Introductory Physics II: Calculus-Based: Exam 2 Name:

Instructions: There are 20 multiple-choice problems worth 1 mark each for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized.

There are **THREE** full-answer problems worth 10 marks each for a total of 30 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may extend over multiple-pages: make sure you have answered everything. And **BOX-IN** your final answers. There are 2 scratch pages for additional calculations or to rewrite answers if needed. In the latter case, make clear which answer is to be marked. You can ask for more paper if needed.

This is a **CLOSED-BOOK: CLOSED-NOTES: NO-ONLINE-ACCESS: NO-AI** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 50 marks altogether and is a 60-minute exam.

Multiple Choice Questions:

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024 qmult 00100 1 1 1 easy memory: Gauss law, high symmetry

1. Gauss's law gives a way to easily obtain an analytic exact formula for the electric field of a charge distribution in cases of:

   a) high symmetry: principally, planar, cylindrical and spherical symmetry.
   b) low symmetry: e.g., irregular objects, charged atmospheric clouds.
   c) indifferent symmetry.
   d) strange symmetry.
   e) charm symmetry.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   b) Oh, c'mon.
   c) Nonsense answer.
   d) Strangeness is a property of fundamental particles, but I don't know if the expression strangeness symmetry makes any sense.
   e) Charm is a property of fundamental particles, but I don't know if the expression charm symmetry makes any sense.

   Redaction: Jeffery, 2001jan01

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024 qmult 00510 2 5 1 moderate thinking memory: force on a spherically sym. distribution

2. From the shell theorem, we know that a system with a spherically symmetric charge distribution of total charge q creates an electric field outside of the distribution which is the same electric field that a (classical) point charge q located at the center of charge

would create. Say we had two such systems and system 1 exerts force on system 2 $\vec{F}_{12}$ and the corresponding force for replacement point charges is $\vec{F}_{1_{\rm eq},2_{\rm eq}}$ We know that:

a) $\vec{F}_{12} = \vec{F}_{1_{\rm eq},2_{\rm eq}}$. b) $\vec{F}_{12} = -\vec{F}_{1_{\rm eq},2_{\rm eq}}$ c) $\vec{F}_{12} = 2\vec{F}_{1_{\rm eq},2_{\rm eq}}$.
d) $\vec{F}_{12} = (1/2)\vec{F}_{1_{\rm eq},2_{\rm eq}}$. e) $\vec{F}_{12} = -(1/2)\vec{F}_{1_{\rm eq},2_{\rm eq}}$.

SUGGESTED ANSWER: (a)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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024 qmult 00600 1 1 2 easy memory: Gauss law cylindrical

3. For a cylindrically symmetric charge distribution, Gauss's law gives the magnitude of the electric field to be

a) $E = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\rm encl}}{2\pi\varepsilon_0 r}$, where $\lambda_{\rm encl}$ is linear charge density.
c) $E = \dfrac{\sigma_{\rm encl}}{2\varepsilon_0}$, where $\sigma_{\rm encl}$ is area charge density. d) $E = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r}$.
e) $V = \dfrac{q_{\rm encl}}{4\pi\varepsilon_0 r}$.

SUGGESTED ANSWER: (b)

Wrong answers:
e) This is for potential, not electric field.

Redaction: Jeffery, 2001jan01

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024 qmult 00900 1 1 3 easy memory: net charge on conductor
Extra keywords: Some day check the long answer for rightness.

4. Where is all the (net) charge on a conductor in an electrostatic situation?

a) Clumped in the middle.
b) Spread uniformly throughout.
c) Spread (in general non-uniformly) over the surface.
d) Clumped into two orbiting mass in the inside.
e) All at the bottom where gravity pulls it.

SUGGESTED ANSWER: (c)

Wrong answers:
e) The gravitational force on the excess charge is usually much, much less than the charge's mutual repulsion. Thus the effect of gravity is usually negligible. But there will be some tendency for the excess charge to be lower on the conductor surfaces because of gravity.

Redaction: Jeffery, 2001jan01

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024 qmult 00910 1 4 5 easy deducto-memory: Faraday cage

5. "Let's play *Jeopardy*! For \$100, the answer is: It is a conducting container used to shield the interior cavity from external electric fields. For perfect shielding, the

container must completely enclose the cavity and the external field must be static, but in practice a non-complete enclosure will shield against even fairly rapidly varying fields to some degree of adequacy."

What is a/an ____________, Alex?

a) Franklin stove b) Whitney cotton gin c) Arkwright spinning frame
d) Carnot engine e) Faraday cage

SUGGESTED ANSWER: (e) Michael Faraday (1791–1867) built a room-sized one in 1836.

Wrong answers:

a) Well, it would probably work as a Faraday cage. Say how did Ben miss the idea.

Redaction: Jeffery, 2008jan01

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024 qmult 00915 1 1 3 easy memory: 3 ball charge exchange

6. Say you had 3 **IDENTICAL** conducting balls. Call them A, B, and C. Initially, A has charge $8q$, B has charge $4q$, and C has charge $-2q$. All other matter and entities are at infinity relative to the balls. Step 1, you bring A and B into (electrical) contact while putting C at infinity and let electrostatic equilibrium be reached between A and B. Step 2, you move A to infinity, bring B into contact with C, and let electrostatic equilibrium be reached between B and C. Step 3, you move B to infinity, bring A into contact with C, and let electrostatic equilibrium be reached between A and C. What is the final charge on A?

a) 0. b) $-4q$. c) $4q$. d) $10q$. e) $6q$.

SUGGESTED ANSWER: (c)

The key word is **IDENTICAL**. When the balls are in contact symmetry dictates that the charge must flow to a symmetric electrostatic equilibrium. Thus, on every contact the balls end up sharing their initial total charge equally.

The charges on the balls are at each step are given in the following table:

Charges on the Ball at Each Step

Step	A	B	C
0	$8q$	$4q$	$-2q$
1	$6q$	$6q$	$-2q$
2	$6q$	$2q$	$2q$
3	$4q$	$2q$	$4q$

So A ends up with $4q$ on it.

What if you repeated the cycle of contacts over and over again? Well, every contact brings the charge amounts on ball closer to equality. The process of charge exchange can only stop when the amounts are equal. I guess that exact equality would only happen in the limit of infinite contacts for the given set of initial conditions. But I'd guess that close to exact equality should happen pretty quickly. An exact mathematical proof eludes me at the moment. Maybe if one needs to imagine a random series of contacts which give the same result on average as the ordered sequence of contacts. In any case, the random contact problem is the one I want to solve right now.

Say you had N identical balls that have total charge Q and mean mean charge $\bar{q}$. The Q and $\bar{q}$ values are constant and never change. You played the equalizing contact game with these balls. The two balls at each step were chosen at random with equal probability among the balls.

You started with some initial set of charges $\{q_{i,0}\}$. At some general point in the procedure, you had a set of charges $\{q_i\}$. The chi-square (per ball) for the sample is

$$\chi^2 = \frac{\sum_i (q_i - \bar{q})^2}{N} = \overline{q^2} - \bar{q}^2$$

(Bev-89), where the sum is over all charges. Then you do one contact between balls j and k. There is a change in the chi-square $\Delta\chi^2$ that satisfies

$$\begin{aligned} N\Delta\chi^2 &= 2\left[\frac{1}{2}(q_j + q_k) - q\right]^2 - (q_j - q)^2 - (q_k - q)^2 \\ &= 2\left[\frac{1}{4}(q_j + q_k)^2 + q^2 - (q_j + q_k)q\right] - q_j^2 - q^2 + 2q_j q - q_k^2 - q^2 + 2q_k q \\ &= \frac{1}{2}(q_j + q_k)^2 - q_j^2 - q_k^2 \\ &= -\frac{1}{2}(q_j - q_k)^2 \ . \end{aligned}$$

The last expression is always less than or equal to zero, and shows that a contact always decreases or leaves unchanged the chi-square. The unchanged case is only when $q_j = q_k$. The decrease/no-change result is true whether the balls are chosen in an ordered sequence or in a random way.

Now we do impose random choices and find the average value of $\Delta\chi^2$: i.e., $\langle\Delta\chi^2\rangle$. First, consider

$$(q_j - q_k)^2 = q_j^2 + q_k^2 - 2q_j q_k \ .$$

We choose a q_j at random for this expression and then a q_k at random and then average over q_k:

$$\begin{aligned} \langle(q_j - q_k)^2\rangle_k &= \langle q_j^2 + q_k^2 - 2q_j q_k\rangle_k \\ &= q_j^2 + \langle q_k^2\rangle_k - 2q_j\langle q_k\rangle_k \\ &= q_j^2 + \frac{\sum_{i,i\neq j} q_i^2}{N-1} - 2q_j\frac{\sum_{i,i\neq j} q_i}{N-1} \\ &= q_j^2 + \left(\frac{\sum_i q_i^2}{N-1} - \frac{q_j^2}{N-1}\right) - 2q_j\left(\frac{\sum_i q_i}{N-1} - \frac{q_j}{N-1}\right) \\ &= q_j^2 + \frac{N}{N-1}\overline{q^2} - \frac{q_j^2}{N-1} - 2q_j\left(\frac{N}{N-1}\bar{q} - \frac{q_j}{N-1}\right) \ . \end{aligned}$$

We now average the last expression over q_j which is the full averaging of $(q_j - q_k)^2$ over all possible pairs of q_j and q_k. Thus

$$\begin{aligned}\langle (q_j - q_k)^2\rangle &= \langle q_j^2 + q_k^2 - 2q_jq_k\rangle \\ &= \overline{q^2} + \frac{N}{N-1}\overline{q^2} - \frac{1}{N-1}\overline{q^2} - 2\left(\frac{N}{N-1}\bar{q}^2 - \frac{1}{N-1}\overline{q^2}\right) \\ &= 2\overline{q^2}\left(1 + \frac{1}{N-1}\right) - 2\bar{q}^2\left(\frac{N}{N-1}\right) \\ &= 2\overline{q^2}\left(\frac{N}{N-1}\right) - 2\bar{q}^2\left(\frac{N}{N-1}\right) \\ &= 2\chi^2\left(\frac{N}{N-1}\right) .\end{aligned}$$

Now we find that

$$\langle \Delta\chi^2\rangle = -\frac{\chi^2}{N-1} .$$

We can now average over the all χ^2 possibilities for the inital set of charges $\{q_{i,0}\}$ values to obtain

$$\langle \Delta\chi^2\rangle_{\text{full}} = -\frac{\langle\chi^2\rangle_{\text{full}}}{N-1} .$$

Let's now drop the averaging notation since we know we are dealing with a full averaging. Thus our result of the change in the average chi-square for one constant is

$$\Delta\chi^2 = -\frac{\chi^2}{N-1} .$$

Say the contact we have been considering is the ℓth contact. Then

$$\chi_\ell^2 = \chi_{\ell-1}^2 + \Delta\chi_\ell^2 = \chi_{\ell-1}^2\left(1 - \frac{1}{N-1}\right) ,$$

where we have used subscripts ℓ to indicate the number of contacts done. Finally, we obtain the explicit formula for the mean chi-square after the ℓth contact: i.e.,

$$\chi_\ell^2 = \chi_0^2\left(1 - \frac{1}{N-1}\right)^\ell ,$$

where χ_0^2 is the inital chi-square.

The formula does not handle the case of $N = 1$ obviously since it gives an indefinite. But that is understandable since we derived it tacitly assuming that there were at least two balls. In the case, of one ball there are no contacts to be made and no equalization.

For $N = 2$, the mean chi-square drops to zero after the first contact. This is just what we know happens for any initial setup. So the formula is consistent for $N = 2$.

For all other cases, the mean chi-square decreases for all ℓ, but only reaches zero at $\ell = \infty$. But for special initial conditions, the chi-square itself can reach zero for finite ℓ and can stay constant for awhile. For example, say all the balls initially had the same charge, except for two of them with charges whose mean was the same as the other charges. If the two odd balls contact first by chance, then they equalize and the chi-square drops to zero at once. On the other hand if any of the other two balls collide, there is no charge exchange and the chi-square stays constant. I rather think that as long as special initial conditions don't give rise to a zero chi-square for finite ℓ, the chi-square will eventually start declining like the mean chi-square, except perhaps for an offset: i.e., the decline is for $\ell - \ell_{\text{offset}}$, where $\ell_{\text{offset}} > 0$. I think this just because charge values will become somewhat randomized after some number of contacts even if they were initially rather ordered, unless the ordering is fine-tuned to dictate a decline to zero chi-square for finite ℓ.

One other illuminating trick is to make the continuum approximation for the antepenultimate equation:

$$\Delta\chi^2 = -\frac{\chi^2}{N-1}$$

changes to

$$\Delta\chi^2 = -\frac{\chi^2}{N-1}\Delta\ell$$

with $\Delta\ell = 1$ which changes to

$$d\chi^2 = -\frac{\chi^2}{N-1}dx$$

since x is a better continuum variable than ℓ. Now we can solve a differential equation to get

$$\chi^2 = \chi_0^2 e^{-x/(N-1)} \ .$$

This result shows that the actual decline in mean chi-square is approximately exponential.

Now what about an ordered sequence of contacts among charged balls. Not now, not ever.

Wrong answers:

a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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025 qmult 00110 1 4 5 easy deducto-memory: electric potential defined 2

7. What is electric potential? It is the electric potential energy:

a) **PER** unit point charge. It is a **VECTOR** field like the electric field.
b) **OF** a charge. It is a **VECTOR** field like the electric field.
c) **PER** unit point charge. It is a **TENSOR** field unlike electric charge.
d) **OF** of a charge. It is a **TENSOR** field unlike electric charge.
e) **PER** per unit point charge. It is a **SCALAR** field unlike the electric field.

SUGGESTED ANSWER: (e)

Note, we usually just say electric potential is potential energy per unit charge, but this actually as shorthand for electric potential is potential energy per unit point charge. This is so because electric potential is a scalar field that depends on position: i.e., $V = V(\vec{r})$. Thus, it clearly can only define potential energy at a single point $\vec{r}$. Of course, you could also define potential by the differential expression $dU = V(\vec{r})\,dq = V(\vec{r})\rho(\vec{r})\,d^3r$, where $\rho(\vec{r})$ is charge density and d^3r is differential volume. The two definitions are actually equivalent, but the second is longwinded.

Wrong answers:

c) Tensor fields are what you get beyond vector fields.

Description: An easy deducto-memory question.
Redaction: D.J. Jeffery, 2000oct01

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025 qmult 00150 1 1 2 easy memory: electric potential scalar addition

8. Since electric potential is a scalar it adds:

   a) like a vector. b) like a scalar. c) like a tensor. d) to zero always. e) not at all.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   a) Oh c'mon.

   Redaction: Jeffery, 2001jan01

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025 qmult 00220 1 4 1 easy deducto-memory: electronvolt calculation

9. If an electron is accelerated through a potential difference of 100 V, what kinetic energy does it acquire in eVs (electronvolts)?

   a) $100\,\text{eV}$. b) $0.01\,\text{eV}$. c) $1.6\times10^{-17}\,\text{eV}$. d) $1.6\times10^{-17}\,\text{J}$. e) $1.6\times10^{17}\,\text{eV}$.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   d) This is the answer in Joules.

   Redaction: Jeffery, 2001jan01

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025 qmult 00310 1 1 3 easy memory: point charge potential

10. The potential of a point charge (relative to infinity) is:

    a) $V = \dfrac{kq}{r^3}$. b) $V = \dfrac{kq}{r^2}$. c) $V = \dfrac{kq}{r}$. d) $V = kqr$. e) $V = kqr^2$.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    d) Oh c'mon.

    Redaction: Jeffery, 2001jan01

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025 qmult 00410 1 2 3 moderate memory: discrete point charge

11. The general formula to calculate the potential of a localized set of point charges (with infinity set to zero potential as is conventional) is:

a) $V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|^3}$. b) $V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|^2}$. c) $V(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|}$.
d) $V(\vec{r}) = \sum_i kq_i|\vec{r}-\vec{r}_i|$. e) $V(\vec{r}) = \sum_i kq_i|\vec{r}-\vec{r}_i|^2$.

SUGGESTED ANSWER: (c)

Wrong answers:
b) The potential is a $1/r$ law not a $1/r^2$ law like the electric field.

Redaction: Jeffery, 2001jan01

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025 qmult 00610 1 1 1 easy memory: gradient of potential

12. The electric field is given by

$$\vec{E} = -\nabla V \ ,$$

where in Cartesian coordinates

$$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = \sum_i \hat{x}_i \frac{\partial V}{\partial x_i}$$

is the ____________ of the potential.

a) gradient b) Laplacian c) divergence d) D'Alembertian
e) Einsteinium

SUGGESTED ANSWER: (a)

Wrong answers:
b) The Laplacian is the divergence of a gradient.
c) The divergence is operation on a vector field: e.g., the divergence of $\vec{E}$ is

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \ .$$

d) The D'Alembertian is the 4-dimensional version of the Laplacian (Ar-128).
e) Einsteinium is element 99 (HRW-A-12).

Redaction: Jeffery, 2001jan01

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025 qmult 00720 1 1 3 easy memory: set of charged conductors

13. Say that you had a localized set of charge conductors with charges $\{q_i\}$ and potentials $\{V_i\}$—and remember each conductor has only one potential since it is an equipotential object. It follows from

$$U = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\vec{r}$$

that the potential energy of the set is given by

a) $\frac{1}{2}\sum_i V_i$. b) $\sum_i q_i V_i$. c) $\frac{1}{2}\sum_i q_i V_i$. d) $\frac{1}{2}\sum_i q_i$. e) $\sum_i q_i$.

SUGGESTED ANSWER: (c)

Behold:

$$U = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\vec{r} = \frac{1}{2}\sum_i \int_i \rho(\vec{r})V_i\,d\vec{r} = \frac{1}{2}\sum_i q_i V_i \ ,$$

where the integrals subscripted i are just over the conductors: there is no charge elsewhere to integrate over. Although a cute formula, it has limited use. Given the charges on the conductors and the geometrical arrangement, you still need to do some work to calculate the V_i values. Of course, one could impose the potentials using some potential source by charging up the conductors simultaneously. This would be an easy thing to do approximately. The charging sources are kept far away and the charging is done by barely perturbing wires. But then you'd have the problem of what would the charges are?

There is special case of interest. Say there were only two conductors with charges q and $-q$. In this case,

$$U = \frac{1}{2}q\Delta V \ ,$$

where ΔV is the potential difference between the high and low potential conductor. This case is actually that of a capacitor in normal operation.

Wrong answers:
a) Dimensionally incorrect.

Redaction: Jeffery, 2008jan01

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026 qmult 00200 1 1 3 easy memory: capacitance geometry

14. The capacitance of a capacitor ideally depends only on:

a) its dielectric environment. b) its geometry. c) its geometry and its dielectric environment. d) the potential put on the capacitor. e) the charge put on the capacitor.

SUGGESTED ANSWER: (c)

Wrong answers:
b) Not the best answer.
d) Not at all.
e) Not at all.

Redaction: Jeffery, 2001jan01

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026 qmult 00300 1 1 4 easy memory: C=Q/V

15. The standard formula for capacitance with standard symbols is:

a) $C = V/Q$. b) $C = V^2/Q$. c) $C = Q^2/V$. d) $C = Q/V$. e) $C = Q - V$.

SUGGESTED ANSWER: (d)

Wrong answers:

e) Not even dimensionally possible.

Redaction: Jeffery, 2001jan01

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026 qmult 00500 1 1 4 easy memory: capacitor symbol

16. The standard circuit diagram symbol for a capacitor is ____.

a) ♀ b) ♅ c) $\odot$ d) $\dashv\vdash$ e) $\oplus$

SUGGESTED ANSWER: (d)

Wrong answers:

a) The Venus symbol approximately.
b) The Uranus symbol approximately. I can't seem to make the Mars symbol with TEX.
c) The Sun symbol.
e) The Earth symbol.

Redaction: Jeffery, 2001jan01

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026 qmult 00700 1 4 3 easy deducto-memory: capacitor formulae

17. The capacity formulae for the three ideal, high-symmetry, vacuum-separated capacitors—planar, cylindrical, and spherical—in order are:

a)

$$C = \frac{q}{V} \qquad C_{\rm eq} = \sum_i C_i \qquad \frac{1}{C_{\rm eq}} = \sum_i \frac{1}{C_i} \ .$$

b)

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad \varepsilon = \kappa\varepsilon_0 \qquad \vec E = \frac{q}{4\pi\kappa\varepsilon_0 r^2}\hat r \ .$$

c)

$$C = \frac{\varepsilon A}{d} \qquad C = \frac{2\pi\varepsilon L}{\ln(r_{\rm outer}/r_{\rm inner})} \qquad C = 4\pi\varepsilon\frac{r_{\rm inner}}{1 - r_{\rm inner}/r_{\rm outer}} \ .$$

d)

$$C = \frac{2\pi\varepsilon L}{\ln(r_{\rm outer}/r_{\rm inner})} \qquad C = \frac{\varepsilon A}{d} \qquad C = 4\pi\varepsilon\frac{r_{\rm inner}}{1 - r_{\rm inner}/r_{\rm outer}} \ .$$

e)

$$C = 4\pi\varepsilon\frac{r_{\rm inner}}{1 - r_{\rm inner}/r_{\rm outer}} \qquad C = \frac{\varepsilon A}{d} \qquad C = \frac{2\pi\varepsilon L}{\ln(r_{\rm outer}/r_{\rm inner})} \ .$$

SUGGESTED ANSWER: (c)

Note that ε is the permittivity of the dielectric between the capacitor plates. It is given by

$$\varepsilon = \kappa\varepsilon_0 \ ,$$

where κ is the dielectric constant and ε is the vacuum permittivity. Ideally,

$$C = \kappa C_0$$

where C_0 is the capacitance in the absence of the dielectric.

Note also that the Coulomb constant is given by

$$k = \frac{1}{4\pi\varepsilon_0} .$$

Wrong answers:

a) These are all good capacitor formulae to know: general expression, in parallel, in series.

b) Energy stored in electric field (in vacuum), the effective permittivity in a material with dielectric constant κ, and the electric field of a point macroscopic charge embedded in material of dielectric constant κ.

Redaction: Jeffery, 2001jan01

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026 qmult 00900 1 1 1 easy memory: capacitors in series

18. If you have a set of $\{C_i\}$ capacitances in series, then the equivalent capacitance $C_{\rm equ}$ is given by:

a) $1/C_{\rm equ} = \sum_i 1/C_i$. b) $1/C_{\rm equ} = \sum_i C_i$. c) $C_{\rm equ} = \sum_i 1/C_i$.
d) $1/C_{\rm equ} = \sum_i 1/C_i^2$. e) $C_{\rm equ} = \sum_i C_i$.

SUGGESTED ANSWER: (a)

Wrong answers:

b) Dimensionally inconsistent: i.e., the units don't work out.

e) This is the formulae for capacitors in parallel.

Redaction: Jeffery, 2001jan01

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026 qmult 01000 1 4 1 easy deducto-memory: E-field energy density

19. "Let's play *Jeopardy*! For \$100, the answer is: The energy density of the electric field in vacuum."

What is ____________, Alex?

a) $u = \frac{1}{2}\varepsilon_0 E^2$ b) $u = \frac{B^2}{2\mu_0}$ c) $u = -kx^2$ d) $u = \frac{1}{2}\varepsilon_0 E$
e) $u = \frac{1}{2}\varepsilon_0 E^3$

SUGGESTED ANSWER: (a)

Wrong answers:

b) This is the energy density of the magnetic field in vacuum (HRW-732).

c) The potential energy of a simple harmonic oscillator.

d) Dimensionally wrong.

Redaction: Jeffery, 2001jan01

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026 qmult 01100 1 1 1 easy memory: dielectric

20. By the word dielectric, one usually means a/an:

a) insulator considered as electrically active. b) conductor.
c) semi-conductor. d) superconductor. e) two-electric-field material.

SUGGESTED ANSWER: (a)

Wrong answers:
e) Sounds plausible, but there is no such term in use that I know of.

Redaction: Jeffery, 2001jan01

Full Answer Problems:

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026 qfull 00300 2 3 0 moderate math: energy stored in a capacitor

21. The potential energy stored in a capacitor is a function of the charge and its distribution only. It does not depend on how the distribution was assembled. The work done to assemble the distribution, however, equals this potential energy. So you can imagine any simple way of assembly you like as means of calculating the potential energy. Say that you just took positive charge from one plate little bit at a time and moved it to the other plate. You started from uncharged state and built up to charge q on the capacitor. The change in potential energy of the system due the transfer of dq' is

$$dU = V'\,dq' \ ,$$

where V' is the potential difference between the plates at the time of transferring dq'. What is the total potential energy of charging the capacitor to Q in terms of Q and C and in terms of final potential V and C? **HINT:** You need to do an integration.

SUGGESTED ANSWER:

Behold:

$$dU = V'\,dq' = \frac{q'}{C}\,dq' \ .$$

This can be integrated at once:

$$U = \int_0^Q \frac{q'}{C}\,dq' = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \ .$$

Redaction: Jeffery, 2001jan01

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025 qfull 00400 2 3 0 moderate math: potential of infinitely thin disk of charge

22. You have an infinitely thin disk of charge q with the charge spread uniformly over the disk of radius R.

a) What is the area charge density σ of the disk?

b) What is the differential charge on a ring in the disk of radius ρ and width $d\rho$, where ρ is the radius coordinate of the disk. raw a diagram of the system.

c) What is the potential dV at a height z above the disk along the axis of the disk due to a ring of charge of width $d\rho$. Remember potential is a scalar and adds like a scalar therefore. Assume the potential is zero at infinity as is conventional for a localizable charge distribution and replace σ by the part (a) result. Draw a diagram of the system.

d) Find the total potential at height z due to the whole ring.

e) Using Taylor's series to 1st order, find the far-field expression for the potential of the disk on the axis.

f) Using Taylor's series, find the first order in small z/R form of the solution for the potential: i.e., the approximate form the solution for being very close to the disk. Eliminate q in favor of σ and k in favor of ε_0. Recall,

$$k = \frac{1}{4\pi\varepsilon_0} \; .$$

g) What is the z-component of the electric field E_z for the small z/R case?

SUGGESTED ANSWER:

a) Behold: $\sigma = q/(\pi R^2)$.

b) Behold: $dq = \sigma(2\pi)\rho\, d\rho$.

c) The potential due to a differential bit of arc of the ring $ds = \rho\, d\theta$ is

$$dV = \frac{k\sigma\rho\, d\theta\, d\rho}{\sqrt{z^2+\rho^2}}$$

using the formula for the potential due to a point charge. Since potential is a scalar, we can integrate over the whole ring without more ado:

$$dV = \frac{k\sigma(2\pi)\rho\, d\rho}{\sqrt{z^2+\rho^2}} \; .$$

d) Behold:

$$V = \int_0^R \frac{k\sigma(2\pi)\rho}{\sqrt{z^2+\rho^2}}\, d\rho = \frac{2kq}{R^2}\sqrt{z^2+\rho^2}\bigg|_0^R = \frac{2kq}{R^2}\left(\sqrt{z^2+R^2}-z\right) \; .$$

e) Behold:

$$V = \frac{2kq}{R^2}z\left[\sqrt{1+\left(\frac{R}{z}\right)^2}-1\right] \approx \frac{2kq}{R^2}z\left[1+\frac{1}{2}\left(\frac{R}{z}\right)^2-1\right] = \frac{2kq}{R^2}z\left[\frac{1}{2}\left(\frac{R}{z}\right)^2\right]$$

$$V^{\rm 1st} = \frac{kq}{z} \; .$$

This is just the potential of a point charge. Once again a localized charge distribution will look like a point charge in the far-field limit.

f) Behold:

$$V = \frac{2kq}{R^2}\left(\sqrt{z^2+R^2}-z\right) = 2k\sigma\pi R\left[\sqrt{1+\left(\frac{z}{R}\right)^2}-\frac{z}{R}\right] \approx \frac{\sigma}{2\varepsilon_0}(R-z)$$

to 1st order in z/R.

g) Now

$$\vec{E} = -\nabla V = -\left(0, 0, -\frac{\sigma}{2\varepsilon_0}\right) ,$$

and so the z-component of the electric field (which is the only non-zero component) is

$$E_z = \frac{\sigma}{2\varepsilon_0} .$$

Not surprisingly this is the electric field about an infinite plane of charge.

Redaction: Jeffery, 2001jan01

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024 qfull 00220 1 3 0 easy math: forces between systems and system equivalents

23. In this problem, we consider the forces between two systems.

a) Consider two systems made of (classical) point particles: system 1 with particles labeled by index i and system 2 with particles labeled by index j. Given that the interparticle forces strictly obey Newtons 3rd law (i.e., $\vec{F}_{ij} = -\vec{F}_{ji}$), prove that the overall systems obey Newton's 3rd law: i.e.,

$$\vec{F}_{12} = -\vec{F}_{21} .$$

HINT: You will need to use a double summation $\sum_{ij}$.

b) Now say that you can replace (literally replace) system k (where k is either of 1 or 2) by system $k_{\rm eq}$ (where eq stands for equivalent) that exerts the same (total) force on any other as system k does: i.e.,

$$\vec{F}_{12} = \vec{F}_{1_{\rm eq},2} \qquad \text{and} \qquad \vec{F}_{21} = \vec{F}_{2_{\rm eq},1} .$$

Note, we are **NOT** saying the individual interparticle forces $\vec{F}_{ij}$ or $\vec{F}_{ji}$ are the same for equivalent systems, just their (total) forces are the same as the original system (total) forces. Note also, in general, the equivalent systems are **NOT** necessarily (classical) point systems, but they likely are in physically realistic cases.

Prove

$$\vec{F}_{12} = \vec{F}_{1_{\rm eq},2_{\rm eq}} .$$

HINT: The proof is short and you will need to use the 3rd law which the equivalent systems obey.

c) For the electrostatic/gravitational force, we know from the shell theorem that

$$\vec{F}_{12} = \vec{F}_{1_{\rm eq},2} ,$$

where system 1 has a spherically symmetric charge/mass distribution and its equivalent system is a (classical) point particle at the center of the distribution with the same total charge/mass. System 2 is completely general. However, assuming system 2 has the same properties as system 1, show what $\vec{F}_{12}$ is equal to in terms of the equivalent systems? Does result still hold if the systems touch or overlap as long as they both magically preserve spherical symmetry? **HINT:** You should make use of the part (b) result.

d) Will the systems of part (c) necessarily interact dynamically like classical point charges/masses provided they maintain their respective symmetries and no other sources of force are present?

e) Try to repeat the part (b) proof with the only change being now use

$$\vec{F}_{12} = \vec{F}_{1,2_{\rm eq}} \qquad \vec{F}_{21} = \vec{F}_{2,1_{\rm eq}} \ .$$

The change means the force system k exerts on the other system is the same as the force system k exerts on the other system's equivalent system. In fact, the required result is not generally true as emerges in the proof. What are the most obvious special cases in which the required result holds?

SUGGESTED ANSWER:

a) Behold:

$$\vec{F}_{12} = \sum_{ij} \vec{F}_{ij} = \sum_{ij} (-\vec{F}_{ji}) = -\sum_{ji} \vec{F}_{ji} = -\vec{F}_{21} \qquad \text{QED.}$$

b) Behold:

$$\vec{F}_{12} = \vec{F}_{1_{\rm eq},2} = -\vec{F}_{2,1_{\rm eq}} = -\vec{F}_{2_{\rm eq},1_{\rm eq}} = \vec{F}_{1_{\rm eq},2_{\rm eq}} \qquad \text{QED.}$$

c) Using the part (b) result, we know that

$$\vec{F}_{12} = \vec{F}_{1_{\rm eq},2_{\rm eq}} \qquad \text{QED.}$$

The result still holds if the systems touch or overlap as long both preserve spherical symmetry—which would be very hard to arrange in physical reality.

d) The spherically symmetric charge systems will not necessarily act interact dynamically like point charges since the centers of charge are not necessarily the centers of mass. However, the spherically symmetric mass systems will since will necessarily act interact dynamically like point masses since the centers of mass are the centers of mass. Recall the dynamical interaction is governed by Newton's 2nd law

$$\vec{F}_{\rm net} = m\vec{a}_{\rm cm} \ ,$$

where $\vec{F}_{\rm net}$ is the net force on a system, m is the system mass, and $\vec{a}_{\rm cm}$ is the system center of mass.

e) Behold:

$$\vec{F}_{12} = \vec{F}_{1,2_{\rm eq}} = -\vec{F}_{2_{\rm eq},1} = -\vec{F}_{2_{\rm eq},1_{\rm eq}} = \vec{F}_{1_{\rm eq},2_{\rm eq}} \ .$$

But note $\vec{F}_{2_{\rm eq},1} = \vec{F}_{2_{\rm eq},1_{\rm eq}}$ is not true in general given the conditions, and so the required result is not true in general given the conditions. Only in special cases is the required result true. In fact, the most obvious case is when the conditions of part (b) also hold. To show that explicitly,

$$F_{2_{\rm eq},1} = -\vec{F}_{1,2_{\rm eq}} = -\vec{F}_{1_{\rm eq},2_{\rm eq}} = F_{2_{\rm eq},1_{\rm eq}} \ .$$

Note, $\vec{F}_{12} = \vec{F}_{1,2_{\rm eq}}$ holds for when $\vec{F}_{12}$ is the net force on system in which case the system 2 equivalent is the system 2 center of mass. So the obvious cases where the required result holds is for two spherically symmetric charge distributions where the center of charge is also the center of mass and two spherically symmetric mass distributions. Of course, we discussed these cases in part (d).

Redaction: Jeffery, 2008jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

---

1 Constants

$$
\begin{aligned}
c &= 2.997\ 924\ 58 \times 10^{8}\,\mathrm{m/s} \approx 2.998 \times 10^{8}\,\mathrm{m/s} \approx 3 \times 10^{8}\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \\
&\quad \text{exact by definition} \\
e &= 1.602\ 176\ 634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition} \\
G &= 6.674\ 30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987\ 551\ 785\ 972(14) \times 10^{9} \approx 8.99 \times 10^{9} \approx 10^{10}\,\mathrm{N\,m^2/C^2} \\
k_{\mathrm{Boltzmann}} &= 1.380\ 649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\ 333\ 262 \times 10^{-4}\,\mathrm{eV/K} \\
&\approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms} \\
m_e &= 9.109\ 383\ 7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\ 998\ 950\ 69(16)\,\mathrm{MeV} \\
m_p &= 1.672\ 621\ 923\ 95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\ 089\ 32(29)\,\mathrm{MeV} \\
\varepsilon_0 &= 8.854\ 187\ 8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity} \\
\mu_0 &= 1.256\ 637\ 061\ 27(20)\,\mathrm{N/A^2} \approx \pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}
\end{aligned}
$$

---

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

---

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

---

4 **Approximation Formulae**

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

---

5 **Quadratic Formula**

If $\quad 0 = ax^2 + bx + c\ ,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b + \mathrm{sgn}(b)\sqrt{b^2-4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

---

6 **Vector Formulae**

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\dfrac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\mathrm{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

---

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x - x_0)f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

---

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\qquad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

---

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

---

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\ \text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\ \text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin\gamma_L \qquad \tau_z = RF_{xy} \sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta K_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}}\, d\theta \qquad \Delta U_{\text{rot}} = -W = -\int \tau_{z,\text{con}}\, d\theta$$

$$\Delta E_{\text{rot}} = K_{\text{rot}} + \Delta U_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta K + K_{\text{rot}} + \Delta U = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736{\times}10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707 \times 10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^8 \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}}g = V_{\text{fluid dis}}\rho_{\text{fluid}}g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\text{max}}\sin[k(x - vt)] = y_{\text{max}}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\text{max}}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\text{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\text{K}} = T_{\text{C}} + 273.15\,\text{K} \qquad T_{\text{F}} = 1.8 \times T_{\text{C}} + 32^\circ\text{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

25 **Electrostatics**

$$\vec{F}_{1 \text{ on } 2} = \frac{kq_1 q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_{\text{i}}^{\text{f}} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_i V_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J} \cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma \vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R} e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}} \left(1 - e^{-t/\tau}\right)$$

27 Magnetic Fields and Forces

$$\mu_0 = 1.256\,637\,061\,27(20)\,\text{N/A}^2 \approx \pi \times 10^{-7}\,\text{N/A}^2 \qquad 1\,\text{N/A}^2 = 1\,\text{T}\cdot\text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad d\vec{F} = I\,d\vec{s} \times \vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I \theta}{4\pi r} \qquad \vec{F}_{1\text{ on }2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r} \hat{r}_{2\text{ to }1}$$

$$B_{\text{solenoid}} = \mu_0 n I \qquad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

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28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2} L I^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con-axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0 \cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_{\pm} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0 \cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right] e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

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29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0 \cos(\omega t) \qquad I = I_0 \cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2} I_0 V_0 \cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \qquad I_{\text{p}} N_{\text{p}} = I_{\text{s}} N_{\text{s}} \qquad I_{\text{p}} \mathcal{E}_{\text{p}} = I_{\text{s}} \mathcal{E}_{\text{s}} \qquad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

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30 Electromagnetic Radiation (EMR)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}}\cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2}I_{\text{inc}}$$