Modern Physics: Physics 305, Section 1

NAME:

Homework 3: Photons Homeworks are due as posted on the course web site. They are **NOT** handed in. The student reports that it is completed and receives one point for this. Solutions are already posted, but students are only permitted to look at the solutions after completion. The solutions are intended to be (but not necessarily are) super-perfect and go beyond a complete answer expected on a test.

1. In general relativity, free-fall with no other forces acting is at least in Gre-67's interpretation (and this interpretation is different from one used in most physics contexts):

   a) accelerated motion. b) unaccelerated motion. c) simple harmonic oscillation.
   d) anharmonic oscillation. e) static equilibrium.

2. "Let's play *Jeopardy*! For \$100, the answer is: It is a theory in which mass-energy determines the geometry of spacetime and in which the geometry of spacetime (which is the cause of gravity) plus other forces determine the motion of mass-energy."

   What is ____________, Alex?

   a) quantum mechanics b) special relativity c) general relativity
   d) Newtonian physics e) Maxwellian electromagnetism

3. Because spacetime responds to mass-energy in general relativity, one can say that in general relativity spacetime is:

   a) static. b) ellipsoidal. c) hyperbolical. d) dynamic. e) flat.

4. "Let's play *Jeopardy*! For \$100, the answer is: It is the emission of electrons from matter caused by the absorption of photons. The effect in some sense includes photoionization as a subcategory since photoionization agrees with the definition, but other cases such as emission of non-localized electrons in materials are also included in the effect and are what one usually thinks of when one says the name of the effect."

   What is the ____________, Alex?

   a) Mössbauer effect b) Hall effect c) quantum Hall effect d) Zeeman effect
   e) photoelectric effect

5. Given that the work function of gold (Au) is 4.8 eV, what is the maximum wavelength of light that will cause the emission of a photoelectron? **HINT:** $hc = 12398.419\,\text{eV}\,\text{Å}$.

   a) 2600 Å. b) 3000 Å. c) 5000 Å. d) 7000 Å. e) 10000 Å.

6. The Compton equation can be derived using the photon picture of electromagnetic radiation and:

   a) the photoelectric effect. b) classical energy and momentum conservation laws.
   c) relativistic energy and momentum conservation laws. d) the Planck spectrum.
   e) the Einstein equation $E = mc^2$.

7. The standard Compton equation is

   $$\Delta\lambda = \lambda_{\rm scat} - \lambda_{\rm inc} = \lambda_{\rm C}(1 - \cos\theta) \;,$$

   where $\lambda_{\rm scat}$ is the wavelength of the scattered photon, $\lambda_{\rm inc}$ is the wavelength of the incident photon, θ is the scattering angle (i.e., the angle between the incident and scattering directions), and $\lambda_{\rm C}$ is the Compton wavelength. Note that

   $$\lambda_{\rm C} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,\text{m}$$

   where h is Planck's constant, m_e is the electron mass, and c is the speed of light. Compton scattering by protons can occur too. What is the proton Compton wavelength?

a) $2.426310238(16) \times 10^{-15}\,\mathrm{m} \approx 24.3\,\mathrm{fm}$.
b) $2.426310238(16) \times 10^{-15}\,\mathrm{m} \approx 2.4\,\mathrm{fm}$.
c) $2.426310238(16) \times 10^{-12}\,\mathrm{m} \approx 0.024\,\text{Å}$.
d) $1.321409855 \times 10^{-15}\,\mathrm{m} \approx 13.2\,\mathrm{fm}$.
e) $1.321409855 \times 10^{-15}\,\mathrm{m} \approx 1.3\,\mathrm{fm}$.

8. "Let's play *Jeopardy*! For \$100, the answer is: A bound state of matter which is usually formed by a positron on its way to annihilation with an electron. It has a mean lifetime of $1.25 \times 10^{-10}\,\mathrm{s}$ if it forms in the singlet ground state."

   What is ____________, Alex?

   a) pragmatium b) plutonium c) protonium d) positronium e) protesium

9. You are given that the probability density for photon removal from a beam along a beam path is

$$\rho(s) = \frac{e^{-s/\ell}}{\ell} ,$$

   where s is the path coordinate from some initial position and ℓ turns out to be the mean free path.

   a) Find the probability for removal by point s.

   b) Find the probability for survival to point s.

   c) Find the moments of the probability distribution.

   d) Find the standard deviation of the distribution.

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

---

1 Geometrical Formulae

$$C_{\rm cir} = 2\pi r \qquad A_{\rm cir} = \pi r^2 \qquad A_{\rm sph} = 4\pi r^2 \qquad V_{\rm sph} = \frac{4}{3}\pi r^3$$

---

2 Trigonometry

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

---

3 Blackbody Radiation

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\left[e^{h\nu/(kT)}-1\right]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\left[e^{hc/(kT\lambda)}-1\right]}$$

$$B_\lambda\, d\lambda = B_\nu\, d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$k = 1.3806505(24)\times 10^{-23}\,{\rm J/K} \qquad c = 2.99792458\times 10^{8}\,{\rm m}$$

$$h = 6.6260693(11)\times 10^{-34}\,{\rm J\,s} = 4.13566743(35)\times 10^{-15}\,{\rm eV\,s}$$

$$\hbar = \frac{h}{2\pi} = 1.05457168(18)\times 10^{-34}\,{\rm J\,s}$$

$$hc = 12398.419\,{\rm eV\,\AA} \approx 10^4\,{\rm eV\,\AA} \qquad E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^4 \qquad \sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2h^3} = 5.670400(40) \times 10^{-8}\,\mathrm{W/m^2/K^4}$$

$$\lambda_{\max} T = \text{constant} = \frac{hc}{kx_{\max}} \approx \frac{1.4387751 \times 10^{-2}}{x_{\max}}$$

$$B_{\lambda,\mathrm{Wien}} = \frac{2hc^2}{\lambda^5} e^{-hc/(kT\lambda)} \qquad B_{\lambda,\mathrm{Rayleigh-Jeans}} = \frac{2ckT}{\lambda^4}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L} n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L} n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^2}{\pi^2}\,dk = \pi \left(\frac{2}{c}\right) \nu^2\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z + \frac{1}{2}\right) \ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^3} + \frac{1}{1260z^5} - \ldots$$

$$\ln(N!) \approx N\ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1 - e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x - vt) \qquad f(kx - \omega t)$$

4 Photons

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\mathrm{scat}} - \lambda_{\mathrm{inc}} = \lambda_{\mathrm{C}}(1 - \cos\theta)$$

$$\lambda_{\mathrm{C}} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,\mathrm{m} \qquad e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$m_e = 9.1093826(16) \times 10^{-31}\,\mathrm{kg} = 0.510998918(44)\,\mathrm{MeV}$$

$$m_p = 1.67262171(29) \times 10^{-27}\,\mathrm{kg} = 938.272029(80)\,\mathrm{MeV}$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m \rangle = \ell^m m!$$

5 Special Relativity

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_{\rm obj} = \beta_{\rm obj} - \beta$	$\beta'_{\rm obj} = \dfrac{\beta_{\rm obj} - \beta}{1 - \beta\beta_{\rm obj}}$

$$\ell = \ell_{\rm proper}\sqrt{1-\beta^2} \qquad \Delta\tau_{\rm proper} = \Delta\tau\sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c\beta \qquad E_0 = m_0c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2 = (\gamma - 1)m_0c^2$$

$$f = f_{\rm proper}\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\rm proper}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\rm trans} = f_{\rm proper}\sqrt{1-\beta^2} \qquad f_{\rm trans}(\beta << 1) = f_{\rm proper}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1}x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\rm intersection}}{\gamma} = x'_{x\ \rm scale}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\rm intersection}}{\gamma} = \tau'_{\tau\ \rm scale}\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\rm Mink} = \tan^{-1}(\beta)$$