formula for linear waves

2. The speed of sound formula for linear waves is:

   a) $v = \sqrt{(\partial P/\partial\rho)_S}$, where the derivative is with constant entropy (i.e., the adiabatic condition).
   b) $v = \sqrt{T/\mu}$, where T is tension and μ is linear mass density
   c) $v = \sqrt{(\partial P/\partial\rho)_T}$, where the derivative is with constant temperature (i.e., the isothermal condition).

d) $v = \sqrt{T/\mu}$, where μ is tension and T is linear mass density
e) $v = \sqrt{(\partial v/\partial \rho)_S}$, where the derivative is with constant entropy (i.e., the adiabatic condition).

SUGGESTED ANSWER: (a) Students do have to remember this particular fact and the adiabatic condition is the right one. Of course, this result applies only to linear waves and the adiabatic condition, albeit the usual one, may not apply in all cases.

Wrong answers:
b) This for wave speed on a string.
c) Nah. Using the isothermal condition was Newton's famous blunder. He "corrected" his result with a fudge factor.
e) This is a phoney non-linear partial differential equation for v.

Redaction: Jeffery, 2001jan01

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017 qmult 00150 1 1 4 easy memory: general traveling wave solution
3. The linear wave equation for 1-dimensional wave motion

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

has the semi-general solution $y = f(x - v_\pm t)$, where $v_\pm = \pm v$ is the phase velocity and v is the phase speed. (It is only semi-general since a general solution requires a linear combination of the v_+ and v_- cases.) The solution is a traveling wave solution. Let $\phi = x - v_\pm t$. For a constant ϕ, one has a constant oscillation deviation $f(\phi)$. But this deviation's x position (i.e., given by x equals something) changes as ____________ with velocity ____________. Since ϕ is called the phase of the wave solution, the name for phase velocity $v_\pm$ follows. A wave solution of the form $f(x-v_\pm t)$ is just a pattern that is traveling to the positive for v_+ and to the negative for v_-. If $v^2 = 0$, the wave equation is not defined and there is no solution. But there can be solutions that do not travel.

a) $\phi - v_\pm/t$; $v_\pm$. b) $\phi + v_\pm/t$; $-v_\pm$. c) $\phi v_\pm t$; $-v_\pm$. d) $\phi + v_\pm t$; $v_\pm$.
e) v; $\phi - v_\pm t$.

SUGGESTED ANSWER: (d)

Since $\phi = x - v_\pm t$ is a constant, we find the location of the deviation $f(\phi)$ evolves according to $x = \phi + v_\pm t$: i.e., the location is a linear function of time.

Wrong answers:
a) As Lurch would say AAAAaarrrgh.

Redaction: Jeffery, 2008jan01

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022 qmult 00200 1 4 5 easy deducto-memory: ordinary matter neutral
Extra keywords: physci
4. "Let's play *Jeopardy*! For $100, the answer is: It is usually approximately electrically neutral."

What is ____________, Alex?

a) an electron b) a proton c) highly charged matter
d) highly negatively charged matter
e) ordinary, terrestrial matter above the atomic scale

SUGGESTED ANSWER: (e)

Wrong answers:
a) C'mon.

Redaction: Jeffery, 2001jan01

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023 qmult 00202 1 1 3 easy memory: electric force and a charge distribution

5. Say that you have a charge distribution of classical point charges with net charge q in an external electric field $\vec{E}$. Since it is external, $\vec{E}$ does **NOT** include contributions from the charge distribution itself. Also $\vec{E}$ is **NOT** necessarily uniform. The electric force on the distribution due to $\vec{E}$ is

$$\vec{F} = \sum_i q_i \vec{E}_i \ ,$$

where q_i is the charge of one member of the distribution and $\vec{E}_i$ is the external electric field at the location of charge q_i. The force $\vec{F}$ is ____________ on the distribution and in the absence of any other net external force dictates ____________ of the distribution.

a) the net electric force; individual charge accelerations
b) not the net electric force; center-of-mass acceleration
c) the net electric force; center-of-mass acceleration
d) not the net electric force; individual charge accelerations
e) half the net electric force; individual charge accelerations

SUGGESTED ANSWER: (c)

The internal electrical forces cancel out pairwise and so the external electric field on the distribution is the net electrical field.

Do all the internal forces cancel out pairwise. By Newton's 3rd law they should and we assume that law is obeyed in many cases. But actually Newton's 3rd law is **NOT** always obeyed between matter particles. This is a fine point that is often skipped over in intro physics classes. Magnetic forces can violate it for swarm of free-flying charges for example (Go-8). But nonetheless once one includes the mass and momentum of the electromagnetic field in the total mass and momentum of the system, it still turns out to be true that $\vec{F}_{\text{net external force}} = ma_{\text{CM}}$—or at least so I believe.

Now what about real point charges? What of their internal forces? Well maybe they don't really exist. But the electron is a point charge as far as we know. So what about the electrons internal electric forces? Are they infinite? There's a special place on the far side of the River Styx for people who ask such questions.

Wrong answers:
e) Half?

Redaction: Jeffery, 2008jan01

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024 qmult 00800 1 1 3 easy memory: Gauss law surface of conductor

6. Gauss's law applied to **JUST** outside of an electrostatic conductor gives the magnitude of electric field to be:

   a) $E = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\rm encl}}{2\pi\varepsilon_0 r}$, where $\lambda_{\rm encl}$ is linear charge density.
   c) $E = \dfrac{\sigma_{\rm encl}}{\varepsilon_0}$, where $\sigma_{\rm encl}$ is surface area charge density. d) $E = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r}$.
   e) $V = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r}$.

   SUGGESTED ANSWER: (c)

   Wrong answers:
   a) For a point charge.
   b) For a line charge.
   e) Potential for a point charge.

   Redaction: Jeffery, 2001jan01

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025 qmult 00150 1 1 2 easy memory: electric potential scalar addition

7. Since electric potential is a scalar it adds:

   a) like a vector. b) like a scalar. c) like a tensor. d) to zero always.
   e) not at all.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   a) Oh c'mon.

   Redaction: Jeffery, 2001jan01

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025 qmult 00700 1 1 5 easy memory: conductor equipotential

8. A conductor in an electrostatic case is a/an ____________ object.

   a) non-equipotential b) pseudo-equipotential c) faux-equipotential
   d) equiporpoise e) equipotential

   SUGGESTED ANSWER: (e)

   Ideal wires are equipotentials even though current is flowing, and thus the situation is not electrostatic. Of course, real wires have some potential drop across them when current flows.

   Wrong answers:
   d) Just seeing if you are awake.

   Redaction: Jeffery, 2001jan01

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026 qmult 00600 1 1 3 easy memory: planar capacitor

9. The simplest of all capacitor capacitance formulae to remember is that for an ideal planar capacitor with vacuum between the plates. (Ideal means that it can be treated

as a segment of an infinite planar capacitor.) The formula involves only A (the area of the plates), d the separation of the plates, and the vacuum permittivity ε_0 (which has units of F/m). By dimensional analysis (i.e., analysis of the units), the formula must be:

a) $C = A + d + \varepsilon_0$. b) $C = \dfrac{\varepsilon_0 d}{A}$. c) $C = \dfrac{\varepsilon_0 A}{d}$. d) $C = \dfrac{A}{d\varepsilon_0}$.
e) $C = \dfrac{(A-d)}{\varepsilon_0}$.

SUGGESTED ANSWER: (c)

Wrong answers:
a) Adding apples and oranges.

Redaction: Jeffery, 2001jan01

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028 qmult 00120 1 4 5 easy deducto-memory: Kirchhoff's voltage law 1

10. The potential change around any closed path is zero, and thus in the special case of an electrical circuit loop, the potential changes summed over all elements in the circuit loop must be:

a) infinite: this is Mozart's 3rd law. b) positive: this is Krankheit's 1st law.
c) negative: this is Wilson's 14th point.
d) indefinite: this is Hemingway's conjecture.
e) zero: this is Kirchhoff's voltage law.

SUGGESTED ANSWER: (e)

Wrong answers:
c) I think it was Clemenceau who said: "Dieu avait dix seul."

Redaction: Jeffery, 2001jan01

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028 qmult 00450 1 1 1 easy memory: electric eel
Extra keywords: HRW-646

11. You are swimming with your pet electric eel Fred—who may not be a terribly realistic electrophorus electricus—and playfully grab his tail and just below his head: the potential difference between these regions is $\sim 750\,\mathrm{V}$. Wet, the typical resistance between human body parts is of order $1000\,\Omega$ and let's assume contrafactually that Fred's internal resistance is negligible. Human resistance is much higher if you are dry: of order $10^5\,\Omega$. Given that death is pretty certain if you carry a current of $0.2\,\mathrm{A}$, what are your chances of living?

a) Rather bad. b) So-so. c) Great. d) You have never been safer.
e) You are under 25, you are immortal.

SUGGESTED ANSWER: (a)

With the given numbers—which may be wildly inaccurate—the current through you would be

$$I = \frac{V}{R} = \frac{750}{1000} = 0.75\,\mathrm{A}$$

and you are in trouble.

I don't know a lot about electrical safety, the numbers are what introductory physics books suggest. I know even less about electric eels. Wikipedia suggests that for adult humans the shock only could be harmful:

`http://en.wikipedia.org/wiki/Electric_eel .`

See also `https://commons.wikimedia.org/wiki/File:IEC_TS_60479-1_electric_shock_graph.svg .`

Fear Factor suggests it's even hilariously fun manhandling electric eels—"It didn't stop me from goin' back for more and more.":

`http://www.nbc.com/nbc/Fear_Factor/tales/319_electric_eel.shtml .`

But the women participants—they had true grit—only used one hand which suggests that they were minimizing the potential difference they were allowing across their bodies.

Wrong answers:

e) There is a strong prejudice in favor of this theory, but experience has never verified it.

Redaction: Jeffery, 2001jan01

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030 qmult 00120 1 1 1 easy memory: Biot-Savart law

12. The Biot-Savart law is:

a) $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{s}\times\hat{r}}{r^2}$. b) $\vec{E}(\vec{r}\,) = \dfrac{kq}{r^2}\hat{r}$. c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\rm linked}$.
d) $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\rm enclosed}}{\varepsilon_0}$. e) $E = mc^2$.

SUGGESTED ANSWER: (a) Google AI question: Can one say current linked by an Amperian loop? Answer: "Yes, one can say a current is "linked" by an Amperian loop, meaning the path of the current passes through any open surface bounded by the closed loop."

Wrong answers:

b) Coulomb's law.
c) Ampère's law.
d) Gauss's law.
e) Einstein's equation.

Redaction: Jeffery, 2001jan01

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031 qmult 00150 1 1 4 easy memory: magnetic flux defined

Extra keywords: physci

13. Magnetic flux is defined by the differential formula:

a) $d\vec{\Phi}_B = \vec{E}\times d\vec{A}$. b) $\oint \vec{E}\cdot d\vec{s} = -\dfrac{d}{dt}\int B\cdot d\vec{A}$. c) $d\Phi_B = \vec{E}\cdot d\vec{A}$.

d) $d\Phi_B = \vec{B} \cdot d\vec{A}$. e) $d\vec{\Phi}_B = \vec{B} \times d\vec{A}$.

SUGGESTED ANSWER: (d)

The MKS unit of magnetic flux is a hoary old EM unit, the weber or $\text{Wb} = \text{T}\cdot\text{m}^2$ It is actually a pretty big unit of flux. It is named for Wilhelm Eduard Weber (1804–1891), a German and so pronounced veber by the cognoscenti. Frankly, I've little use for the weber. I just use the MKS unit of magnetic flux. But maybe there are fields where it's webers this and webers that and "Chuck him out, the brute!"

Wrong answers:
b) This is Faraday's law of induction itself.
c) This is electric flux.

Redaction: Jeffery, 2001jan01

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031 qmult 00300 1 1 3 easy memory: univeral flux rule

14. The universal flux rule (or just plain flux rule if its meaning is understood) relative to an inertial frame is

$$\mathcal{E} = \oint [\vec{E} + (\vec{v} \times \vec{B})] \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A} ,$$

where $\mathcal{E}$ is the induced emf around a closed path formed by a thin conducting loop (exactly only for an ideal infinitely thin conducting loop), $\vec{E}$ is the induced electric field due to Faraday's law (of induction), $\vec{v}$ is the velocity of the loop in the observer's inertial frame (note: the velocity of the charge carriers relative to the loop path is not included), $\vec{B}$ is the magnetic field, Φ_B is the magnetic flux through the surface area linked by the loop, the integral for Φ_B includes the changes in loop shape and position and changes in the magnetic field at each fixed point in space relative to the inertial frame (which is the source of the induced electric field due to Faraday's law), and the sense of the differential area vectors $d\vec{A}$ is given by a right-hand rule: curl your right-hand fingers in the fiducial direction of loop path integration and your thumb gives the sense. The universal flux rule is not a fundamental law of classical electromagnetism: it is derived from Faraday's law (which is a fundamental law of classical electromagnetism), the assumption of an infinitely thin conducting loop, and the:

a) drag force. b) electric force. c) Lorentz force. d) normal force.
e) pressure force.

SUGGESTED ANSWER: (c)

Wrong answers:
a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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032 qmult 00100 1 1 3 easy memory: inductance and Faraday's law

15. The quantity ____________ given the symbol L or M is a coefficient that appears in the formula relating emf to the derivative of the current in a system. The formula

itself is

$$\mathcal{E} = -L\frac{dI}{dt} \qquad \text{or} \qquad \mathcal{E} = -M\frac{dI}{dt} \ .$$

The formula is derived from Faraday's law of induction, and so the emf is only due to an induced electric field with no contribution from a motional emf at least not in what one ordinarily means by ____________.

a) resistance b) capacitance c) inductance d) emf e) reluctance

SUGGESTED ANSWER: (c)

Wrong answers:
e) A nonsense answer—well maybe not.

Redaction: Jeffery, 2008jan01

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033 qmult 00100 1 1 3 easy memory: AC definition 1

16. Alternating current or AC means that current and potential vary about a mean value (usually zero) and usually (but not always)

a) trapezoidally. b) parabolically. c) sinusoidally. d) geometrically. e) wretchedly.

SUGGESTED ANSWER: (c)

Wrong answers:
e) A nonsense answer.

Redaction: Jeffery, 2008jan01

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033 qmult 00230 1 1 2 easy memory: phase angle

17. For a circuit branch (which could be a device or circuit system), you have potential drop across and current through given by, respectively,

$$V = V_0\cos(\omega t) \qquad \text{and} \qquad I = I_0\cos(\omega t - \phi)$$

The non-complex number impedance (or modulus of the complex number impedance) is ____________ and the phase angle is ____________. Formally, the current lags the potential by ϕ, but if $\phi = 0$, the two are in phase angle and if ϕ is negative, then the current leads the potential.

a) $\dfrac{I_0}{V_0}$; ϕ b) $\dfrac{V_0}{I_0}$; ϕ c) $\dfrac{V_0}{I_0}$; ωt d) $\dfrac{V_0}{I_0}$; ω e) $\dfrac{I_0}{V_0}$; ω

SUGGESTED ANSWER: (b)

Wrong answers:
d) ω is the angular frequency

Redaction: Jeffery, 2008jan01

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034 qmult 00130 1 1 5 easy memory: displacement current, electric flux

18. The displacement current is:

a) a displaced current of electric charge.

b) the magnetic flux.
c) the electric flux.
d) $\mu_0\varepsilon_0(\partial\vec{B}/\partial t)$: a term Maxwell added to Ampère's law in order to account for the creation of **ELECTRIC** fields by time-varying magnetic fields.
e) $\mu_0\varepsilon_0(\partial\vec{E}/\partial t)$: a term Maxwell added to Ampère's law in order to account for the creation of **MAGNETIC** fields by time-varying electric fields.

SUGGESTED ANSWER: (e)

The name displacement current is historical and does not have a useful explanation in terms of its meaning in modern physics. Virtual current would be more consistent with modern jargon.

Wrong answers:

Redaction: Jeffery, 2001jan01

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035 qmult 01100 1 1 1 easy memory: law of refraction

19. Snell's law or the law of refraction is:

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$. b) $\theta_{\text{incident}} = \theta_{\text{reflected}}$. c) $\dfrac{1}{f} = \dfrac{1}{d_{\text{o}}} + \dfrac{1}{d_{\text{i}}}$.
d) $m = \dfrac{h_{\text{i}}}{h_{\text{o}}} = -\dfrac{d_{\text{i}}}{d_{\text{o}}}$. e) $\theta_{\text{reflected}} = \sin\theta_{\text{incident}}$.

SUGGESTED ANSWER: (a)

Wrong answers:
b) This is the law of reflection.
c) This is the mirror equation and also the thin lens equation.
d) This is the magnification equation for both spherical mirrors and thin lenses.

Redaction: Jeffery, 2008jan01

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022 qmult 00210 1 1 2 easy memory: nucleus small and massive
Extra keywords: Sun-question

20. The nucleus occupies ____________ of the volume of an atom and has ____________ of the atomic mass.

a) a small part; none b) a small part; almost all c) most; almost all
d) most; none e) most; half

SUGGESTED ANSWER: (b)

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

Full Answer Problems:

017 qfull 00330 1 3 0 easy math: a sinusoidal traveling wave

21. The equation of a transverse wave on a string is

$$y(x,t) = 9.0\sin(-0.01\pi x - 4.0\pi t) \ ,$$

where amplitude and x are **CENTIMETERS**, t is in seconds, and the argument of the sine is in radians. **HINT:** Do not forget the π values in calculating results and give results with correct units. Note, the standard form for a 1-dimensional wave solution adopted by the author is

$$y(x,t) = A\sin(kx - \omega t) \ ,$$

where ω is always taken as positive.

Find the (a) amplitude A, (b) wavelength $\lambda = 2\pi/|k|$, (c) frequency f (which is always positive), (d) direction of propagation along the x axis (which is determined by the sign of k) and (e) maximum oscillation speed of the string (i.e., its vertical maximum speed). Also (f), what is the oscillation displacement at $x = 3.5\,\text{cm}$ and time $t = 0.26\,\text{s}$? Also (g), given the phase of the wave of this form is $\phi = kx - \omega t$, determine the formula for the phase (propagation) velocity for height $y(\phi)$ for constant phase and the numerical value for the phase velocity for the given values.

SUGGESTED ANSWER:

a) $A = 9.0\,\text{cm}$.

b) $\lambda = 2\pi/|k| = 2\pi/(0.01\pi) = 200\,\text{cm}$.

c) $f = \omega/(2\pi) = (4\pi/2\pi) = 2.0\,\text{Hz}$.

d) Since $k = -0.01\pi < 0$, the direction of propagation is in the negative x-direction.

e) $v_{y,\max} = \omega A = 4.0\pi \times 9.0 \approx 110\,\text{cm/s}$. To better accuracy, $v_{y,\max} = 113\,\text{cm/s}$.

f) Behold:

$$\begin{aligned} y(x=3.5, t=0.26) &= 9.0 \times \sin(-0.035\pi - 1.0400\pi) = 9.0 \times \sin(-1.075\pi) \\ &= -9.0 \times \sin(1.075\pi) = -9.0 \times \sin(\pi + 0.075\pi) \\ &= -9.0 \times [\sin(\pi)\cos(0.075\pi) + \cos(\pi)\sin(0.075\pi)] \\ &= 9.0 \times \sin(0.075\pi) \approx 9.0 \times (0.075\pi) \approx 28.5 \times 0.075 \approx 2.1\,\text{cm} \end{aligned}$$

to about 1-digit accuracy. To better accuracy, $y(x = 3.5, t = 0.26) = 2.10\,\text{cm}$.

g) Behold:

$$1)\quad \phi = kx - \omega t \qquad 2)\quad x = \frac{\phi + \omega t}{k} \qquad 3)\quad v = \frac{dx}{dt} = \frac{\omega}{k} = \frac{4.0\pi}{(-0.01\pi)} = -400\,\text{cm/s} \ .$$

Fortran-95 Code

```
pi=3.14159265358979323846264338327950288419716939937510_np
    !
!!23456789a123456789b123456789c123456789d123456789e123456789f123456789g12
    !            !
https://en.wikipedia.org/wiki/Pi#Approximate_value_and_digits 51
digits
          v=4.0_np*pi*9.0_np
          x=3.5_np
          t=0.26_np
          arg=(-0.01_np*x-4.0_np*t)
          y=9.0_np*sin(arg*pi)
          print*,'pi,v,x,t,y'
          print*,pi,v,x,t,arg,y
    !    3.1415926535897932385         113.09733552923255659
3.5000000000000000000
    !        0.25999999999999999999        -1.0749999999999999999
2.1010082747031487
```

Redaction: Jeffery, 2001jan01

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026 qfull 00300 2 3 0 moderate math: energy stored in a capacitor

22. The potential energy stored in a capacitor is a function of the charge and its distribution only. It does not depend on how the distribution was assembled. The work done to assemble the distribution, however, equals this potential energy. So you can imagine any simple way of assembly you like as means of calculating the potential energy. Say that you just took positive charge from one plate little bit at a time and moved it to the other plate. You started from the uncharged state and built up to charge q on the capacitor. The change in potential energy of the system due the transfer of dq' is

$$dU = V'\,dq' \ ,$$

where V' is the potential difference between the plates at the time of transferring dq'.

Using an integration, derive the formula for the potential energy stored in a capacitor when charged to charge Q in terms of Q and C, Q and final potential V, **AND** final potential V and C? **HINT:** This is easy.

SUGGESTED ANSWER:

Behold:

$$1) \quad dU = V'\,dq' = \frac{q'}{C}\,dq' \qquad 2) \quad U = \int_0^Q \frac{q'}{C}\,dq' = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \ ,$$

where we have used the basic capacitor formula $C = Q/V$.

Redaction: Jeffery, 2001jan01

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027 qfull 00270 2 3 0 moderate math: atmospheric current

Extra keywords: HRW-631-26

23. The Earth's lower atmosphere contains positively and negatively charged ions produced by radioactive elements in the soil and cosmic rays. Assume their densities are $620\,\text{cm}^{-3}$ and $550\,\text{cm}^{-3}$, respectively, and that they are **SINGLY** charged. (Lower atmospheric molecule density is about $2.5 \times 10^{19}\,\text{cm}^{-3}$ for comparison and overwhelmingly the most molecules are neutral.) Assume also an atmospheric electric field of $120\,\text{V/m}$ pointed downward.

   a) In which direction does each kind of ion move?

   b) Given that the conductivity of the air is given by $\sigma = 2.70\times10^{-14}\,\Omega^{-1}\,\text{m}^{-1}$, what is the current density? Remember to give its direction.

   c) Assume that the ions have the **SAME DRIFT SPEED** v_{d} which is plausible since they are in the same electric field, have about the same mass on average, the same absolute value of charge, and about the same collision time on average. Recall that

$$\vec{J} = nq\vec{v}_{\text{d}} \ ,$$

   where n is carrier density, q is carrier charge, and $\vec{v}_{\text{d}}$ is the drift velocity.

   What is v_{d} in our case? (Note that current is net flux of charge, not flux of one kind or another of charge. Be careful about signs.)

SUGGESTED ANSWER:

   a) The positive ions move down (i.e., in the direction of the E-field) and the negative ions move up (i.e., opposite to the direction of the E-field).

   b) Given σ,

$$\vec{J} = \sigma\vec{E} = 2.70 \times 10^{-14} \times 120\hat{y} = 3.24 \times 10^{-12}\hat{y}\,\text{A/m}^2 \ ,$$

   where we have taken downward as the positive y direction.

   c) Behold:

$$1)\quad \vec{J} = \sigma\vec{E} = nq\vec{v}_{\text{d}} = n_+e\vec{v}_{\text{d}} + n_-(-e)(-\vec{v}_{\text{d}}) = (n_+ + n_-)e\vec{v}_{\text{d}} \qquad 2)\quad v_{\text{d}} = |\vec{v}_{\text{d}}| = \frac{\sigma|\vec{E}|}{(n_+ + n_-)e} = 1.73\times10^{-2}\,\text{m/s}$$

   to 1-digit accuracy. Note, one had to convert densities from cm^{-3} to m^{-3} using the factor of unity $1 = (100\,\text{cm}/1\,\text{m})^3$.

   Note, the numbers and assumptions for this problem come from HRW-631-26 and are probably only approximately correct. The problem is illustrative not definitive.

Fortran Code

```
print*
echarge=1.602e-19
sig=2.70e-14
efield=120.
xnp=620.e+6
xnm=550.e+6
xj=sig*efield
```

```
      vd=xj/( (xnp+xnm)*echarge )
      print*,'xj,vd'
      print*,xj,vd
*     3.24000007E-12  0.0172860846
```

Redaction: Jeffery, 2001jan01

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029 qfull 00410 2 3 0 mod math: mass spectrometer, analytic and numerical

Extra keywords: mass spectrometer, analytic and numerical

24. A particle of charge q is accelerated through an electric potential V. Note, initial potential energy of the particle qV is positive no matter what signs of q and V are individually.

a) What is the formula for its final speed assuming it started from rest?

b) The particle is then injected into a mass spectrometer which is a chamber with a uniform magnetic field $\vec{B}$. The field is perpendicular to the particle motion. The particle travels for half a circle and strikes a wall. The radius of the circle is measured. What is the formula (with velocity eliminated) for the mass of the particle?

c) Assume the particle is a doubly charged positive ion (i.e., positive charge $2e$), the potential was 10^5 V, the magnetic field strength was 0.1 T, and the **DIAMETER** of the circle was 1.5 m. What is the mass of the particle?

SUGGESTED ANSWER:

a) The potential energy qV the particle originally had is turned into kinetic energy. Since the particle started from rest, the final kinetic energy equals the initial potential energy. Thus,

$$1)\quad K = \frac{1}{2}mv^2 = qV \qquad 2)\quad v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2|qV|}{m}} \ ,$$

where the absolute value signs are just for clarity.

b) The magnetic force must supply the centripetal force. Thus,

$$1)\quad m\frac{v^2}{r} = |q|vB \qquad 2)\quad r_{\rm c} = \frac{mv}{|q|B} = \frac{1}{B}\sqrt{\frac{2Vm}{q}} \qquad 3)\quad m = \frac{B^2 r_{\rm c}^2 q}{2V} = \frac{B^2 r_{\rm c}^2 |q|}{2|V|} \ ,$$

where $r_{\rm c}$ is the cyclotron radius (Wikipedia: Cyclotron motion) and the absolute value signs are just for clarity.

c) For the given values

$$m = \frac{0.1^2 \times 0.75^2 \times 3.2 \times 10^{-19}}{2 \times 10^5} \approx 10^{-26}\,\text{kg}$$

to 1-digit accuracy. More accurately the particle mass is 9.01×10^{-27} kg or 5.42 u, where u is the unified atomic mass unit (AKA dalton (Da)). There arn't any known isotopes of this mass in fact. It is rather too far from being a whole number for that to be possible I would think.

Fortran Code

```
          print*
          volt=1.e+5
          charge=2*1.602e-19
          bfield=.1
          r=1.5/2.
          amu=1.66e-27
          xmass=bfield**2*r**2*charge/(2.*volt)
          print*,'xmass,xmass/amu'
          print*,xmass,xmass/amu
* 9.01125056E-27  5.42846394
```

Description: An moderate math question.
Redaction: Jeffery, 2001jan01

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031 qfull 00510 2 3 0 moderate math: bike electric generator

25. Consider a bike electric generator that powers a headlight. The (motional) emf of the generator is given by

$$\mathcal{E} = NAB\omega_{\rm gen}\sin(\omega_{\rm gen}t) ,$$

where N is the number of turns in the generator coil, A is the coil area, B is the magnitude of the magnetic field in the generator, $\omega_{\rm gen}$ is the angular velocity of the the generator coil, and t is time. We assume $\omega_{\rm gen} > 0$ to avoid useless generality.

a) What is $\mathcal{E}_{\rm max}$ (i.e., the maximum value of $\mathcal{E}$)?

b) What is $\omega_{\rm gen}$ in terms of $\mathcal{E}_{\rm max}$, N, A, and B? Note, $\omega_{\rm gen}$ depends on the bike speed, and so $\mathcal{E}_{\rm max}$ is not a fixed parameter.

c) The generator is cranked by a small wheel of radius $r_{\rm gen}$ that is driven by the bike wheel. The outermost radii of the two wheels directly contact each other and there is a no-slip condition between the two wheels. What is the formula for the speed of the bike wheel (i.e., $v_{\rm bike}$) in terms of the emf $\mathcal{E}_{\rm max}$?

d) What is the bike translational speed $v_{\rm trans}$ in terms of the emf $\mathcal{E}_{\rm max}$ given the no-slip condition between ground and bike wheel?

e) What is the bike translational speed $v_{\rm trans}$ given $r_{\rm gen} = 0.010\,{\rm m}$, $\mathcal{E}_{\rm max} = 5.0\,{\rm V}$, $N = 100$, $B = 0.10\,{\rm T}$, $A = 1.0 \times 10^{-3}\,{\rm m}^2$? Note, the emf has been measured somehow.

SUGGESTED ANSWER:

a) Clearly,

$$\mathcal{E}_{\rm max} = NAB\omega_{\rm gen}$$

since $\sin(\omega_{\rm gen}t)$ reaches a maximum of 1 once each period. The absolute value $|\mathcal{E}|$ reaches a maximum of $\mathcal{E}_{\rm max}$ twice per cycle since $|\sin(\omega_{\rm gen}t)|$ reaches a maximum of 1 twice per period.

b) Behold:

$$\omega_{\rm gen} = \frac{\mathcal{E}_{\rm max}}{NAB} .$$

c) The no-slip condition implies

$$v_{\rm bike} = v_{\rm gen} = r_{\rm gen}\omega_{\rm gen} \ , \qquad \text{and thus} \qquad v_{\rm bike} = \frac{r_{\rm gen}\mathcal{E}_{\rm max}}{NAB} \ .$$

d) The no-slip condition implies that the bike wheel and ground at point of contact are at rest relative to each other. The wheel velocity at contact is at $-v_{\rm bike}$ relative to the wheel center, and thus the ground is at $-v_{\rm bike}$ relative to the wheel center. Thus, the wheel center relative to the ground is moving at velocity $v_{\rm bike}$. But the wheel center velocity is just the translational velocity of the whole bike. Thus

$$v_{\rm trans} = v_{\rm bike} = \frac{r_{\rm gen}\mathcal{E}_{\rm max}}{NAB} \ .$$

e) Behold:

$$v_{\rm trans} = \frac{r_{\rm gen}\mathcal{E}_{\rm max}}{NAB} = 5.0\,{\rm m/s} \ .$$

Fortran-95 Code

```
print*
r=.01d0
emf=5.d0
xn=100.d0
a=.001d0
b=.1d0
v=(r*emf)/(xn*a*b)
print*,'v=',v  !  4.999999999999999
```

Redaction: Jeffery, 2008jan01

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032 qfull 00420 3 3 0 tough math: solving a DE for an RL circuit 1

26. It's about time you-all solved a differential equation (DE). Consider the simple one-loop RL circuit DE with a constant driving emf V (i.e., the DE for a simple one-loop circuit with an emf, resistor and inductor in series). The DE derived from Kirchhoff's voltage law is

$$V = LI' + IR \ .$$

a) Draw a circuit diagram.

b) The DE is a 1st order DE and can be solved by integration by isolating the dependent variable I and the independent variable t on different sides of an equal sign. Solve the DE that way with an integral and imposing the initial condition $I(t=0)=0$ necessary for an LR circuit. Write the time constant (or e-folding time) as $\tau = L/R$. What is the final current $I_{\rm final}$ as time goes to infinity? **HINT:** The integral for I is well known.

c) Draw a rough plot of $I/I_{\rm final}$ as a function of t/τ.

d) What is the formula for the power inflow to the inductor? Simplify as much as possible while leaving the formula intelligible for looking at and do not use the parameter $I_{\rm final}$ in the formula. **HINT:** The signs are always a bit tricky in such cases. Here we are counting the power inflow as positive, and so the potential drop $V_{\rm drop}$ (which is not the power source V) is counted as a positive quantity.

SUGGESTED ANSWER:

a) You will have to imagine the diagram.

b) Behold:

$$1)\quad V = LI' + IR \qquad 2)\quad LI' = V - IR \qquad 3)\quad \frac{I'}{V-IR} = \frac{1}{L} \qquad 4)\quad \frac{dI}{V-IR} = \frac{dt}{L}$$

$$5)\quad \int_0^I \frac{di}{V-iR} = \frac{1}{L}\int_0^t dt' \qquad 6)\quad \frac{1}{(-R)}\ln(V-iR)\Big|_0^I = \frac{t}{L} \qquad 7)\quad \ln\left(\frac{V-IR}{V}\right) = -\frac{t}{L/R}$$

$$8)\quad \frac{V-IR}{V} = \exp\left(-\frac{t}{\tau}\right) \qquad 9)\quad V-IR = V\exp\left(-\frac{t}{\tau}\right) \qquad 10)\quad I = I_{\rm final}\left[1-\exp\left(-\frac{t}{\tau}\right)\right] ,$$

where we have defined the LR time constant (or LR e-folding time) by $\tau = L/R$ and $I_{\rm final} = V/R$.

c) You will have to imagine the plot. However $I/I_{\rm final}$ will rise from zero at time zero and asymptotically approach 1 as time goes to infinity.

d) Behold:

$$P = IV_{\rm drop} = ILI' = L\frac{V}{R}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]\frac{V}{R}\left[\frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)\right] = \frac{V^2}{R}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]\left[\exp\left(-\frac{t}{\tau}\right)\right] .$$

To go beyond the required answer, note

$$1)\quad \frac{dP}{dt} = \frac{V^2}{R}\left(-\frac{1}{\tau}\right)\left[1-2\exp\left(-\frac{t}{\tau}\right)\right]\left[\exp\left(-\frac{t}{\tau}\right)\right] \qquad 2)\quad 0 = 1-2\exp\left(-\frac{t}{\tau}\right) \qquad 3)\quad \exp\left(-\frac{t}{\tau}\right) = \frac{1}{2}$$

$$4)\quad t_{\rm max} = \tau\ln(2) = \tau\times(0.693\,147\ldots) \qquad 5)\quad P_{\rm max} = \frac{V^2}{R}\left(\frac{1}{4}\right) .$$

Thus, we find

$$P = \begin{cases} \dfrac{V^2}{R}\left[1-\exp\left(-\dfrac{t}{\tau}\right)\right]\left[\exp\left(-\dfrac{t}{\tau}\right)\right] & \text{in general.} \\ \dfrac{V^2}{R}\left(\dfrac{t}{\tau}\right) & \text{for } t/\tau << 1. \\ P_{\rm max} = \dfrac{V^2}{R}\left(\dfrac{1}{4}\right) & \text{for } t_{\rm max} = \tau\ln(2). \\ \dfrac{V^2}{R}\exp\left(-\dfrac{t}{\tau}\right) & \text{for } t/\tau >> 1. \\ 0 & \text{in the asymptotic limit as } t\to\infty. \end{cases}$$

Redaction: Jeffery, 2008jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$
\begin{aligned}
c &= 2.997\ 924\ 58 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \\
&\quad \text{exact by definition} \\
e &= 1.602\ 176\ 634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition} \\
G &= 6.674\ 30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987\ 551\ 785\ 972(14) \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2} \\
k_{\mathrm{Boltzmann}} &= 1.380\ 649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\ 333\ 262 \times 10^{-4}\,\mathrm{eV/K} \\
&\approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms} \\
m_e &= 9.109\ 383\ 7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\ 998\ 950\ 69(16)\,\mathrm{MeV} \\
m_p &= 1.672\ 621\ 923\ 95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\ 089\ 32(29)\,\mathrm{MeV} \\
\varepsilon_0 &= 8.854\ 187\ 8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity} \\
\mu_0 &= [1.256\ 637\ 061\ 27(20)] \times 10^{-6}\,\mathrm{N/A^2} \approx 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}
\end{aligned}
$$

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2 Geometrical Formulae

$$
C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3
$$

$$
\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi
$$

---

3 Trigonometry Formulae

$$
\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1
$$

$$
\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}
$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

---

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1+x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

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5 Quadratic Formula

If $\quad 0 = ax^2+bx+c\ ,\quad$ then $\quad x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = -\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b+\mathrm{sgn}(b)\sqrt{b^2-4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

---

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2+a_y^2} \qquad \vec{a}+\vec{b} = (a_x+b_x, a_y+b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\frac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\operatorname{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

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7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

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8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $g = 9.8\,\mathrm{m/s^2}$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\rm cm} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\rm flux} - \vec{v}_{\rm cm})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\rm rel}\frac{dm}{dt}$$

$$v = v_0 + v_{\rm ex}\ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\rm avg} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\rm net}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\rm cm} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\rm total}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\rm rel'} = -v_{\rm rel} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\ \text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\ \text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin\gamma_L \qquad \tau_z = RF_{xy} \sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin\theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 **Gravity**

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707\times10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} &\quad p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} &\quad F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g \\
\text{equation of continuity for ideal fluid} &\quad R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} &\quad p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\text{max}}\sin[k(x - vt)] = y_{\text{max}}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\text{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\text{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\text{K}} = T_{\text{C}} + 273.15\,\text{K} \qquad T_{\text{F}} = 1.8 \times T_{\text{C}} + 32^\circ\text{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

25 **Electrostatics**

$$\vec{F}_{1\text{ on }2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_{\text{i}}^{\text{f}} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_i V_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

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26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J} \cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma \vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R} e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}} \left(1 - e^{-t/\tau}\right)$$

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27 Magnetic Fields and Forces

$$\mu_0 = [1.256\ 637\ 061\ 27(20)] \times 10^{-6}\,\text{N/A}^2 \approx 4\pi \times 10^{-7}\,\text{N/A}^2 \qquad u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2$$

$$1\,\text{N/A}^2 = 1\,\text{T} \cdot \text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad d\vec{F} = I\,d\vec{s} \times \vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I\theta}{4\pi r} \qquad \vec{F}_{1 \text{ on } 2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r} \hat{r}_{2 \text{ to } 1}$$

$$B_{\text{solenoid}} = \mu_0 n I \qquad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

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28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2} L I^2 \qquad u_B = \frac{1}{2\mu_0} B^2$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con-axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0\cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_\pm = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0\cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right]e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0\cos(\omega t) \qquad I = I_0\cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2}I_0V_0\cos\phi = I_{\text{rms}}V_{\text{rms}}\cos\phi$$

$$\frac{\mathcal{E}_\text{p}}{N_\text{p}} = \frac{\mathcal{E}_\text{s}}{N_\text{s}} \qquad I_\text{p}N_\text{p} = I_\text{s}N_\text{s} \qquad I_\text{p}\mathcal{E}_\text{p} = I_\text{s}\mathcal{E}_\text{s} \qquad \frac{\mathcal{E}_\text{p}}{\mathcal{E}_\text{s}} = \frac{N_\text{p}}{N_\text{s}} = \frac{I_\text{s}}{I_\text{p}}$$

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B}\cdot d\vec{A} = 0 \qquad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0\oint \vec{J}\cdot d\vec{A} + \varepsilon_0\mu_0\frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$