Introductory Physics II: Calculus-Based: Exam 1 Name:

Instructions: There are 20 multiple-choice problems worth 1 mark each for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized.

There are **THREE** full-answer problems worth 10 marks each for a total of 30 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may extend over multiple-pages: make sure you have answered everything. And **BOX-IN** your final answers. There are 2 scratch pages for additional calculations or to rewrite answers if needed. In the latter case, make clear which answer is to be marked. You can ask for more paper if needed.

This is a **CLOSED-BOOK: CLOSED-NOTES: NO-ONLINE-ACCESS: NO-AI** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 50 marks altogether and is a 60-minute exam.

Multiple Choice Questions:

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017 qmult 00110 1 4 1 easy deducto-memory: 3 kinds of waves

Extra keywords: physci

1. In one method of physically classifying waves, one has:

   a) mechanical, electromagnetic, and quantum mechanical waves.
   b) left, right, and middle waves.
   c) S, P, and Middle Earth waves.
   d) American waves, British waves, and Australian wives.
   e) mechanical, financial, and emotional waves.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   e) Not the best answer in the context of physics.

   Redaction: Jeffery, 2001jan01

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017 qmult 00150 1 1 4 easy memory: general traveling wave solution

2. The linear wave equation for 1-dimensional wave motion

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

   has the semi-general solution $y = f(x - v_{\pm}t)$, where $v_{\pm} = \pm v$ is the phase velocity and v is the phase speed. (It is only semi-general since a general solution requires a linear combination of the v_+ and v_- cases.) The solution is a traveling wave solution. Let $\phi = x - v_{\pm}t$. For a constant ϕ, one has a constant oscillation deviation $f(\phi)$. But this deviation's x position changes as ____________ with velocity ____________. Since ϕ is called the phase of the wave solution, the name for phase velocity $v_\pm$ follows. A wave solution of the form $f(x - v_\pm t)$ is just a pattern that is traveling to the positive for v_+ and to the negative for v_-. If $v^2 = 0$, the wave equation is not defined and there is no solution. But there can be solutions that do not travel.

a) $\phi - v_\pm/t$; $v_\pm$. b) $\phi + v_\pm/t$; $-v_\pm$. c) $\phi v_\pm t$; $-v_\pm$. d) $\phi + v_\pm t$; $v_\pm$.
e) v; $\phi - v_\pm t$.

SUGGESTED ANSWER: (d)

Since $\phi = x - v_\pm t$ is a constant, we find the location of the deviation $f(\phi)$ evolves according to $x = \phi + v_\pm t$: i.e., the location is a linear function of time.

Wrong answers:
a) As Lurch would say AAAAaarrrgh.

Redaction: Jeffery, 2008jan01

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017 qmult 00222 1 1 4 easy memory: wavelength frequency relation
Extra keywords: physci KB-212-9

3. The higher the frequency of a wave, the:

a) remoter its source. b) grander its bearing. c) lower its pitch.
d) shorter its wavelength. e) closer its receiver.

SUGGESTED ANSWER: (d)

Wrong answers:
b) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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017 qmult 00232 1 1 4 easy memory: wavenumber wavelength, angular frequency frequency

4. A sinusoid's shape repeats every time its argument increases by 2π. Thus, sinusoidal waves repeat as spatial coordinate alone varies by $|k|\Delta x = 2\pi$ and as time coordinate alone varies by $\omega\Delta t = 2\pi$ Immediately, one sees that:

a) $|k| = \lambda$ and $f = \omega$. b) $|k| = \pi\lambda$ and $f = \pi\omega$.
c) $|k| = 2\pi\lambda$ and $f = 2\pi\omega$. d) $|k| = 2\pi/\lambda$ and $f = \omega/(2\pi)$.
e) $|k| = f$ and $f = \lambda$.

SUGGESTED ANSWER: (d)

Wrong answers:
a) This is the easy answer.
c) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00240 1 1 3 easy memory: sinusoidal waves and phase velocity

5. Sinusoidal waves, given by

$$y = A\cos(kx - \omega t + \phi) \qquad \text{or} \qquad y = B\cos(kx - \omega t) + C\sin(kx - \omega t) \ ,$$

satisfy the linear wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \ .$$

This can be proven by direct subsitution into the linea wave equation or by recognizing the sinusoidal wave function is of the form of the semi-general general wave solution of the linear wave equation: i.e., $f(x - v_{\pm})$ where $v_{\pm} = \pm v$ is the phase velocity and v is the phase speed. Either way we find the relationship between phase speed v and k and ω to be:

a) $v = \omega k^2 = f\lambda$. b) $v = \omega|k| = f\lambda$. c) $v = \omega/|k| = f\lambda$.
d) $v = |k/|\omega = f\lambda$. e) $v = \omega/k^2 = f\lambda$.

SUGGESTED ANSWER: (c)

Wrong answers:
b) Exactly wrong.

Redaction: Jeffery, 2008jan01

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017 qmult 00620 1 4 3 easy deducto-memory: what's an antinode?

6. What is an antinode?

a) A point of no motion in standing waves.
b) A point of minimum amplitude in standing waves.
c) A point of maximum amplitude in standing waves.
d) That which proceeds a node.
e) That which follows a node.

SUGGESTED ANSWER: (c)

Wrong answers:
a) That's a node.
b) Nah.
d) Sounds plausible.
e) Not even specious.

Redaction: Jeffery, 2001jan01

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018 qmult 00100 1 4 2 easy deducto-memory: sound waves defined

7. "Let's play *Jeopardy*! For $100, the answer is: They are pressure waves in a medium that can be any of gas, liquid, and solid. In gases and fluids, the waves can only be longitudinal. In solids, tranverse waves can occur."

What are ____________, Alex?

a) gravitational waves b) sound waves c) water waves
d) seismic waves e) quantum mechanical waves

SUGGESTED ANSWER: (b)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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018 qmult 00120 1 1 1 easy memory: sound wave longitudinal I

8. Sound waves in fluids are always:

   a) longitudinal. b) latitudinal. c) transverse. d) reverse. e) curse.

   SUGGESTED ANSWER: (a)

   Wrong answers:

   e) "Curses, foiled again." Who used to say that? My memory is going.

   Redaction: Jeffery, 2001jan01

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018 qmult 00200 1 4 3 easy deducto-memory: linear wave equation for sound

9. "Let's play *Jeopardy*! For \$100, the answer is: For the 1-dimensional case,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} ,$$

   where the phase speed is given by

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} ,$$

   where S means the derivative is taken with entropy held constant or is with the adiabatic condition.

   What is the ____________, Alex?

   a) Shrödinger equation b) linear wave equation for electromagnetic radiation
   c) linear wave equation for sound d) Dirac equation
   e) non-linear wave equation for sound

   SUGGESTED ANSWER: (c)

   Wrong answers:

   a) This is quantum mechanical wave equation.
   a) In classical electromagnetism, electromagnetic waves do obey the linear wave equation the speed of light is the phase velocity (Wikipedia: Electromagnetic wave equation).
   d) This is relativistic quantum mechanical wave equation.

   Redaction: Jeffery, 2008jan01

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018 qmult 00700 1 4 1 easy deducto-memory: Doppler effect defined

Extra keywords: physci

10. The Doppler effect is a/an:

   a) shift in wave frequency caused by the motions of sources and receivers.
   b) effect of wave frequency on the motion of sources and receivers.
   c) decrease of intensity as the inverse square of distance from a point source of sound or light.

d) cause of entropy.
e) cause of heat capacity.

SUGGESTED ANSWER: (a)

Note, for sound, the Doppler effect does not just depend on the relative motion of source and observer in general. It does depend on their absolute motions relative to the medium in general and only in the low velocity limit for both source and observer does it depend on relative motion of source and observer. A key point is that the wavelength of a set of waves is observer frame independent while frequency and phase velocity are observer frame dependent. To be clear, all observers observing the same set of waves in one place and time observe the same wavelength. However, the wavelength of sound from a source does depend on the velocity of the source relative to the medium and the direction emission. The motion of observer has no effect on wavelength, except for source observer.

Wrong answers:
b) A cart-before-the-horse answer.

Redaction: Jeffery, 2001jan01

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018 qmult 00710 1 1 2 easy memory: Doppler effect generally
Extra keywords: physci KB-212-11

11. The Doppler effect occurs ____________ waves:

a) only for sound b) for many kinds of c) only for light d) only for transverse e) for no kinds of

SUGGESTED ANSWER: (b) Actually, I think that some kind of Doppler effect can happen for all kinds of waves. But there may be some weird exception of some kind.

Wrong answers:
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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022 qmult 00100 1 4 3 easy deducto-memory: clasical electromagnetism and Maxwell's equations

12. "Let's play *Jeopardy*! For $100, the answer is: The equations that summarize most of classical electromagnetism are

$$1)\quad \nabla\cdot\vec E=\frac{\rho}{\varepsilon_0}\qquad 2)\quad \nabla\cdot\vec B=0\qquad 3)\quad \nabla\times\vec E=-\frac{\partial\vec B}{\partial t}\qquad 4)\quad \nabla\times\vec B=\mu_0\vec J+\mu_0\varepsilon_0\frac{\partial\vec E}{\partial t}$$

which are here givn in their the vacuum form. To complete summary, one also needs the Lorentz force formula

$$\vec F=q\left(\vec E+\vec v\times\vec B\right)\ .$$

What are ____________, Alex?

a) Ehrenfest's equations b) Friedmann's equations
c) Maxwell's equations d) Maxwell's relations
e) the Schrödinger and Dirac equations

SUGGESTED ANSWER: (c)

Wrong answers:
b) As Lurch would say AAAARGH.
d) Yes, they exist and are not the same as Maxwell's equations.

Redaction: Jeffery, 2008jan01

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022 qmult 00160 1 1 5 easy memory: Ben Franklin named charges
Extra keywords: physci KB-137
13. Who named positive and negative electric charge?

a) George Washington (1732–1799). b) John Adams (1735–1826).
c) Benedict Arnold (1741–1801). d) Aaron Burr (1756–1836).
e) Benjamin Franklin (1706–1790).

SUGGESTED ANSWER: (e) KB-137 and WP-513 confirm it was old Ben.

Wrong answers:
c) Oh, c'mon.

Redaction: Jeffery, 2001jan01

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022 qmult 00210 1 4 5 easy memory: no microscopic charge neutralization
Extra keywords: physci
14. Why don't positive and negative charge largely neutralize each other at the microscopic level just like they mostly do at the macroscopic level? The short answer is it is forbidden in most circumstances by ____________.

a) classical physics b) James Clerk Maxwell c) Michael Faraday
d) Ben Franklin e) quantum mechanics

SUGGESTED ANSWER: (e): Quantum mechanics is formulated in terms of differential equations and a lot of other rules that have been found to work. When you have elementary positive and negative charges in proximity and they don't have the energy to escape each other and they don't annihilate each other or undergo some other metamorphosis (which require special circumstances), they can form stable, charge-separated structures (e.g., atoms) according to quantum mechanical rules.

Wrong answers:
a) No it doesn't and the stability of atoms began to puzzle people at the end of the 19th century.

Redaction: Jeffery, 2001jan01

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022 qmult 00400 1 1 4 easy memory: conduction classes
Extra keywords: physci
15. There are four main charge conduction categories for materials:

a) insulator, conductor, demiconductor, orchestra conductor.
b) insulator, conductor, semiconductor, infraconductor.
c) insulator, conductor, semiconductor, Superman-conductor.
d) insulator, conductor, semiconductor, superconductor.
e) insulator, conductor, semiconductor, demisemiconductor.

SUGGESTED ANSWER: (d) We usually think only of metals as conductors. But ionic fluids and ionized gases can also conduct. Ionic fluids can be pure fluids, but the more strongly conducting ones have solutes in them.

Wrong answers:
a) Orchestra conductor?
c) The last one on trains is a very difficult man get away with fooling.

Redaction: Jeffery, 2001jan01

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022 qmult 00640 1 1 2 easy memory: electric and gravity force inverse-square
Extra keywords: physci KB-140 sort of

16. The electrostatic force and gravity are both ____________ force laws.

a) inverse-linear b) inverse-square c) inverse-cube d) linear
e) quadratic

SUGGESTED ANSWER: (b)

Wrong answers:
d) This is the Hooke's law force or the linear restoring force: $F = -kx$.

Redaction: Jeffery, 2001jan01

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023 qmult 00150 1 1 3 easy memory: electric field line
Extra keywords: physci

17. A curve through space that is tangent to the electric field vector at each point and points in the direction of the electric field vector is a/an:

a) streamline. b) stream curve. c) electric field line.
d) magnetic field line. e) straight line always.

SUGGESTED ANSWER: (c)

Wrong answers:
a) Streamlines turn up in fluid dynamics.
e) Electric field lines emanating from a point charge are straight lines. But in general electric field, they are curved.

Redaction: Jeffery, 2001jan01

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023 qmult 00202 1 1 3 easy memory: electric force and a charge distribution

18. Say that you have a charge distribution of classical point charges with net charge q in an external electric field $\vec{E}$. Since it is external, $\vec{E}$ does not include contributions from the charge distribution. Also $\vec{E}$ is **NOT** necessarily uniform. The electric force

on the distribution due to $\vec{E}$ is

$$\vec{F} = \sum_i q_i \vec{E}_i \ ,$$

where q_i is the charge of one member of the distribution and $\vec{E}_i$ is the external electric at the location of charge q_i. The force $\vec{F}$ is ____________ on the distribution and in the absence of any other net external force dictates ____________ of the distribution.

a) the net electric force; individual charge accelerations
b) not the net electric force; center-of-mass acceleration
c) the net electric force; center-of-mass acceleration
d) not the net electric force; individual charge accelerations
e) half the net electric force; individual charge accelerations

SUGGESTED ANSWER: (c)

The internal electrical forces cancel out pairwise and so the external electric field on the distribution is the net electrical field.

Do all the internal forces cancel out pairwise. By Newton's 3rd law they should and we assume that law is obeyed in many cases. But actually Newton's 3rd law is **NOT** always obeyed between matter particles. This is a fine point that is often skipped over in intro physics classes. Magnetic forces can violate it for swarm of free-flying charges for example (Go-8). But nonetheless once one includes the mass and momentum of the electromagnetic field in the total mass and momentum of the system, it still turns out to be true that $\vec{F}_{\text{net external force}} = ma_{\text{CM}}$—or at least so I believe.

Now what about real point charges? What of their internal forces? Well maybe they don't really exist. But the electron is a point charge as far as we know. So what about the electrons internal electric forces? Are they infinite? There's a special place on the far side of the River Styx for people who ask such questions.

Wrong answers:
e) Half?

Redaction: Jeffery, 2008jan01

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023 qmult 00230 1 4 4 easy deducto-memory: E-field for arbitrary charge distribution

19. "Let's play *Jeopardy*! For \$100, the answer is: The formula for a discrete set of charges is

$$E(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \ ,$$

where the sum is over all charges q_i, $\vec{r}_i$ is the charge location, and $\vec{r}$ is the point of evaluation."

What is the discrete charge electric field formula for ____________, Alex?

a) an overall neutral charge distribution
b) a point charge
c) a dipole charge distribution.
d) an arbitrary charge distribution
e) a huge charge distribution

SUGGESTED ANSWER: (d)

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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023 qmult 00330 1 1 3 easy memory: dipole far-field

20. The far-field electric field of an electric dipole falls off as:

a) $1/r$. b) $1/r^2$. c) $1/r^3$. d) $1/r^4$. e) $1/r^5$.

SUGGESTED ANSWER: (c) The far-field electric field of an electric dipole falls off as: $1/r^3$.

Wrong answers:

a) A line charge.
b) A point charge or monopole.
d) A quadrupole.
e) An octupole.

Redaction: Jeffery, 2001jan01

Full Answer Problems:

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017 qfull 00330 1 3 0 easy math: a sinusoidal traveling wave

21. The equation of a transverse wave on a string is

$$y(x,t) = 9.0\sin(-0.01\pi x - 4.0\pi t) \ ,$$

where amplitude and x are centimeters, t is in seconds, and the argument of the sine is in radians. **HINT:** Do not forget the π values in calcutaling results and give results with correct units. Note, the standard form for a 1-dimensional wave solution adopted by the author is

$$y(x,t) = A\sin(kx - \omega t) \ ,$$

where ω is always taken as positive.

Find the (a) amplitude A, (b) wavelength $\lambda = 2\pi/|k|$, (c) frequency f (which is always positive), (d) direction of propagation along the x axis (which is determined by the sign of k) (e) phase velocity $v_{\rm ph} = \omega/k$ (which can be positive or negative), and (f) maximum oscillation speed of the string (i.e., its vertical maximum speed). Also (g), what is the oscillation displacement at $x = 3.5\,\text{cm}$ and time $t = 0.26\,\text{s}$?

SUGGESTED ANSWER:

a) $A = 9.0\,\text{cm}$.

b) $\lambda = 2\pi/|k| = 2\pi/(0.01\pi) = 200\,\text{cm}$.

c) $f = \omega/(2\pi) = (4\pi/2\pi) = 2.0\,\text{Hz}$.

d) Since $k = -0.01 < 0$, the direction of propagation is in the negative x-direction.

e) Behold:

$$v_{\rm ph} = \frac{\omega}{k} = \frac{4.0\pi}{(-0.01\pi)} = -400\,{\rm cm/s} \ .$$

f) $v_{y,\rm max} = \omega A = 4.0\pi \times 9.0 \approx 110\,{\rm cm/s}$. To better accuracy, $v_{y,\rm max} = 113\,{\rm cm/s}$.

g) Behold:

$$\begin{aligned} y(x=3.5,t=0.26) &= 9.0\times\sin(-0.035\pi-1.0400\pi) = 9.0\times\sin(-1.075\pi) \\ &= -9.0\times\sin(1.075\pi) = -9.0\times\sin(\pi+0.075\pi) \\ &= -9.0\times[\sin(\pi)\cos(0.075\pi)+\cos(\pi)\sin(0.075\pi)] \\ &= 9.0\times\sin(0.075\pi)\approx 9.0\times(0.075\pi)\approx 28.5\times0.075\approx 2.1\,{\rm cm} \end{aligned}$$

to about 1-digit accuracy. To better accuracy, $y(x=3.5,t=0.26) = 2.10\,{\rm cm}$.

Note: To go beyond the required answer for part (e), we will prove that $v_{\rm ph}$ is the velocity at which the shape of the wave propagates and that the sign of k determines the propagation. There are multiple ways of doing the proof. We will do a proof that is cogent to the cognoscienti, but rather obscure to intro physics students. Say that we follow y on path through $x(\ell)$ and $t(\ell)$ space where ℓ is a path parameter: i.e., $x(\ell)$ and $t(\ell)$ are functions of ℓ. In general, we could follow any path, but we want to follow a path where y is constant. Therefore along the whole path, we require

$$0 = \frac{dy}{d\ell} = \frac{dy}{d\phi}\frac{\partial\phi}{\partial x}\frac{dx}{d\ell} + \frac{dy}{d\phi}\frac{\partial\phi}{\partial t}\frac{dt}{d\ell} = \frac{dy}{d\phi}\left(k\frac{dx}{d\ell} - \omega\frac{dt}{d\ell}\right) \ ,$$

where we have used the chain rule, partial differentiation, and the fact that y is a function of ϕ alone in a direct sense (i.e., that $y = y(\phi)$) where phase $\phi = kx - \omega t$. Nothing forbids us from making t itself the path parameter and then we find

$$k\frac{dx}{dt} = \omega$$

which implies that $x(t)$ (i.e., x along the path of constant y) has velocity

$$v_{\rm ph} = \frac{\omega}{k} \ .$$

This velocity is a constant and it is the same for all y: these facts actually do require a proof for cogency. Thus, the whole shape of the wave propagates with velocity $v_{\rm ph}$ and (since $\omega > 0$ always) $v_{\rm ph}$ always has the sign of k. Of course, $v_{\rm ph}$ is what we call the phase velocity.

Fortran-95 Code

```
pi=3.1415926535897932384626433832795028841971693993751O_np
     !
!!23456789a123456789b123456789c123456789d123456789e123456789f123456789g12
```

```
    !           !
https://en.wikipedia.org/wiki/Pi#Approximate_value_and_digits 51
digits
          v=4.0_np*pi*9.0_np
          x=3.5_np
          t=0.26_np
          arg=(-0.01_np*x-4.0_np*t)
          y=9.0_np*sin(arg*pi)
          print*,'pi,v,x,t,y'
          print*,pi,v,x,t,arg,y
    !    3.1415926535897932385         113.09733552923255659
3.5000000000000000000
    !        0.25999999999999999999        -1.0749999999999999999
2.1010082747031487
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Redaction: Jeffery, 2001jan01

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017 qfull 00510 1 3 0 easy math: interference of two solutions

22. In this problem, we consider the interference of two basis solutions of the linear wave equation. The two solutions are

$$y_1 = A_1 \sin(k_1 x - \omega_1 t + \phi_1) \qquad \text{and} \qquad y_2 = A_2 \sin(k_2 x - \omega_2 t + \phi_2) \ ,$$

where the A's are amplitudes, the k's are wavenumbers, the ω's are angular frequencies, and the ϕ's are phases. Note, $\omega_i = k_i v_i$, where the v_i is the phase velocity for which there are only two possibilities $v_i = v$ and $v_i = -v$, where v is the phase speed.

Note, the linear wave equation is a linear differential equation, and so the sum of solutions is a solution, but in general not a basis solution.

Note, there are no numbers in this problem in order to be general, and so all answers are qualitative or formulae.

a) Assume A_1 and A_2 are both A. Determine $y = y_1 + y_2$ in the form of a product of factors of trigonometric functions. **HINT:** You will need the trigonometric identity

$$\sin(u) + \sin(v) = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

(Wikipedia: List of trigonometric identities: Sum-to-product identities). Note, creating product solutions from linear combinations of solutions (as in this problem) can only be done in very few special cases. But those special cases are very useful for insight. However, the two factors (i.e., the two factor trigonometric functions) are not necessarily themselves solutions of the linear wave equation as we elucidate in part (d).

b) Note, the effective wavelengths for the two factors (i.e., the two factor trigonometric functions) are given by

$$|k_1 + k_2| = \frac{2\pi}{\lambda_{\text{additive case}}} \qquad \text{and} \qquad |k_1 - k_2| = \frac{2\pi}{\lambda_{\text{subtractive case}}} \ ,$$

and therefore the effective slow-varying-scale and fast-varying-scale wavenumbers are, respectively,

$$k_{\text{slow}} = \min(|k_1 + k_2|, |k_1 - k_2|) \qquad \text{and} \qquad k_{\text{fast}} = \max(|k_1 + k_2|, |k_1 - k_2|) \ .$$

Draw rough a diagram illustrating the product solution as a function of x. How would you describe the relationship of the slow-varying-scale factor to the fast-varying-scale factor?

c) Give formulae for the highest and lowest absolute value angular frequency of oscillations at a fixed point in space for the factor functions: label these, respectively, ω_{high} and ω_{low}. Note, the factors can formally have negative angular frequencies, but here we are only interested in the highest and lowest oscillation rate without regard to the formal sign of their angular frequencies. Recall, for basis solutions of the linear wave equation the angular frequencies are always defined as positive quantities, but for the factors they can be positive or negative. **HINT:** You will need to use the maximum function (e.g., $2 = \max(1, 2)$ and the minimum function (e.g., $1 = \min(1, 2)$).

d) Define effective phase velocities for the two factors by

$$v_{\text{additive case}} = \frac{\omega_1 + \omega_2}{k_1 + k_2} = \frac{k_1 v_1 + k_2 v_2}{k_1 + k_2} = \begin{cases} v_1 & \text{for } v_2 = v_1; \\ \left(\dfrac{k_1 - k_2}{k_1 + k_2}\right) v_1 & \text{for } v_2 = -v_1; \end{cases}$$

$$v_{\text{subtractive case}} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{k_1 v_1 - k_2 v_2}{k_1 - k_2} = \begin{cases} v_1 & \text{for } v_2 = v_1; \\ \left(\dfrac{k_1 + k_2}{k_1 - k_2}\right) v_1 & \text{for } v_2 = -v_1. \end{cases}$$

Write the product solution in terms of $x - v_{\text{additive case}}t$ and $x - v_{\text{subtractive case}}t$.

In which direction do factors travel depending on the sign of $v_{\text{additive case}}$ and $v_{\text{subtractive case}}$. (**HINT:** The travel is to positive/negative if the sign is positive/negative). In what direction do the factors travel relative to the direction implied by v_1 (**HINT:** In some cases, you may not have enough information to answer since you have no values for k_1 and k_2.) When are the factors also solutions of the linear wave equation and when are they not? (**HINT:** To be a solution, the absolute value of an effective phase velocities has to be the actual phase speed v.) itemitem What does it mean if $k_1 + k_2$ or $k_1 - k_2$ is zero?

e) Write the formula for the product solution for the special case where $k = k_1 = k_2$ and $\omega = \omega_1 = \omega_2$. How would you describe the cases where and $\phi_1 - \phi_2 = 0$ and $\phi_1 - \phi_2 = \pi$?

f) Write the formula for the product solution for the special case where $k_2 = -k_1$ and relabel k_1 as k. Given the condition on k_1 and k_2, we know from general formula $k_\pm = v_\pm \omega$ that $\omega_2 = \omega_1$. Relabel ω_1 as ω in the formula. In order to make the formula look clean, we choose the phases to obey $\phi_1 - \phi_2 = \pi$ and $\phi_1 + \phi_2 = 4\pi$ (which implies $\phi_1 = (5/2)\pi$ and $\phi_2 = (3/2)\pi$, but this implication is of no interest). Then use the trigonometric identity $\sin(u + v) = \sin(u)\cos(v) + \cos(u)\sin(v)$ and the result $\cos(\pi) = -1$ to replace the cosine function by a sine function. Then simplify as much as possible.

What is the resulting solution called wave-motion jargon? What does the solution require for the basis that were originally summed to construct it.

SUGGESTED ANSWER:

a) Behold:

$$y = 2A \sin\left[\frac{(k_1+k_2)x - (\omega_1+\omega_2)t + \phi_1 + \phi_2}{2}\right] \cos\left[\frac{(k_1-k_2)x - (\omega_1-\omega_2)t + \phi_1 - \phi_2}{2}\right] .$$

b) You will have to imagine the diagram. The slow-varying-scale factor will appear as an envelope to the fast-varying-scale factor. The envelope appearance will be more pronounced the greater the difference between k_{slow} and k_{fast}

c) Behold:

$$\omega_{\text{high}} = \max(|\omega_1+\omega_2|, |\omega_1-\omega_2|) \qquad \text{and} \qquad \omega_{\text{low}} = \min(|\omega_1+\omega_2|, |\omega_1-\omega_2|) .$$

d) When you write the product solution

$$y = 2A \sin\left[\frac{(k_1+k_2)(x - v_{\text{additive case}}t) + \phi_1 + \phi_2}{2}\right] \cos\left[\frac{(k_1-k_2)(x - v_{\text{subtractive case}}t) + \phi_1 - \phi_2}{2}\right] ,$$

it is clear that director of factor's propagation direction depends in the effective phase velocity. A factor propagates to the positive/negative direction for the effective phase velocity positive/negative.

If $v_2 = v_1$, the factors travel in the direction implied by v_1. If $v_2 = -v_1$, you cannot tell travel direction of the factors without knowing the values of k_1 and k_2?

If $v_2 = v_1$, then both factors have the physical phase speed, and so are solutions of the linear wave equation?

If $k_1 + k_2$ or $k_1 - k_2$ is zero, the corresponding factor does not propagate, it just oscillates with time as can be seen from part (a)?

e) Behold:

$$y = \begin{cases} 2A \sin\left[kx - \omega t + \dfrac{\phi_1+\phi_2}{2}\right] \cos\left[\dfrac{\phi_1-\phi_2}{2}\right] & \text{in general: this case is often called partial interf} \\ & \quad \text{if } \phi_1 - \phi_2 \text{ is neither 0 nor } \pi. \\ 2A \sin\left[kx - \omega t + \dfrac{\phi_1+\phi_2}{2}\right] & \text{for } \phi_1 - \phi_2 = 0\text{: this is constructive} \\ & \quad \text{or in-phase interference.} \\ 0 & \text{for } \phi_1 - \phi_2 = \pi\text{: this is destructive} \\ & \quad \text{or completely-out-of-phase interference.} \end{cases}$$

Note, the partial interference factor $\cos[(\phi_1 - \phi_2)/2]$ only varies between -1 and 1 and there is no change for adding $2\pi n$ to the argument where n is any integer, and so effectively the argument $(\phi_1 - \phi_2)/2$ only varies between 0 and 2π or $-\pi$ and π.

f) Behold:

$$
\begin{aligned}
y &= 2A\sin\left(-\omega t + \frac{\phi_1+\phi_2}{2}\right)\cos\left(kx + \frac{\phi_1-\phi_2}{2}\right) \\
&= 2A\sin\left(-\omega t + 2\pi\right)\cos\left(kx + \frac{\pi}{2}\right) \\
&= 2A\sin(-\omega t)\left[\cos(kx)\cos\left(\frac{\pi}{2}\right) - \sin(kx)\sin\left(\frac{\pi}{2}\right)\right] \\
&= 2A\sin(kx)\sin(\omega t) \ ,
\end{aligned}
$$

and so the final solution is

$$y = 2A\sin(kx)\sin(\omega t) \ .$$

The solution is a standing-wave solution. There is a spatial sinusoidal variation and at each point x a sinusoidal oscillation in time with amplitude $y(x,t)$. The standing-wave solution implies that summand basis solutions y_1 and y_1 had to be waves traveling in the opposite directions with equal amplitudes and equal absolute values of the wave number (which implies equal angular frequencies). There is no requirements on the phases. We chose the phases in this part just to get a clean result.

Redaction: Jeffery, 2008jan01

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023 qfull 00280 1 3 0 tough math: E-field of a circular disk

23. The goal of this problem is to determine the electric field on the symmetry axis of an infinitely thin circular disk of charge. The disk has radius R and uniform surface charge density σ. Let the symmetry axis be the z axis with the origin at the disk center.

a) Draw a diagram of the system.

b) Determine the formula for the electric field on the axis at a general z due to a differential ring of charge that is part of the disk. Count z as positive in both the up and down directions for simplicity The ring is at general r from the disk center with $r \leq R$. **HINT:** Since the electric field is a vector, you will need the unit vector $\hat{z}$ which points away from the disk in both up and down directions. You will also need to determine a cosine function in terms of z and r.

c) Integrate the part (b) result over the whole disk. Write the formula with the vacuum permittivity ε_0 (instead of Coulomb's constant k) and as a function of z/R.

d) Give the the formula from part (c) in the limit that $z \to 0$ and the asymptotic formula for large z/R.

SUGGESTED ANSWER:

a) You will have to imagine the diagram.

b) Behold:

$$d\vec{E} = \left[\frac{k\sigma}{r^2+z^2}(2\pi r)\,dr\right]\cos\theta\,\hat{z} = \left[\frac{k\sigma}{r^2+z^2}(2\pi r)\,dr\right]\frac{z}{\sqrt{r^2+z^2}}\hat{z} = \left[\frac{k\sigma}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}(2\pi r)\,dr\right]\hat{z}$$
$$= 2\pi k\sigma z\left[\frac{r}{(r^2+z^2)^{3/2}}\,dr\right]\hat{z}\ .$$

Note, the $\cos\theta$ is needed because only the z components of the electric field contribute to the electric field on the z axis. All other components cancel out by symmetry.

c) Behold:

$$\vec{E} = 2\pi k\sigma z\left[\int_0^R \frac{r}{(r^2+z^2)^{3/2}}\,dr\right]\hat{z} = 2\pi k\sigma z(-2)\left(\frac{1}{2}\right)\frac{1}{\sqrt{r^2+z^2}}\Bigg|_0^R\hat{z} = 2\pi k\sigma z\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)\hat{z}$$
$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z/R}{\sqrt{1+(z/R)^2}}\right]\hat{z}\ .$$

d) Behold:

$$\vec{E} = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{z/R}{\sqrt{1+(z/R)^2}}\right]\hat{z} = \dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{1}{\sqrt{1+(R/z)^2}}\right]\hat{z} & \text{in general.} \\[2ex] \dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{z}{R}+\dfrac{1}{2}\left(\dfrac{z}{R}\right)^2\right]\hat{z} & \text{to 2nd order in small } z/R. \\[2ex] \dfrac{\sigma}{2\varepsilon_0}\hat{z} & \text{to zeroth order in in small } z/R. \\ & \quad\text{This is the infinite plane limit.} \\[2ex] \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{2}\right)\left(\dfrac{R}{z}\right)^2 = \dfrac{kQ}{z^2} & \text{in the asymptotically small } R/z \text{ limit with } Q=\pi R^2\sigma. \\ & \quad\text{This is the point charge limit.} \end{cases}$$

Note, there is a discontinuity of the electric field at the origin. If we had evaluated the electric field at the origin by symmetry, we would have found that it was zero. In fact, for a finite disk with a uniform 3-dimensional charge density ρ, the electric field would be zero at the origin with electric field lines starting there and extending to infinity in all directions.

Our result is of interest for infinitely thin surfaces of charge. Consider a general point on the surface. Over a small enough region of the surface, the surface can be regarded as planar with disk inscribed in the planar region. At a location close enough to the general point on a axis through the point normal to the surface, there will be an E-field contribution given approximately by our limiting formula. Of course, all the rest of the charge distribution will make a contribution to the net E-field at that location. If by some means we knew the net E-field (and there are special cases where we do), then we could find the contribution of the rest the distribution by subtracting off the local contribution of the inscribed disk. I think there are ambiguities in doing this for any finite distance off the surface. But the limit for $z/R \to 0$ for the subtraction seems clear. But how accurate that limit value would be in any finite case takes more analysis than I care to do right now.

Redaction: Jeffery, 2008jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$
\begin{aligned}
c &= 2.997\ 924\ 58 \times 10^{8}\,\mathrm{m/s} \approx 2.998 \times 10^{8}\,\mathrm{m/s} \approx 3 \times 10^{8}\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \\
&\quad \text{exact by definition} \\
e &= 1.602\ 176\ 634 \times 10^{-19}\,\mathrm{C} \qquad \text{exact by definition} \\
G &= 6.674\ 30(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987\ 551\ 785\ 972(14) \times 10^{9} \approx 8.99 \times 10^{9} \approx 10^{10}\,\mathrm{N\,m^2/C^2} \\
k_{\mathrm{Boltzmann}} &= 1.380\ 649 \times 10^{-23}\,\mathrm{J/K} = 0.8617\ 333\ 262 \times 10^{-4}\,\mathrm{eV/K} \\
&\approx 10^{-4}\,\mathrm{eV/K} \qquad \text{exact by definition in both forms} \\
m_e &= 9.109\ 383\ 7139(28) \times 10^{-31}\,\mathrm{kg} = 0.510\ 998\ 950\ 69(16)\,\mathrm{MeV} \\
m_p &= 1.672\ 621\ 923\ 95(52) \times 10^{-27}\,\mathrm{kg} = 938.272\ 089\ 32(29)\,\mathrm{MeV} \\
\varepsilon_0 &= 8.854\ 187\ 8188(14) \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11}\,\mathrm{C^2/(N\,m^2)} \qquad \text{vacuum permittivity} \\
\mu_0 &= 1.256\ 637\ 061\ 27(20)\,\mathrm{N/A^2} \approx \pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{vacuum permeability}
\end{aligned}
$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1+x \;:\; (x << 1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

If $\quad 0 = ax^2+bx+c\ ,\quad$ then $\quad x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = -\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

Numerically robust solution (Press-178):

$$q = -\frac{1}{2}\left[b+\mathrm{sgn}(b)\sqrt{b^2-4ac}\right] \qquad x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2+a_y^2} \qquad \vec{a}+\vec{b} = (a_x+b_x, a_y+b_y)$$

$$\theta = \begin{cases} \arctan(a_x, a_y) + n\pi = \arctan_{\text{computational}}\left(\dfrac{a_y}{a_x}\right) + \pi \mathrm{H}_{\text{Heaviside}}(-a_x)\mathrm{sgn}(a_y) + n\pi & \text{for } a_x \neq 0. \\ \dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y > 0. \\ -\dfrac{\pi}{2} + n\pi & \text{for } a_x = 0 \text{ and } a_y < 0. \end{cases}$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

---

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

---

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Five 1-dimensional equations of kinematics

Equation No.	Equation	Unwanted variable
1	$v = at + v_0$	Δx
2	$\Delta x = \frac{1}{2}at^2 + v_0 t$	v
3 (timeless eqn)	$v^2 - v_0^2 = 2a\Delta x$	t
4	$\Delta x = \frac{1}{2}(v + v_0)t$	a
5	$\Delta x = vt - \frac{1}{2}at^2$	v_1

Fiducial acceleration due to gravity (AKA little g) $\quad g = 9.8\,\mathrm{m/s^2}$

$$x_{\mathrm{rel}} = x_2 - x_1 \qquad v_{\mathrm{rel}} = v_2 - v_1 \qquad a_{\mathrm{rel}} = a_2 - a_1$$

$$x' = x - v_{\mathrm{frame}}t \qquad v' = v - v_{\mathrm{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\mathrm{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\mathrm{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\mathrm{for}\ y\ \mathrm{max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\mathrm{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\mathrm{for\ max}} = \frac{\pi}{4} \qquad x_{\mathrm{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

---

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

---

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

---

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

---

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad K = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = K + U$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta K = W_{\text{net}} \quad \Delta U_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dU}{dx} \qquad \vec{F} = -\nabla U \qquad U = \frac{1}{2}kx^2 \qquad U = mgy$$

---

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta K_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\rm cm} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\rm flux} - \vec{v}_{\rm cm})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\rm rel}\frac{dm}{dt}$$

$$v = v_0 + v_{\rm ex}\ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t}\vec{F}(t)\,dt \qquad \vec{F}_{\rm avg} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\rm net}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\rm cm} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\rm total}}$$

$$K_{\text{total } f} = K_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\rm rel'} = -v_{\rm rel} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots\ \text{deg/rad} \approx 60\,\text{rad/deg} \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots\ \text{rad/deg} \approx \frac{1}{60}\,\text{rad/deg}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\mathrm{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy}\sin\gamma_L \qquad \tau_z = RF_{xy}\sin\gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\mathrm{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho\, dV \qquad I_{\text{parallel axis}} = I_{\mathrm{cm}} + mR_{\mathrm{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\mathrm{cyl,shell,thin}} = MR^2 \qquad I_{\mathrm{cyl}} = \frac{1}{2}MR^2 \qquad I_{\mathrm{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\mathrm{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\mathrm{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\mathrm{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g\sin\theta}{1 + I/(mr^2)}$$

$$K_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z\, d\theta \qquad P = \frac{dW}{dt} = \tau_z\omega$$

$$\Delta K_{\mathrm{rot}} = W_{\mathrm{net}} = \int \tau_{z,\mathrm{net}}\, d\theta \qquad \Delta U_{\mathrm{rot}} = -W = -\int \tau_{z,\mathrm{con}}\, d\theta$$

$$\Delta E_{\mathrm{rot}} = K_{\mathrm{rot}} + \Delta U_{\mathrm{rot}} = W_{\mathrm{non,rot}} \qquad \Delta E = \Delta K + K_{\mathrm{rot}} + \Delta U = W_{\mathrm{non}} + W_{\mathrm{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\mathrm{ext,net}} = 0 \qquad \vec{\tau}_{\mathrm{ext,net}} = 0 \qquad \vec{\tau}_{\mathrm{ext,net}} = \tau'_{\mathrm{ext,net}} \quad \text{if } F_{\mathrm{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 **Gravity**

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$U = -\frac{Gm_1m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1370\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$1\,\text{AU} = 1.495978707 \times 10^{11}\,\text{m} \qquad \text{exact by definition}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^8 \ \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \qquad y = f(x - vt)$$

$$y = y_{\max}\sin[k(x - vt)] = y_{\max}\sin(kx - \omega t) \qquad v = \frac{\omega}{k}$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\max}^2$$

$$y = 2y_{\max}\sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \ : \ n = 1, 3, 5, \ldots \qquad f_{\text{medium}} = \frac{f_0}{1 - v_0/v_{\text{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$

24 Thermodynamics

$$dE = dQ - dW = T\,dS - p\,dV$$

$$T_{\text{K}} = T_{\text{C}} + 273.15\,\text{K} \qquad T_{\text{F}} = 1.8 \times T_{\text{C}} + 32^\circ\text{F}$$

$$Q = mC\Delta T \qquad Q = mL$$

$$PV = NkT \qquad P = \frac{2}{3}\frac{N}{V}K_{\text{avg}} = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}mv_{\text{RMS}}^2\right)$$

$$v_{\text{RMS}} = \sqrt{\frac{3kT}{m}} = 2735.51\ldots \times \sqrt{\frac{T/300}{A}}$$

$$PV^\gamma = \text{constant} \qquad 1 < \gamma \leq \frac{5}{3} \qquad v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{-V(\partial P/\partial V)_S}{m(N/V)}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{W} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \qquad \eta_{\text{heating}} = \frac{Q_{\text{H}}}{W} = \frac{Q_{\text{H}}}{Q_{\text{H}} - Q_{\text{C}}} = \frac{1}{1 - Q_{\text{C}}/Q_{\text{H}}} = \frac{1}{\varepsilon}$$

$$\eta_{\text{cooling}} = \frac{Q_{\text{C}}}{W} = \frac{Q_{\text{H}} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\text{heating}} - 1$$

$$\varepsilon_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} \qquad \eta_{\text{heating,Carnot}} = \frac{1}{1 - T_{\text{C}}/T_{\text{H}}} \qquad \eta_{\text{cooling,Carnot}} = \frac{T_{\text{C}}/T_{\text{H}}}{1 - T_{\text{C}}/T_{\text{H}}}$$

25 Electrostatics

$$\vec{F}_{1 \text{ on } 2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad \vec{F} = q\vec{E} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \qquad \vec{E}(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,')\,d\vec{r}\,' \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\vec{E}(\vec{r}\,) = \sum_i \frac{kq_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i + \ldots\right)$$

$$q = \sum_i q_i \qquad \vec{p} = \sum_i \vec{r}_i q_i \qquad \vec{p} = 2q\vec{a} \qquad \vec{E}(\vec{r}\,) = \frac{kq}{r^2}\hat{r} + k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right]$$

$$\vec{E}(\vec{r}\,) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] = \frac{kp}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Delta U = q\Delta V \qquad \Delta V = -\int_{\text{i}}^{\text{f}} \vec{E}\cdot d\vec{s} \qquad dV = -\vec{E}\cdot d\vec{s}$$

$$\vec{E} = -\nabla V \qquad \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$V = \frac{kq}{r} \qquad V(\vec{r}\,) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad V(\vec{r}\,) = \int_{\text{all space}} \frac{k\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\,d\vec{r}\,' \qquad V(\vec{r}\,) = \frac{k\vec{p}\cdot\hat{r}}{r^2}$$

$$U = \frac{1}{2}\sum_{i,j,i\neq j} \frac{kq_iq_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_i q_iV_i$$

$$U = \frac{1}{2}\int\int \frac{k\rho(\vec{r}\,)\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\,d\mathcal{V}\,d\mathcal{V}' = \frac{1}{2}\int \rho(\vec{r}\,)V(\vec{r}\,)\,d\mathcal{V}$$

$$C = \frac{Q}{V} \qquad C = \frac{\varepsilon_0 A}{d} \qquad C_{\text{parallel}} = \sum_i C_i \qquad \frac{1}{C_{\text{series}}} = \sum_i \frac{1}{C_i}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$$

$$\vec{E}_{\text{macroscopic charge}} = \kappa \vec{E}_{\text{net}} \qquad \oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

26 Current and Circuits

$$I = \frac{dq}{dt} \qquad I = \int \vec{J} \cdot d\vec{A} \qquad J = \frac{I}{A} \qquad \vec{J} = nq\vec{v}_{\text{drift}} \qquad \vec{J} = \sigma \vec{E} \qquad V = IR$$

$$\sigma = \frac{nq^2\tau}{m} \qquad \rho = \frac{1}{\sigma} \qquad V = IR \qquad R = \rho\frac{L}{A} \qquad P = VI \qquad P = VI = I^2R = \frac{V^2}{R}$$

$$0 = \sum_i^{\text{node}} I_i \qquad 0 = \sum_i^{\text{loop}} \Delta V_i \qquad R_{\text{series}} = \sum_i R_i \qquad \frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i}$$

$$\tau = RC \qquad I = \frac{\Delta V_{\text{initial}}}{R} e^{-t/\tau} \qquad \Delta V = \Delta V_{\text{final}} \left(1 - e^{-t/\tau}\right)$$

27 Magnetic Fields and Forces

$$\mu_0 = 1.256\,637\,061\,27(20)\,\text{N/A}^2 \approx \pi \times 10^{-7}\,\text{N/A}^2 \qquad 1\,\text{N/A}^2 = 1\,\text{T}\cdot\text{m/A} \qquad 1\,\text{tesla (T)} = 10^4\,\text{gauss (G)}$$

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad d\vec{F} = I\,d\vec{s} \times \vec{B}$$

$$\vec{r}_{\text{cyclotron}} = \frac{mv}{|q|B} \qquad \omega_{\text{cyclotron}} = \frac{|q|B}{m} \qquad f_{\text{cyclotron}} = \frac{|q|B}{2\pi m} \qquad P = \frac{2\pi m}{|q|B}$$

$$\vec{\mu} = NIA\hat{n} \qquad \tau = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \qquad B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \qquad B_{\text{arc}} = \frac{\mu_0 I \theta}{4\pi r} \qquad \vec{F}_{1\text{ on }2} = \frac{\mu_0 \ell I_1 I_2}{2\pi r} \hat{r}_{2\text{ to }1}$$

$$B_{\text{solenoid}} = \mu_0 n I \qquad B_{\text{toroid}} = \frac{\mu_0 N I}{2\pi r} \qquad \vec{B}_{\text{circ. loop}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \hat{z} \qquad \vec{B}_{\text{dipole}} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad \Phi_B = \int_{\text{linked area}} \vec{B} \cdot d\vec{A} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d}{dt} \int_{\text{linked area}} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

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28 LR, LC, and RLC Circuits

$$L = \frac{\Phi_B}{I} \qquad L_{\text{solenoid}} = \mu_0 n^2 V_{\text{vol}} \qquad \mathcal{E} = -L\frac{dI}{dt} \qquad U_L = \frac{1}{2} L I^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = \sum_i L_i \qquad \frac{1}{L_{\text{parallel}}} = \sum_i \frac{1}{L}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \qquad M = M_{12} = M_{21} \qquad M_{\text{con}-\text{axial solenoids}} = \mu_0 n_1 n_2 V_{\text{inner}}$$

$$\tau_L = \frac{L}{R} \qquad I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = I_\infty \left(1 - e^{-t/\tau_L}\right) \qquad I = I_0 e^{-t/\tau_L}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad q = A\cos(\omega t) + B\sin(\omega t) \qquad q = q_0 \cos(\omega t) \qquad U = \frac{q_0^2}{2C}$$

$$\alpha_{\pm} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad R_{\text{critical}} = 2\sqrt{\frac{L}{C}}$$

$$q = \left[q_0 \cos(\omega t) + \frac{q_0}{\tau\omega}\sin(\omega t)\right] e^{-t/\tau} \qquad \tau = \frac{2L}{R}$$

29 Alternating Current, Driven RLC Circuits, and Transformers

$$V = V_0 \cos(\omega t) \qquad I = I_0 \cos(\omega t - \phi) \qquad Z = \frac{V_0}{I_0}$$

$$Z = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2} \qquad \phi = \tan^{-1}\left[\frac{\omega L - 1/(\omega C)}{R}\right] \qquad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}$$

$$P = IV \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \qquad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad P_{\text{avg}} = \frac{1}{2} I_0 V_0 \cos\phi = I_{\text{rms}} V_{\text{rms}} \cos\phi$$

$$\frac{\mathcal{E}_{\text{p}}}{N_{\text{p}}} = \frac{\mathcal{E}_{\text{s}}}{N_{\text{s}}} \qquad I_{\text{p}} N_{\text{p}} = I_{\text{s}} N_{\text{s}} \qquad I_{\text{p}} \mathcal{E}_{\text{p}} = I_{\text{s}} \mathcal{E}_{\text{s}} \qquad \frac{\mathcal{E}_{\text{p}}}{\mathcal{E}_{\text{s}}} = \frac{N_{\text{p}}}{N_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}}$$

30 Electromagnetic Radiation (EMR)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \oint \vec{J} \cdot d\vec{A} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \qquad f(x,t) = f(x - vt)$$

$$\text{Period} = \frac{1}{f} \qquad c = f\lambda = \frac{\omega}{|k|} \qquad \lambda = \frac{2\pi}{|k|} \qquad k = \frac{2\pi}{\lambda} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\vec{E} = \vec{E}_0 \sin(kx - wt) \qquad \vec{B} = \vec{B}_0 \sin(kx - wt) \qquad \frac{B}{E} = \frac{1}{c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} \qquad I = S_{\text{avg}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{cB_{\text{max}}^2}{2\mu_0}$$

$$u_{\text{avg}} = \frac{I}{c} = \frac{\varepsilon E_{\text{max}}^2}{2} = \frac{B_{\text{max}}^2}{2\mu_0} \qquad I_{\text{point}} = \frac{L}{4\pi r^2}$$

$$\lambda = \frac{C_{\text{wien}}}{T} = \frac{2897.7685\,\mu\text{m-K}}{T} \approx \frac{3000\,\mu\text{m-K}}{T}$$

$$I_{\text{trans. polarized}} = I_{\text{inc}} \cos^2\theta \qquad I_{\text{trans. non-polarized}} = \frac{1}{2} I_{\text{inc}}$$