Introductory Physics II: Calculus-Based: Exam 3 Name:

Instructions: There are 20 multiple-choice problems worth 1 mark each for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized.

There are **THREE** full-answer problems worth 10 marks each for a total of 30 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may extend over multiple-pages: make sure you have answered everything. And **BOX-IN** your final answers. There are 2 scratch pages for additional calculations or to rewrite answers if needed. In the latter case, make clear which answer is to be marked. You can ask for more paper if needed.

This is a **CLOSED-BOOK: CLOSED-NOTES: NO-ONLINE-ACCESS: NO-AI** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 50 marks altogether and is a 60-minute exam.

Multiple Choice Questions:

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027 qmult 00100 1 1 1 easy memory: electrical current partially defined

Extra keywords: physci

1. An electrical current, partially defined, is:

   a) moving charge. b) static charge. c) electrical potential.
   d) moving water. e) static water.

   SUGGESTED ANSWER: (a)

   Wrong answers:

   e) As Lurch would say: "Aaaarh."

   Redaction: Jeffery, 2001jan01

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027 qmult 00200 1 1 4 easy memory: metal charge carriers

2. The actual charge carriers in metals are:

   a) tightly bound electrons. b) bound electrons. c) positrons.
   d) free electrons. e) holes.

   SUGGESTED ANSWER: (d)

   Wrong answers:

   e) An electron hole is an absence of an electron and does act like a positive charge carrier. The term hole does a apply to conductors, but is largely used for semiconductors where holes can be tracked whereas in conductor they can't. Google AI question: Why is the concept of electron hole usually used only for semiconductors?: Answer: The concept of an electron hole is mostly used for semiconductors because their nearly full valence bands make tracking a few missing spaces easier than tracking billions of electrons, whereas metals have a chaotic sea of free electrons where vacancies vanish instantly

Redaction: Jeffery, 2008jan01

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027 qmult 00300 2 4 4 moderate deducto-memory: Ohm's law

Extra keywords: physci KB-149

3. Ohm's law (in usualy meaning) is:

   a) $P = VI$. b) $C = q/V$. c) $V = I/R$. d) $V = IR$.
   e) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\rm enc}}{\varepsilon_0}$.

   SUGGESTED ANSWER: (d)

   Wrong answers:
   e) This is Gauss's law (HRW-547).

   Redaction: Jeffery, 2001jan01

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028 qmult 00120 1 4 5 easy deducto-memory: Kirchhoff's voltage law 1

4. The potential change around any closed path is zero, and thus in the special case of an electrical circuit the potential changes summed over all elements in the circuit must be:

   a) infinite: this is Mozart's 3rd law. b) positive: this is Krankheit's 1st law.
   c) negative: this is Wilson's 14th point.
   d) indefinite: this is Hemingway's conjecture.
   e) zero: this is Kirchhoff's voltage law.

   SUGGESTED ANSWER: (e)

   Wrong answers:
   c) I think it was Clemenceau who said: "Dieu avait dix seul."

   Redaction: Jeffery, 2001jan01

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028 qmult 00300 1 1 1 easy memory: resistors series/parallel

5. The equivalent resistor formulae for resistors in series and parallel are, respectively:

   a) $R_{\rm eq} = \sum_i R_i$ and $\frac{1}{R_{\rm eq}} = \sum_i \frac{1}{R_i}$.

   b) $\frac{1}{R_{\rm eq}} = \sum_i \frac{1}{R_i}$ and $R_{\rm eq} = \sum_i R_i$.

   c) $R_{\rm eq} = \prod_i R_i$ and $\frac{1}{R_{\rm eq}} = \prod_i \frac{1}{R_i}$.

   d) $R_{\rm eq} = \sqrt{\sum_i R_i^2}$ and $R_{\rm eq} = \sum_i R_i$.

e) $R_{\rm eq} = \sqrt{\sum_i R_i^2}$ and $R_{\rm eq} = \sum_i \frac{1}{R_i}$.

SUGGESTED ANSWER: (a)

Wrong answers:
c) Not dimensionally correct.

Redaction: Jeffery, 2001jan01

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028 qmult 00450 1 1 1 easy memory: electric eel
Extra keywords: HRW-646

6. You are swimming with your pet electric eel Fred—who may not be a terribly realistic electrophorus electricus—and playfully grab his tail and just below his head: the potential difference between these regions is $\sim 750\,\text{V}$. Wet, the typical resistance between human body parts is of order $1000\,\Omega$ and let's assume contrafactually that Fred's internal resistance is negligible. Human resistance is much higher if you are dry: of order $10^5\,\Omega$. Given that death is pretty certain if you carry a current of $0.2\,\text{A}$, what are your chances of living?

a) Rather bad. b) So-so. c) Great. d) You have never been safer.
e) You are under 25, you are immortal.

SUGGESTED ANSWER: (a)

With the given numbers—which may be wildly inaccurate—the current through you would be

$$I = \frac{V}{R} = \frac{750}{1000} = 0.75\,\text{A}$$

and you are in trouble.

I don't know a lot about electrical safety, the numbers are what introductory physics books suggest. I know even less about electric eels. Wikipedia suggests that for adult humans the shock only could be harmful:

`http://en.wikipedia.org/wiki/Electric_eel` .

See also `https://commons.wikimedia.org/wiki/File:IEC_TS_60479-1_electric_shock_graph.svg` .

Fear Factor suggests it's even hilariously fun manhandling electric eels—"It didn't stop me from goin' back for more and more.":

`http://www.nbc.com/nbc/Fear_Factor/tales/319_electric_eel.shtml` .

But the women participants—they had true grit—only used one hand which suggests that they were minimizing the potential difference they were allowing across their bodies.

Wrong answers:
e) There is a strong prejudice in favor of this theory, but experience has never verified it.

Redaction: Jeffery, 2001jan01

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029 qmult 00110 1 4 5 easy deducto-memory: magnetic field lines
Extra keywords: mathematical physics

7. "Let's play *Jeopardy*! For \$100, the answer is: They have no ends, except at rather rare points of high symmetry where the zero magnetic field (unless magnetic monopoles exist after all) and a every point along them the magnetic field vector is tangent to them and points in their direction."

   What are ____________, Alex?

   a) transmission lines b) telephone lines c) streamlines
   d) electric field lines e) magnetic field lines

SUGGESTED ANSWER: (e)

Wrong answers:
b) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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029 qmult 00140 1 1 5 easy memory: north and south pole

8. In the view of the instructor, a region where magnetic field lines diverge can be called a/an ____________ and a region where they converge can be called a/an ____________.

   a) up pole; down pole b) left pole; right pole c) magnet; magnetite
   d) south pole; north pole e) north pole; south pole

SUGGESTED ANSWER: (e)

Most people would agree with this definition. It's just a qualitative definition.

Wrong answers:
a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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029 qmult 00400 1 4 3 easy deducto-memory: magnetic force law
Extra keywords: magnetic force law.

9. The magnetic force law for a point charge is:

   a) $\vec{F} = q\vec{E}$. b) $\vec{F} = q\vec{v} \times \vec{E}$. c) $\vec{F} = q\vec{v} \times \vec{B}$. d) $\vec{F} = \vec{v} \times \vec{B}$.
   e) $\vec{F} = \vec{v} \times \vec{E}$.

SUGGESTED ANSWER: (c)

Wrong answers:
a) This is the law for an electric force.

Description: An easy memory question.
Redaction: Jeffery, 2001jan01

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029 qmult 00410 1 1 5 easy memory: magnetic force does no work on point charges

10. The point-charge magnetic force law ____________ can ____________ work on a point charge since ____________. Thus, a useful potential energy ____________ be defined for the magnetic force for the case of single point charge.

    a) $\vec{F} = \vec{v} \times \vec{B}$; do; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can

    b) $\vec{F} = q\vec{v} \times \vec{B}$; do; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can

    c) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can

    d) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt = 0$; can

    e) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt = 0$; cannot

    SUGGESTED ANSWER: (e)

    Wrong answers:
    a) Everything is wrong.

    Redaction: Jeffery, 2008jan01

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029 qmult 00430 1 4 5 easy deducto-memory: circling charges or helixing charges
Extra keywords: circling charges.

11. Charged particles moving in magnetic fields tend to:

    a) travel in straight lines. b) travel in sinusoidal curves.
    c) move in zigzags. d) stop and start constantly.
    e) move in circles or helixes.

    SUGGESTED ANSWER: (e) Helix is really the right term. A helix is a curve that spirals around a cylinder: it has a constant radius from the axis but increases constantly in the axial direction in the simplest case. A spiral is a curve that is the locus of a point that rotates around a center with constantly increasing radius. But informally a spiral is also a helix (Wiktionary: spiral, 2008mar19).

    Wrong answers:

    Description: An easy memory question.
    Redaction: Jeffery, 2001jan01

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029 qmult 00630 1 4 2 easy deducto-memory: magnetic moment
Extra keywords: magnetic moment

12. For a magnetic moment:

    a) $\vec{\tau} = \vec{\mu} \times \vec{E}$ and $U = -\vec{\mu} \cdot \vec{E}$. b) $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\vec{\mu} \cdot \vec{B}$.
    c) $\vec{\tau} = -\vec{\mu} \cdot \vec{B}$ and $U = \vec{\mu} \times \vec{B}$. d) $\vec{\tau} = -\vec{\mu} \cdot \vec{E}$ and $U = \vec{\mu} \times \vec{E}$.
    e) $\vec{\tau} = U$ and $U = \vec{\mu} \times \vec{E}$.

    SUGGESTED ANSWER: (b)

    Wrong answers:

    Description: An easy memory question.
    Redaction: Jeffery, 2001jan01

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029 qmult 00730 1 4 1 easy deducto-memory: Hall effect

Extra keywords: Hall effect

13. The Hall effect showed that:

    a) the charge carriers in metals had negative charge.
    b) the charge carriers in metals had positive charge.
    c) the charge carriers in metals had neutral charge.
    d) the charge carriers in metals had both negative and positive charge.
    e) metals had no charge carriers.

SUGGESTED ANSWER: (a)

Wrong answers:

Description: An easy memory question.
Redaction: Jeffery, 2001jan01

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030 qmult 00100 1 1 5 easy memory: electro-magnetic connection Oersted
Extra keywords: physci if magnetism is covered

14. Hans Christian Ørsted (1777–1851) in 1820 found that a current (of electric charge) generated:

    a) a Bessel function. b) a Hermite polynomial. c) Michael Faraday.
    d) a Hooke's law force. e) a magnetic force.

SUGGESTED ANSWER: (e) See HRW-506. This discovery proved that there was some intrinsic connection between electricity and magnetism.

Wrong answers:
c) C'mon.

Redaction: Jeffery, 2001jan01

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030 qmult 00120 1 1 1 easy memory: Biot-Savart law

15. The Biot-Savart law is:

    a) $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{s}\times\hat{r}}{r^2}$. b) $\vec{E}(\vec{r}) = \dfrac{kq}{r^2}\hat{r}$. c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{linked}}$.
    d) $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$. e) $E = mc^2$.

SUGGESTED ANSWER: (a) Google AI question: Can one say current linked by an Amperian loop? Answer: "Yes, one can say a current is "linked" by an Amperian loop, meaning the path of the current passes through any open surface bounded by the closed loop."

Wrong answers:
b) Coulomb's law.
c) Ampère's law.
d) Gauss's law.
e) Einstein's equation.

Redaction: Jeffery, 2001jan01

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030 qmult 00130 1 4 3 easy deducto-memory: Ampere's law

Extra keywords: Ampère's law

16. Ampère's law is:

a) $dB\vec{} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$. b) $\vec{E}(\vec{r}) = \frac{kq}{r^2}\hat{r}$. c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\rm enc}$.
d) $\oint \vec{E}\cdot d\vec{A} = \frac{q_{\rm enc}}{\varepsilon_0}$. e) $E = mc^2$.

SUGGESTED ANSWER: (c)

Wrong answers:

c) Biot-Savart law.
b) Coulomb's law.
d) Gauss's law.
e) Einstein's equation.

Redaction: Jeffery, 2001jan01

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030 qmult 00200 2 1 1 moderate memory: straight wire B-field 1

17. The magnitude of the magnetic field outside of an axially-symmetric, infinite straight wire (with current I, coordinate r perpendicular to the wire, and coordinate z along the wire) which can be found directly found easily by an application of Ampère's law is:

a) $B = \frac{\mu_0 I}{2\pi r}$. b) $B = \frac{\mu_0 I\theta}{4\pi r}$. c) $B = \mu_0 nI$. d) $B = \frac{\mu_0 NI}{2\pi r}$.
e) $\vec{B} = \frac{\mu_0\vec{\mu}}{2\pi z^3}$.

SUGGESTED ANSWER: (a)

Wrong answers:

b) Arc of circle.
c) Ideal solenoid.
d) Toroid.
e) Dipole field on axis and this is a vector expression contrary to the question.

Redaction: Jeffery, 2001jan01

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030 qmult 00230 1 1 4 easy memory: wire attraction

18. Parallel current-carrying wires will attract if the currents are parallel and repel if the currents are antiparallel. It's possible to remember this by imagining deforming the wires into circular loops with your right hand both times and then you get an north pole facing a south which should attract. Your thinking is making use of:

a) geometry. b) symmetry. c) a left-hand rule. d) a right-hand rule.
e) an up-down rule.

SUGGESTED ANSWER: (d)

Wrong answers:

e) A nonsense answer.

Redaction: Jeffery, 2008jan01

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030 qmult 00242 1 4 1 easy deducto-memory: ampere defining force post-2019

19. "Let's play *Jeopardy*! For \$100, the answer is: Exactly by post-2019 definition

$$1\,\mathrm{C/s} = \frac{10^{19}\,e}{1.609\ 176\ 634\,s} \approx 0.6 \times 10^{19}\,e/\mathrm{s}\ ,$$

where $e = (1.602\ 176\ 634) \times 10^{-19}\,\mathrm{C}$ is the elementary charge which is exact by definition."

What is the ____________, Alex?

a) ampere (A) b) farad (F) c) gauss (G) d) tesla (T)
e) weber (Wb)

SUGGESTED ANSWER: (a) Note, if you substitute for e in the formula with its value in coulombs, you get one $1\,\mathrm{C/s} = 1\,\mathrm{C/s}$ which is satisfactory.

Wrong answers:

e) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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030 qmult 00340 1 1 1 easy memory: right-rule for solenoids

20. The magnetic field lines running through the middle region of a solenoid have their sense determined the direction of the thumb for:

a) a right-hand rule in which the fingers curl in the direction of current flow.
b) a left-hand rule in which the fingers curl in the direction of current flow.
c) a right-hand rule in which the fingers curl in the opposite to the direction of current flow.
d) wind speed.
e) guesswork.

SUGGESTED ANSWER: (a)

Wrong answers:

e) In this case, it works 50 % of the time. This also the accuracy of lie-detectors. To quote Richard Nixon: "I don't care if lie-detectors work: they scare the heck out of people." Of course, he didn't actually say heck.

Redaction: Jeffery, 2001jan01

Full Answer Problems:

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028 qfull 00520 1 3 0 easy math: 1 loop circuit

Extra keywords: (HR-654:5E)

21. There is a single loop circuit with two reversible emfs ($\mathcal{E}_1 = 12\,\mathrm{V}$ and $\mathcal{E}_2 = 6\,\mathrm{V}$) and two resistors ($R_1 = 4\,\Omega$ and $R_2 = 8.0\,\Omega$). (Reversible in this context means they can supply or absorb energy.) Consider the loop to be rectangular as usual: R_1 is on the

top segment; R_2 is on the right segment; $\mathcal{E}_1$ is on the bottom segment with positive terminal aimed right; $\mathcal{E}_2$ is on the left segment with positive terminal aimed up.

a) Draw a schematic circuit diagram with correct symbols for the components.

b) Determine the current in the circuit and its direction.

c) What is the power dissipated in each resistor?

d) What is the power of each battery and is it emitted or absorbed?

SUGGESTED ANSWER:

a) You will have to imagine the diagram.

b) Assume the current flows counterclockwise and use Kirchhoff's voltage law:

$$\mathcal{E}_1 = \mathcal{E}_2 + IR_1 + IR_2 \ .$$

Solving for I gives

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = 0.5\,\mathrm{A} \ .$$

Since the current is positive, our assumption that it flows counterclockwise is correct.

c) Behold:

$$P_1 = I^2 R_1 = 1\,\mathrm{W} \qquad \text{and} \qquad P_2 = I^2 R_2 = 2\,\mathrm{W} \ .$$

d) Behold:

$$P_{\text{emf, }1} = \mathcal{E}_1 I = 6\,\mathrm{W} \qquad \text{and} \qquad P_{\text{emf, }2} = \mathcal{E}_2 I = 3\,\mathrm{W} \ ,$$

where emf 1 is supplying power and emf 2 is absorbing it.

Description: An easy math question.
Redaction: Jeffery, 2001jan01

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029 qfull 00400 2 3 0 mod math: electron in crossed E and B fields

22. A free electron is in a region of uniform electric and magnetic fields. (Recall $e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}$ exactly and $m_e = 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg}$.) The system is described by Cartesian coordinates. The electron's initial velocity is $(0.0, 5.0, 5.0) \times 10^3\,\mathrm{m/s}$ and it has a constant acceleration in the x-direction of $2.0 \times 10^{12}\,\mathrm{m/s^2}$. In the y-z plane, the electron is merely circling. The magnetic field is $(400, 0, 0)\,\mu\mathrm{T}$: note, the given units are **MICROTESLAS**.

a) Draw a diagram of the system.

b) What is the electric field, magnitude and direction? **HINT:** If any component of the electric field were in the y-z plane, could the electron just be circling there?

c) What is the speed in the y-z plane at any time?

d) What is the radius of the circular motion and what is the direction of circling?

SUGGESTED ANSWER:

a) You must imagine the diagram for yourself.

b) Since the electron is merely circling in the y-z plane the entire force acting in that plane must be the magnetic force. Any electric field component in that plane would cause some rectilinear acceleration. (There may be some tricky way of putting in some electric field component in the y-z plane, but I don't think so. Certainly there is no obvious way.) Therefore the electric field must act entirely in the x-direction. Since the acceleration in the x-direction is entirely rectilinear, there can be no magnetic force in the x-direction (i.e., the magnetic field is entirely in the x-direction), and so the x-direction acceleration is entirely due to the electric field. The electric field must be

$$\vec{E} = \frac{ma_x}{-e}\hat{x} \approx -12\hat{x}\,\mathrm{V/m}$$

to 2-digit accuracy. To better accuracy

$$\vec{E} = \frac{ma_x}{-e}\hat{x} \approx -11.4\hat{x}\,\mathrm{V/m}\ .$$

c) The magnetic force can't increase the speed of a free charge. Thus the speed in the y-z plane is a constant and is just initial speed $\sqrt{5^2+5^2}\times 10^3 \approx 7\times 10^3\,\mathrm{m/s}$ to 1-digit accuracy. To better accuracy the speed is $7070\,\mathrm{m/s}$.

d) The radius of the helixing motion is given by the cyclotron radius formula

$$r_{\mathrm{c}} = \frac{mv}{|q|B} \approx 10^{-4}\,\mathrm{m}$$

to about 1-digit accuracy. To better accuracy,

$$r_{\mathrm{c}} = 1.01\times 10^{-4}\,\mathrm{m}\ .$$

It is easiest to think of a positive charge and view in the direction of the B-field. In this case, cross product $\vec{v}\times\vec{B}$ gives gives counterclockwise rotation for the positive charge. Therefore, the negatively charge electron is circling clockwise looking in the B-field or positive x direction.

Fortran Code

```
print*
echarge=1.602176634e-19_np
emass=9.1093837139e-31
acc=2.e+12_np
vy=5.e+3_np
vz=5.e+3_np
bfield=400.e-6_np
efield=emass*acc/echarge
speed=sqrt(vy**2+vz**2)
rad=emass*speed/(echarge*bfield)
print*,'efield,speed,rad'
print*,efield,speed,rad
```

```
! 11.3732834   7071.06787   0.00010052658
```

Description: An easy math question.
Redaction: Jeffery, 2001jan01

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030 qfull 00100 3 3 0 tough math: finite straight wire magnetic field

23. An infinitely thin, straight wire of length L carries current I. The current comes out of nowhere and goes into nowhere which is really unrealistic.

a) Draw a diagram of the system with the wire aligned with the z direction and the center of the wire at $z = 0$. Use ρ for the cylindrical radial coordinate and note $r = \sqrt{\rho^2 + z^2}$, where r is the (spherical) radius from the center. Describe the magnetic field (i.e., B-field) field lines everywhere. **HINT:** Remember the Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2} \ .$$

If we make the gross approximation that the integrand can be treated as a constant, then

$$\vec{B}_{\rm approx} = \frac{\mu_0}{4\pi}\frac{IL\sin(\theta)}{r^2}\hat{\phi} \ ,$$

where θ is the angle from the $\hat{z}$ direction which is the direction of current flow and $\hat{\phi}$ is the azimuthal angle of cylindrical coordinates. Note, $\theta = 0$ specifies the polar direction and $\theta = \pi/2$ the equator direction. For your diagram, consider how $\vec{B}_{\rm approx}$ behaves for fixed r.

b) Determine the formula for B-field for the $\theta = \pi/2$ (the $\hat{\rho}$ direction). **HINTS:** Ampère's law can't be used because there is not exact cylindrical symmetry, except for the case of $L = \infty$. You will need the table integral

$$\int \frac{dx}{(ax^2+b)^{3/2}} = \frac{x}{b\sqrt{ax^2+b}} \qquad \text{or alternatively} \qquad \int \frac{dx}{(ax^2+b^2)^{3/2}} = \frac{x}{b^2\sqrt{ax^2+b^2}}$$

(Hu-7).

c) If $L \to \infty$, what does the magnetic field become and where along the wire does it apply?

SUGGESTED ANSWER:

a) You will have to imagine the diagram. But from $\vec{B}_{\rm approx}$, we see the B-field field lines should be circles parallel to the x-y plane and that for fixed r, the magnitude $|\vec{B}|$ increases from zero at $\theta = 0$ (the $\hat{z}$ direction) to a maximum at $\theta = \pi/2$ (the $\hat{\rho}$ direction). Below the equator, you have the mirror image behavior.

b) Behold:

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-L/2}^{L/2}\frac{I\,dz\sin(\theta)}{r^2}\hat{\phi} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2}\left(\frac{1}{r^2}\right)\left(\frac{\rho}{r}\right)dz\,\hat{\phi} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2}\frac{\rho}{(\rho^2+z^2)^{3/2}}\,dz\,\hat{\phi}$$

$$= \frac{\mu_0 I\rho}{4\pi}(2)\int_0^{L/2}\frac{1}{(\rho^2+z^2)^{3/2}}\,dz\,\hat{\phi} = \frac{\mu_0 I\rho}{2\pi}\left(\frac{1}{\rho^2}\right)\left.\frac{z}{\sqrt{\rho^2+z^2}}\right|_0^{L/2}\hat{\phi} = \frac{\mu_0 I}{2\pi\rho}\frac{L/2}{\sqrt{\rho^2+(L/2)^2}}\hat{\phi}$$

or compactly

$$\vec{B} = \begin{cases} \dfrac{\mu_0 I}{2\pi\rho} \dfrac{L/2}{\sqrt{\rho^2 + (L/2)^2}} \hat{\phi} & \text{for general } L. \\ \dfrac{\mu_0 I}{2\pi\rho} \dfrac{L/2}{\rho} \hat{\phi} = \dfrac{\mu_0 I L}{4\pi\rho^2} \hat{\phi} & \text{for } L << \rho \text{ and this agrees} \\ & \quad \text{with the gross approximation of part (a) for } \theta = \pi/2 \\ & \quad \text{which gives } r(\theta = \pi/2) = \rho. \end{cases}$$

c) If $L \to \infty$, then

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho} \hat{\phi} \ .$$

This result applies everywhere along the wire since everywhere along the wire is infinitely far from the ends. The formula is exactly what you get from Ampère's law for a case of cylindrical symmetry.

Note : Some left-over stuff from an earlier version of this problem:

If the point for evaluating the field hadn't been symmetrically placed, the result would be almost the same. Say the point was at a along axis of the wire, then

$$\begin{aligned} \vec{B} &= \frac{\mu_0 I}{4\pi} \int_{-a}^{L-a} \frac{R}{(R^2 + s^2)^{3/2}} \, ds \, \hat{n} = \frac{\mu_0 I}{4\pi R} \frac{s}{\sqrt{R^2 + s^2}} \Bigg|_{-a}^{L-a} \hat{n} \\ &= \frac{\mu_0 I}{4\pi R} \left[\frac{L - a}{\sqrt{R^2 + (L - a)^2}} + \frac{a}{\sqrt{R^2 + a^2}} \right] \hat{n} \ . \end{aligned}$$

If one doesn't want to use the table integral, one can transform to trigonometric functions. First,

$$s = -\frac{R}{\tan(\theta)} \qquad \text{and} \qquad ds = \frac{R}{\sin^2(\theta)} \, d\theta \ .$$

Note we don't worry about the infinity at $\theta = \pi/2$. If everything is fine up the point of the infinity and no infinity appears in the end result, then usually in physics the infinity is taken care of in a limit sense. Also

$$r = \frac{R}{\sin\theta} \ .$$

Thus we a transformed integral we have

$$\vec{B} = \frac{\mu_0 I}{4\pi R} \int_{\theta_\ell}^{\pi - \theta_\ell} \sin\theta \, d\theta \hat{n} \ ,$$

where

$$\theta_\ell = \sin^{-1}\left(\frac{R}{\sqrt{R^2 + (L/2)^2}} \right) \ ,$$

Doing the integration, we find

$$\vec{B} = \frac{\mu_0 I}{4\pi R} \left(2\cos\theta_\ell\right) \hat{n} \ .$$

Now

$$\cos\theta_\ell = \frac{L/2}{\sqrt{R^2 + (L/2)^2}} \ ,$$

and so

$$\vec{B} = \frac{\mu_0 I}{2\pi R} \frac{L/2}{\sqrt{R^2 + (L/2)^2}} \hat{n} \ .$$

Leaving the integral in terms of s and using a table was easier.

Redaction: Jeffery, 2001jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

---

1 Constants

$$
\begin{aligned}
c &= 2.997\,924\,58 \times 10^{8}\,\mathrm{m/s} \approx 2.998 \times 10^{8}\,\mathrm{m/s} \approx 3 \times 10^{8}\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \\
&\quad \text{exact by definition} \\
e &= 1.602\,176\,634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition} \\
G &= 6.674\,30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987\,551\,785\,972(14) \times 10^{9} \approx 8.99 \times 10^{9} \approx 10^{10}\,\mathrm{N\,m^2/C^2} \\
k_{\mathrm{Boltzmann}} &= 1.380\,649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\,333\,262 \times 10^{-4}\,\mathrm{eV/K} \\
&\approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms} \\
m_e &= 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\,998\,950\,69(16)\,\mathrm{MeV} \\
m_p &= 1.672\,621\,923\,95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\,089\,32(29)\,\mathrm{MeV} \\
\varepsilon_0 &= 8.854\,187\,8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity} \\
\mu_0 &= 1.256\,637\,061\,27(20)\,\mathrm{N/A^2} \approx \pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}
\end{aligned}
$$

---

2 Geometrical Formulae

$$
C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3
$$

$$
\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi
$$

---

3 Trigonometry Formulae

$$
\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1
$$

$$
\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}
$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

---

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

---

5 Quadratic Formula

If $\quad 0 = ax^2 + bx + c \ ,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2 - 4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

---

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\frac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\operatorname{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\frac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

---

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

---

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\qquad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 **Energy and Work**

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\rm cm} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\rm flux} - \vec{v}_{\rm cm})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\rm rel}\frac{dm}{dt}$$

$$v = v_0 + v_{\rm ex}\ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

---

16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\rm avg} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\rm net}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\rm cm} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\rm total}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\rm rel'} = -v_{\rm rel} \qquad \text{1-d Elastic Collision Expressions}$$

---

17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\ \text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\ \text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin\gamma_L \qquad \tau_z = RF_{xy} \sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin\theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1\text{ on }2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g}\cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736\times10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707\times10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875\times10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5\times10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955\times10^{8} \approx 109\times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891\times10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\text{max}}\sin[k(x - vt)] = y_{\text{max}}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\text{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\text{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

---

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\text{K}} = T_{\text{C}} + 273.15\,\text{K} \qquad T_{\text{F}} = 1.8 \times T_{\text{C}} + 32^\circ\text{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \le \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

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25 Electrostatics

$$\vec{F}_{1\text{ on }2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_{\text{i}}^{\text{f}} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_i V_i$$

$$U = \frac{1}{2}\int\!\!\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

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26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J} \cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma \vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R} e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}} \left(1 - e^{-t/\tau}\right)$$

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27 Magnetic Fields and Forces

$$\mu_0 = 1.256\,637\,061\,27(20)\,\text{N/A}^2 \approx \pi \times 10^{-7}\,\text{N/A}^2 \qquad 1\,\text{N/A}^2 = 1\,\text{T} \cdot \text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad d\vec{F} = I\,d\vec{s} \times \vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I \theta}{4\pi r} \qquad \vec{F}_{1 \text{ on } 2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r} \hat{r}_{2 \text{ to } 1}$$

$$B_{\text{solenoid}} = \mu_0 n I \qquad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

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28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2} L I^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con-axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0 \cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_{\pm} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0 \cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right] e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0 \cos(\omega t) \qquad I = I_0 \cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2} I_0 V_0 \cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \qquad I_{\text{p}} N_{\text{p}} = I_{\text{s}} N_{\text{s}} \qquad I_{\text{p}} \mathcal{E}_{\text{p}} = I_{\text{s}} \mathcal{E}_{\text{s}} \qquad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$