Modern Physics: Physics 305, Section 1

NAME:

Homework 1: Special Relativity Homeworks are due as posted on the course web site. They are **NOT** handed in. The student reports that they are completed and receives one point for each. Solutions are already posted, but students are only permitted to look at the solutions after completion. The solutions are intended to be (but not necessarily are) super-perfect and go beyond a complete answer expected on a test.

1. In physics jargon, the word "clearly" means:

   a) clearly. b) unclearly. c) after 4 pages of algebra. d) wrongly. e) all of the above.

2. In physics jargon, the phrase "must be" means:

   a) is. b) just accept it that. c) not necessarily so. d) can't be. e) all of the above.

3. "Let's play *Jeopardy*! For $100, the answer is: In Akira Kurosawa's film *The Seven Samurai* in the misremembering of popular memory, what the samurai leader said when one of the seven asked why they were going to defend this miserable village from a horde of marauding bandits."

   What is "____________," Alex?

   a) For honor. b) It is the way of the samurai. c) It is the Tao. d) For a few dollars more. e) For the fun of it.

4. Brian Greene probably titled his popular book on modern physics *The Fabric of the Cosmos* mostly maybe because:

   a) he's a proponent of superstring theory.
   b) he's **NOT** a proponent of superstring theory.
   c) in imitation of Stephen Toulmin and June Goodfield's *The Fabric of the Heavens*.
   d) in the modern age every book has to have a farfetched metaphorical title like *The God Particle* or *The Snail's Ear*: a title like *A Popular Account of Modern Particle Physics and Cosmology* just doesn't cut it.
   e) of random processes.

5. On the back cover of Brian Greene's *The Fabric of the Cosmos* (pocket-size paperback), Brian Greene (one supposes) is in front of an ocean. Which ocean and why?

   a) Greene lives in New York state, and so it's probably the Atlantic Ocean.
   b) It's not an ocean. One can descry Port Colborne, Ontario on the horizon. He's in front of Lake Erie on the New York State side. In fact, he's probably at Angola-on-the-Lake—which is not in Africa whatever you may think.
   c) Well . . .

      > . . . like stout Cortez when with eagle eyes
      > He star'd at the Pacific—and all his men
      > Look'd at each other with a wild surmise—
      > Silent, upon a peak in Darien.

   d) Quoting Newton:

      > I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

That's the one.

e) I've no idea.

6. "Let's play *Jeopardy*! For \$100, the answer is: In this physical theory (circa 2004 at least), the basic element of matter is a string/filament/little-thingy which vibrates in different ways to make the fundamental particles (e.g., electron, neutrino, quark). The theory requires 9 or 10 space dimensions plus 1 time dimension and thus 10 or 11 spacetime dimensions. The higher numbers are for the version called M-theory."

   What is ____________, Alex?

   a) Aristotelian physics b) Newtonian physics c) Einsteinian relativistic physics
   d) quantum mechancis e) superstring theory

7. The Planck units are quantities constructed by dimensional analysis from 5 fundamental constants:

$$\begin{cases}
c = 2.99792458\times10^{8}\,\mathrm{m/s} \approx 3\times10^{8}\,\mathrm{m/s} & \text{the vacuum light speed;} \\
G = 6.67428(67)\times10^{-11}\,\mathrm{J\,m/kg^2} \approx 7\times10^{-11}\,\mathrm{J\,m/kg^2} & \text{the gravitational constant which can also be given the units } \mathrm{m^5/(J\,s^4)}; \\
\hbar = 1.054571628(53)\times10^{-34}\,\mathrm{J\,s} \approx 1\times10^{-34}\,\mathrm{J\,s} & \text{which is h-bar or Dirac's constant or Planck's constant divided by } 2\pi; \\
k_{\mathrm{C}} = \dfrac{1}{4\pi\epsilon_0} = 8.987551787\times10^{9}\,\mathrm{J\,m/C^2} \approx 9\times10^{9}\,\mathrm{J\,m/C^2} & \text{the Coulomb force constant where } \epsilon_0 = 8.854187817\times10^{12}\,\mathrm{F/m} \text{ is the vacuum permittivity;} \\
k = 1.3806504(24)\times10^{-23}\,\mathrm{J/K} \approx 1.4\times10^{-23}\,\mathrm{J/K} & \text{which is Boltzmann's constant,}
\end{cases}$$

   where the values have been taken from Wikipedia (2007oct21). The Planck units (originally proposed by Max Planck) are based only on general universal physics and not arbitrary human choices. They should have some fundamental significance and are often ingredients in advanced theory. Its helpful in constructing Planck units to note that G/c^4 has units of m/J and G/c^5 has units of s/J (which incidentally makes it the inverse of the Planck power).

   Brian Greene (Gre-17), in the customary arcane jargon of grand high theorists, refers to a length "some hundred billion billion times smaller than a single atomic nucleus." Atomic nuclei have a size scale of order $10^{-15}\,\mathrm{m}$. Evidently, he is referring to the Planck length. Construct the Planck length from the above constants and evaluate it approximately.

   a) $\sqrt{\hbar c^5/G} \approx 1.8\times10^{9}\,\mathrm{m}$ b) $\sqrt{\hbar G/c^5} \approx 5\times10^{-44}\,\mathrm{m}$
   c) $\sqrt{\hbar G/c^3} \approx 1.6\times10^{-35}\,\mathrm{m}$ d) $\sqrt{\hbar c/G} \approx 2\times10^{-8}\,\mathrm{m}$
   e) $\sqrt{c^5/(\hbar G)} \approx 2\times10^{43}\,\mathrm{m}$

8. Nowadays takeoffs on cultural detritus of all kinds frequently occur without any acknowledgement. The section title *Coming of Age in Space and Time* of Greene's chapter 1, section 8 is probably a takeoff on:

   a) both of (b) and (c) maybe.
   b) *Coming of Age in the Milky Way* (1989) by Timothy Ferris (1944–).
   c) *Coming of Age in Somoa* (1928) by Margaret Mead (1901–1978).
   d) *The Waning of the Middle Ages* (1924) by Johan Huizinga (1872–1945).
   e) *Coming of Nonage in Fermullan* by Thomas Caskey, Sr. (1883–1964 or so).

9. Classical Newtonian laws of physics are inertial frame covariant under:

   a) the Lorentz transformations. b) the Galilean transformations. c) no transformations at all. d) the Keplerian transformations. e) the Maxwellian transformations.

10. Heinrich Hertz (1857–1894) experimentally verified the existence of:

    a) electromagnetic waves. b) light. c) an upper limit to all physical velocities. d) gravity. e) gravitational radiation.

11. Einstein first published his theory of special relativity in:

    a) 1905. b) 1879. c) 1955. d) 1776. e) 1066.

12. Einstein developed special relativity starting from ____________ basic postulates—at least in the accounts of most textbooks—which may be thinking of the ideal Einstein rather than of the historical Einstein.

    a) zero b) two c) three d) four e) infinite

13. According to the principle (or postulate) of relativity of special relativity, the laws of physics or, more exactly, the expressions of physical law are the same in:

    a) all frames of reference.
    b) most frames of reference.
    c) all inertial frames (i.e., fundamentally unaccelerated frames) of reference.
    d) all non-inertial frames (i.e., fundamentally accelerated frames) of reference.
    e) no frames of reference

14. The vacuum speed of light has the same value, labeled by c, in all directions in:

    a) some inertial frames. b) frames moving with the Earth only. c) no frames. d) all inertial frames. e) no inertial frames.

15. Einstein's two special relativity postulates implied that classical Newtonian physics:

    a) needed no revision. b) needed revision. c) was wildly wrong in all cases of interest. d) was not correct now, but would be in the future. e) was completely useless.

16. In special relativity, the spacetime coordinates in different inertial frames are related by the ____________ transformations.

    a) Galilean b) Keplerian c) Newtonian d) Maxwellian e) Lorentz

17. The proper or rest-frame length L_0 parallel to the direction of motion of an object moving uniformly relative to an inertial frame S is related to the length L measured by an observer at rest in S by the Fitzgerald contraction formula:

    a) $L = L_0\sqrt{1-\beta^2}$. b) $L = L_0/\sqrt{1-\beta^2}$. c) $L = L_0$. d) $L = 1/L_0$. e) $L = L_0^2$.

18. As an object's velocity increases relative to some observer, the mass of the object as measured by that observer:

    a) goes to zero. b) decreases. c) stays constant. d) increases and approaches infinity as the speed approaches the vacuum light speed. e) is infinite.

19. The mnemonic for the time dilation effect is "moving clocks:

a) run fast." b) run slow." c) are stopped." d) are right twice a day."
e) run backward."

20. In the twins paradox, the twin who ages least is the one who is:

a) not accelerated. b) not Fitzgerald contracted. c) accelerated. d) Fitzgerald contracted. e) hermetically sealed in a time capsule.

21. "Let's play *Jeopardy*! For \$100, the answer is: It can be divided into Past, Future, Now, and Elsewhen, and on it world-lines evolve."

a) What is ____________, Alex?

a) a conformal mapping b) a Feynman diagram c) a Minkowski diagram d) history e) life

22. A world line in a Minkowski diagram is:

a) the trajectory of an object in spacetime.
b) the trajectory of an object in space alone.
c) trajectory of an object in time alone.
d) the arrow of time.
e) a telephone that is a party line for the entire world—recommended for exhibitionists.

23. We know already that a tricky political situation will arise on Alpha Centauri 6 (4 light-years distant) in a election 3 years hence. In our Terracentric way, we the people of Earth are going to make our views known to the Alpha Centaurs. What is our chance of influencing their decision?

a) It's possible: Alpha Centauri 3 years hence is **INSIDE** our light cone. A radio signal can be sent and received and the Alpha Centaurs will conclude that there is life out there—arguably intelligent.
b) So-so.
c) Immense: nothing is more cherished than unasked for advice from those incompetent to give it.
d) Nil: Alpha Centaurs would rather take their own bad advice, than someone else's good advice: they're pretty normal that way—explaining the horse's hind end is another matter.
e) Nil: Alpha Centauri 3 years hence is **OUTSIDE** of our light cone. Nothing we do now can have the slightest effect on them then. Jump up and down, scream, shout, giggle inopportunely—nothing can help.

24. If you could travel instantly over a finite distance in any inertial reference frame, you could:

a) do anything. b) write the great American novel. c) play a didjeridu. d) ride a unicycle and spit backwards at the same time. e) time travel.

25. Einstein's equation (or mass-energy equivalence equation) is

a) $E = mc^3$. b) $E = m/c^3$. c) $E = m/c^2$. d) $E = mc^4$. e) $E = mc^2$.

26. Einstein's equation $E = mc^2$:

a) applies only to rest mass.
b) applies only to electromagnetic radiation.
c) applies only to nuclear reactions.
d) is only valid in non-inertial frames.
e) is general. All forms of energy have inertial mass and a gravitational effect. Rest mass is itself a special form of energy like electromagnetic field, thermal, or kinetic energies.

27. The mass of an electron is 9.1×10^{-31} kg. The energy equivalent of this mass is approximately:

a) 10^{-13} J. b) 10^{-47} J. c) 3×10^{-22} J. d) 3×10^{-38} J. e) 1 J.

28. Do the following.

    a) Given
    $$\gamma = \frac{1}{\sqrt{1-\beta^2}} ,$$
    find β as a function of the Lorentz factor γ.

    b) Given $p = \gamma m_0 v = \gamma m_0 c\beta$, find β as a function of $p/(m_0 c)$.

    c) Given $E = \gamma m_0 c^2$, show that $E = \sqrt{p^2c^2 + (m_0c^2)^2}$.

    d) Given $E = \gamma m_0 c^2$, find β as a function of $E/(m_0c^2)$.

    e) Given the result of part (d), find β as a function of kinetic energy KE.

    f) If you know Taylor's series, verify that $\beta = \sqrt{2KE/(m_0c^2)}$ to 1st order in $KE/(m_0c^2)$. The 1st order expression is, of course, the classical (i.e., non-relativistic result) result.

29. Stacy Dragila, Pocatello's pride and 2000 Olympic gold-medal winner, is barrelling along at a relativistic speed with her pole. The pole is 15 ft long in rest length. She runs through a barn 10 ft long in rest length and open at both ends. Stacy's going so fast that to an observer at rest in the ground inertial frame, her pole is Fitzgerald contracted to less than 10 ft. Thus the ground-frame observer sees Stacy and pole entirely vanish into the barn for a brief time. But to Stacy it's the barn that is Fitzgerald contracted and she and pole are never entirely inside the barn. Resolve the paradox. **HINT:** The answer isn't very long.

30. The Fitzgerald contraction formula is
    $$\ell = \ell_0 \sqrt{1-\beta^2} ,$$
    where ℓ_0 is the proper length of object (i.e., length measured in the rest frame of the object) moving at speed β relative to some other inertial frame (let's call it the lab frame) and ℓ is the corresponding length in the lab frame: note the lengths are both parallel to the direction of motion. The formula is derived from the Lorentz transformations assuming that the measurement of the location of the ends of the length was simultaneous in the lab frame. But there is another way one can measure the length of the moving object in the lab frame: measure the time $\Delta\tau$ (i.e., the time in the "length form" equal to $c\Delta t$) between the front end of the object passing a fixed point in the lab frame and the rear end passing the same point, and multiply by the speed of the object. Call this length ℓ_*, where
    $$\ell_* = \beta\Delta\tau .$$
    Using the Lorentz transformations determine ℓ_* as a function of ℓ_0. **HINT:** The ends of the object passing the fixed point constitute two events with different, but related, spacetime coordinates. Also, observers in both frames see the measurements happening on the end points of the object. In the frame of the object, the measurement events are on an object at rest.

31. Federation star ships Egregious and Execrable are on a head-on collision course. Egregious is moving at velocity $\beta_1 = 0.6$ and Execrable at velocity $\beta_2 = -0.8$: both velocities are relative to the more-or-less inertial frame of the Galaxy.

    a) What is the velocity β_2' of Execrable in the inertial frame of the Egregious?

    b) Good ship Execrable has proper length 500 m. What are it's lengths in the Galaxy frame and in the Egregious frame?

    c) In the Galaxy frame Egregious and Execrable start off a light-year apart (1 lyr $= 9.460 \times 10^{15}$ m). How long in **YEARS** till they collide in the Galaxy frame? **HINT:** Calculate their relative velocity as seen in the Galaxy frame.

d) In part (c), you obtained a superluminal velocity, but in special relativity we say c (speed of light in vacuum) is the ultimate physical speed. Both the part (c) answer and saying c is the ultimate physical speed are correct. Resolve the paradox? **HINT:** Note the word "physical" before speed.

e) By the simplest means calculate the time to collision in the rest frame of the Egregious.

f) Now for the challenge. Calculate the time to collision in the rest frame of the Egregious, using velocities, lengths, and times as measured in the Egregious frame. Is the answer the same as that in part (e)? **HINT:** First, find Execrable's spacetime coordinates in the Egregious frame.

32. You and your loyal hound Frodo are traveling through space in your star ship Smaug. You are limited to going at less than the ultimate physical speed $c = 1\,\text{lyr/yr}$ or $\beta = 1$, of course. But other than that Smaug can go at any speed and you and Frodo are real tough and can stand any acceleration.

    a) Say Frodo wants to travel from Earth to Sirius (distance $d = 8.7\,\text{lyr}$) and you want to make the trip in 1 year for **YOU**. At what β value would Smaug have to go? **HINTS:** You will have to do some algebra. Recall the Galaxy-frame travel time is $d/(c\beta)$.

    b) Stirred by your first conquest of space, you and Frodo—well Frodo is less eager—now want to cross the bulge to the far end of the disk of Galaxy: distance $100,000\,\text{lyr}$. But you want to live to see the journey's end. What physical effect makes it possible to survive the journey?

    c) How much **LESS** than the speed of light (in units of c: i.e., in terms of β) do you need to go at to make the pan-Galactic journey of $100,000\,\text{lyr}$ in 1 year for **YOU**? **HINT:** Recall the first order Taylor expansion

    $$\frac{1}{\sqrt{1+x}} \approx 1 - \frac{1}{2}x$$

    which is valid for $x << 1$.

    d) How long are each of your the trips in dog years for Frodo? And why does Frodo want to stay at Sirius?

33. Once again you and your trusty hound Frodo are voyaging through Galactic space on your star ship Smaug.

    a) Unbeknownst to you, Frodo has an inner life in which he in comtemplates the universe. He asks himself—since it's no use asking you–"if the gravitational force is always attractive and dominates large-scale structure, why doesn't the whole Galaxy of stars collapse to form a super-massive blackhole?" Assuming Frodo answers himself correctly, what is his answer?

    b) While Frodo is musing, you are doing a few calculations. Your speed is a lazy $3.000 \times 10^4\,\text{km/s}$—kilometers per second mark you—relative to the more-or-less inertial frame of the Galaxy and Smaug's rest mass is $1.000 \times 10^5\,\text{kg}$. What is Smaug's **NON-RELATIVISTIC** kinetic energy?

    c) Calculate the kinetic energy of Smaug in special relativity. Does the special relativistic calculation of kinetic energy yield a significantly different value (say $0.1\,\%$ different) from the non-relativistic calculation?

    d) Just as Frodo is begging to go for a space walk, you realize from your space chart that you are on a straight line path for a black hole—your controls are jammed—could be Frodo was gnawing on some vital connection—the mass of the black hole is 1 solar mass (1 solar mass is $1.9891 \times 10^{30}\,\text{kg}$) and it is $1.5 \times 10^{11}\,\text{m}$ away. From a Newtonian calculation at what distance will you reach the speed of light! **NOTE**: Because of special and general relativistic effects this calculation is not really valid, but it does lead to some insight into what must happen.

e) There's only one thing left to do—get the last word. So when your Lorentz factor reaches 10^{10}—just before your gravitational shielding (a pure plot device) fails and your atoms become permanently uncooperative—you send a proper time 3 minute video message to all your friends. Just from special relativistic physics, how long will your video message run in **YEARS** in the more-or-less inertial frame of the Galaxy? **NOTE**: General relativity will also affect time and in reality you won't be at a constant velocity.

34. The proton mass in energy units is 938.272029 MeV or approximately 1 GeV. The proton synchrotron at Fermilab near Chicago can (or could) accelerate a proton to total energy (i.e., kinetic energy plus rest mass energy) of about 500 Gev.

a) What is the kinetic energy of the proton to the same order of accuracy that the total energy was quoted to?

b) What is the Lorentz factor of the proton?

c) What is the velocity β of the proton?

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

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2 Trigonometry

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

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3 Blackbody Radiation

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\left[e^{h\nu/(kT)}-1\right]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\left[e^{hc/(kT\lambda)}-1\right]}$$

$$B_\lambda\,d\lambda = B_\nu\,d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$k = 1.3806505(24)\times10^{-23}\,\text{J/K} \qquad c = 2.99792458\times10^{8}\,\text{m}$$

$$h = 6.6260693(11)\times10^{-34}\,\text{J s} = 4.13566743(35)\times10^{-15}\,\text{eV s}$$

$$\hbar = \frac{h}{2\pi} = 1.05457168(18) \times 10^{-34}\,\mathrm{J\,s}$$

$$hc = 12398.419\,\mathrm{eV\,\AA} \approx 10^4\,\mathrm{eV\,\AA} \qquad E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^4 \qquad \sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2h^3} = 5.670400(40) \times 10^{-8}\,\mathrm{W/m^2/K^4}$$

$$\lambda_{\max} T = \text{constant} = \frac{hc}{kx_{\max}} \approx \frac{1.4387751 \times 10^{-2}}{x_{\max}}$$

$$B_{\lambda,\mathrm{Wien}} = \frac{2hc^2}{\lambda^5} e^{-hc/(kT\lambda)} \qquad B_{\lambda,\mathrm{Rayleigh-Jeans}} = \frac{2ckT}{\lambda^4}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L}n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L}n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^2}{\pi^2}\,dk = \pi\left(\frac{2}{c}\right)\nu^2\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z + \frac{1}{2}\right)\ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^3} + \frac{1}{1260z^5} - \dots$$

$$\ln(N!) \approx N\ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1 - e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x - vt) \qquad f(kx - \omega t)$$

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4 Photons

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\mathrm{scat}} - \lambda_{\mathrm{inc}} = \lambda_{\mathrm{C}}(1 - \cos\theta)$$

$$\lambda_{\mathrm{C}} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,\mathrm{m} \qquad e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$m_e = 9.1093826(16) \times 10^{-31}\,\mathrm{kg} = 0.510998918(44)\,\mathrm{MeV}$$

$$m_p = 1.67262171(29) \times 10^{-27}\,\mathrm{kg} = 938.272029(80)\,\mathrm{MeV}$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m \rangle = \ell^m m!$$

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5 Special Relativity

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_{\mathrm{obj}} = \beta_{\mathrm{obj}} - \beta$	$\beta'_{\mathrm{obj}} = \dfrac{\beta_{\mathrm{obj}} - \beta}{1 - \beta\beta_{\mathrm{obj}}}$

$$\ell = \ell_{\mathrm{proper}}\sqrt{1-\beta^2} \qquad \Delta\tau_{\mathrm{proper}} = \Delta\tau\sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c\beta \qquad E_0 = m_0 c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0 c^2)^2} - m_0 c^2 = (\gamma - 1) m_0 c^2$$

$$f = f_{\mathrm{proper}}\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\mathrm{proper}}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\mathrm{trans}} = f_{\mathrm{proper}}\sqrt{1-\beta^2} \qquad f_{\mathrm{trans}}(\beta << 1) = f_{\mathrm{proper}}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1} x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\text{intersection}}}{\gamma} = x'_{x\ \text{scale}} \sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\text{intersection}}}{\gamma} = \tau'_{\tau\ \text{scale}} \sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\text{Mink}} = \tan^{-1}(\beta)$$