Magnetic Forces on Currents

1) The <u>magnetic force</u> is more complex than the <u>electric force</u>.

The force on a charged particle q is

$$\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B} \qquad \text{(HR-660)}$$

- $\underset{\sim}{F}$: force
- q: charge
- $\underset{\sim}{v}$: particle velocity
- $\times$: cross product
- $\underset{\sim}{B}$: magnetic field.

~~From this we ...~~

Recall

$$\underset{\sim}{A} \times \underset{\sim}{B} = AB\sin\theta\,\hat{n},$$

for cross product

where $\hat{n}$ is a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$ and whose direction is determined by a right-hand rule

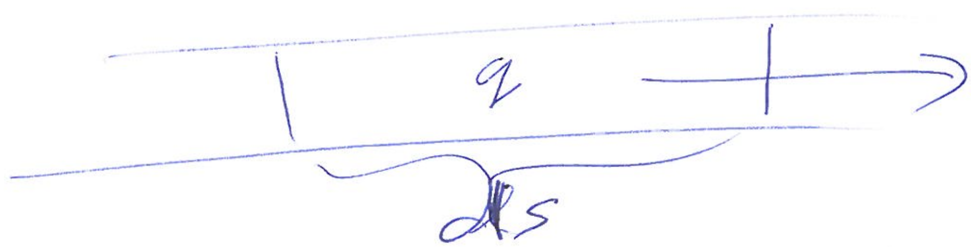
sweep fingers of right hand from $\underset{\sim}{A}$ to $\underset{\sim}{B}$ and the thumb gives the $\hat{n}$ direction.

$$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{cases} AB\sin\theta\,\hat{n} \\ AB\,\hat{n} & \theta = 90^\circ \\ 0 & \theta = 0^\circ \text{ or } 180^\circ \\ -\underset{\sim}{B} \times \underset{\sim}{A} \end{cases}$$

cross product multiplication
anti commutes

2) B-force on a current element

Consider a thin wire



Let q be the free charge in length ds.

This charge all moves right with velocity $\underset{\sim}{v}$

The current is $I = \frac{q}{dt}$ where dt is the time for all the change to clear out of ds

$$I\,ds = q\,\frac{ds}{dt}$$

$$I\,d\underset{\sim}{s} = q\,\frac{d\underset{\sim}{s}}{dt}$$

where we have vectorized $ds \rightarrow d\underset{\sim}{s}$

$d\underset{\sim}{s}$ points along the thin wire in the direction of the current.

$$I\,d\underset{\sim}{s} = q\,\frac{d\underset{\sim}{s}}{dt} = q\,\underset{\sim}{v}$$

The total magnetic force on the current element is

$$d\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

or

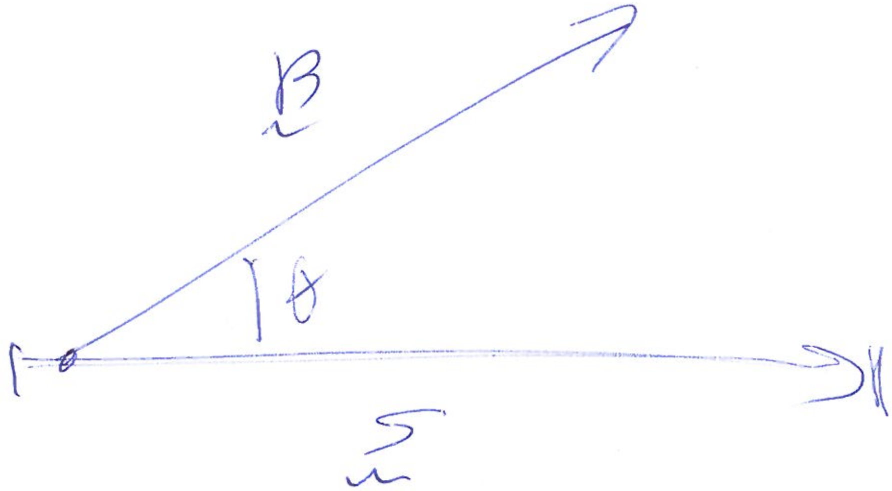
$$d\mathbf{F} = I\,d\mathbf{s} \times \mathbf{B} \quad \text{(HR-680)}$$

If one integrates over the ~~whole wire~~, ~~F~~ a finite ~~bit~~ segment of wire

$$\mathbf{F} = \int I\,d\mathbf{s} \times \mathbf{B}$$

If the wire segment is straight and $\underset{\sim}{B}$ is constant over the segment

$$\underset{\sim}{F} = I \, \underset{\sim}{s} \times \underset{\sim}{B}$$



3) B-fields of Long straight wires

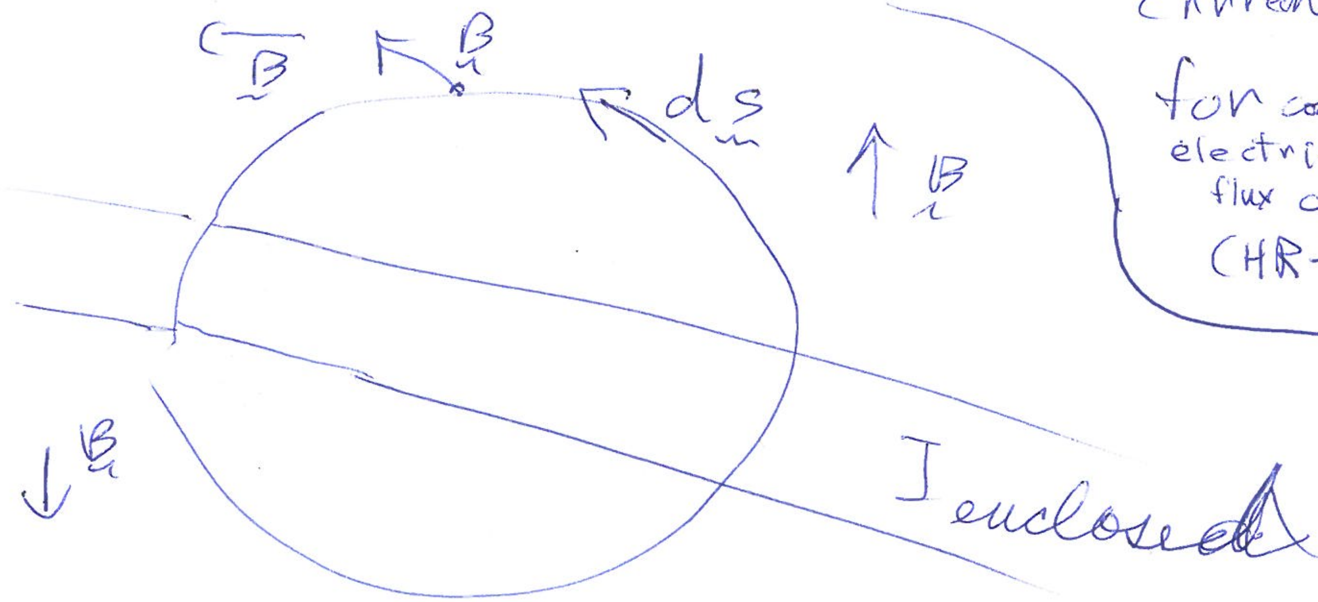
Ampère's law

(HR-694)

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{s} = \mu_0 I_{enclosed}$$

current

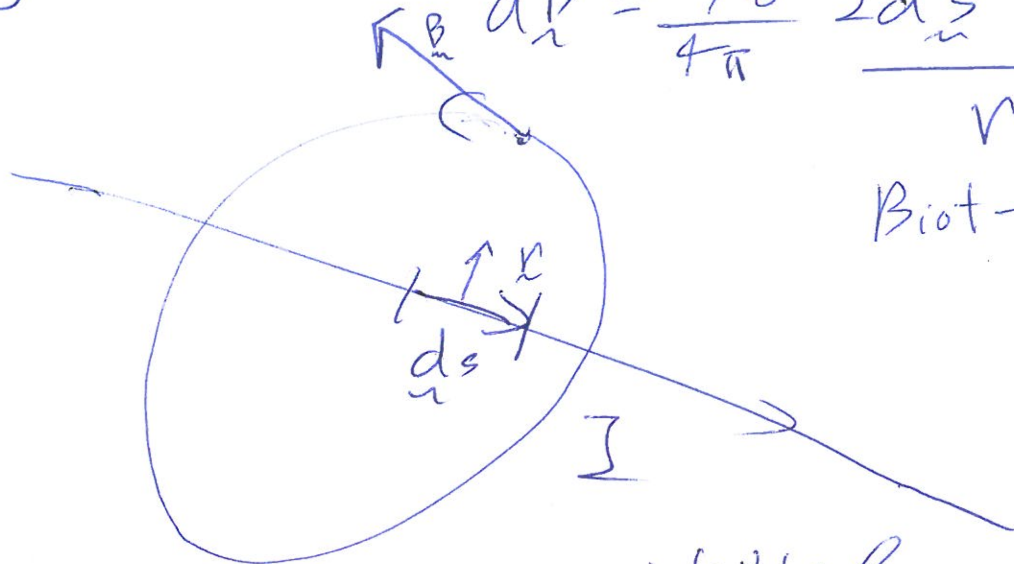
for constant electric flux cases (HR-759)



In a few cases of high symmetry, one can use Ampère's law to solve the B-field directly.

$$d\underset{\sim}{B} = \frac{\mu_0}{4\pi} \frac{I d\underset{\sim}{s} \times \hat{r}}{r^3}$$

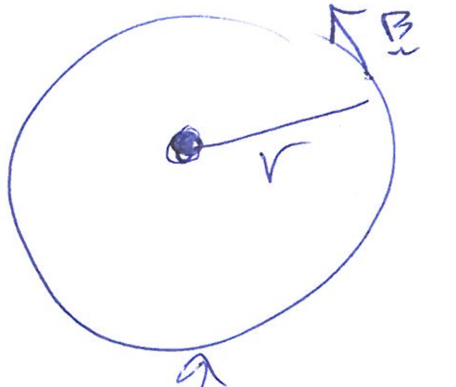
Biot-Savart law (HR-688)



infinitely long thin wire.

$d\underset{\sim}{s} \times \hat{r}$

suggests ~~that~~ $\underset{\sim}{B}$ points counterclockwise around the wire and is tangent to a circle centered.



Wire coming straight out of page

field line

Symmetry tells us that $|\mathbf{B}|$ should be constant on circle & the Biot-Savart law that it should be tangent and the field line run counterclockwise.

$$\therefore \quad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{encl}$$

$$B \, 2\pi r = \mu_0 I$$

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

The direction of $\hat{\theta}$ is determined in general by another right-hand rule!

Put the thumb of your right hand in the current direction and your fingers curl in the direction of the B-field line.

4) Forces between Infinite thin Wires

Conventions

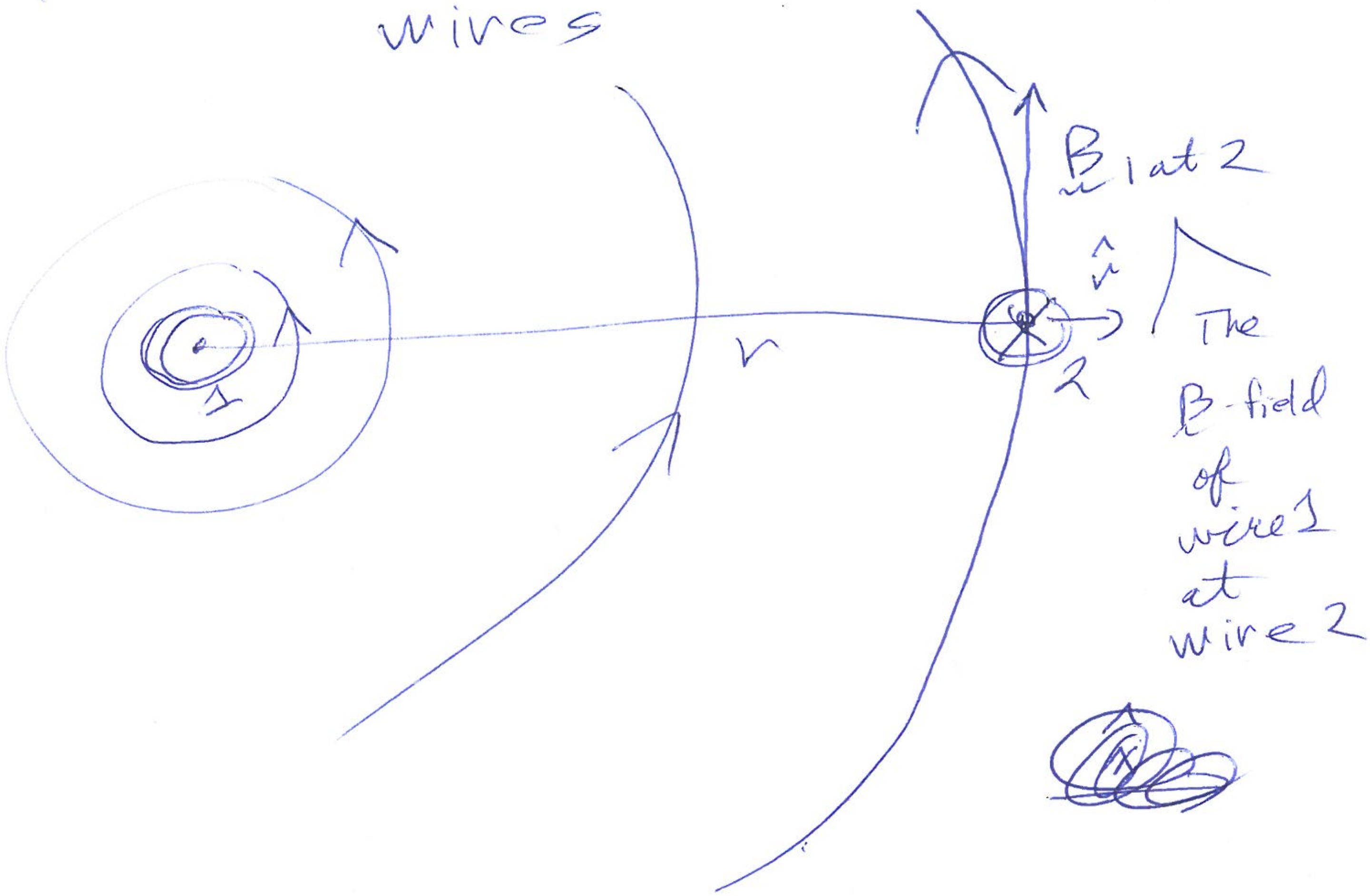
$\odot$

Vector pointing out of page (a head of an arrow coming at you)

$\otimes$

Vector pointing into page (tail feathers of arrows going away from you)

Two current in infinite long wires



Wire 2 only feels wire 1's B-field, not its own.

[Recall Biot-Savart law (HR-688)

$$d\underline{B} = \frac{\mu_0}{4\pi} \frac{I\, d\underline{s} \times \hat{r}}{r^3}$$

if $d\underline{s}$ and $\hat{r}$ are parallel $d\underline{s} \times \hat{r} = 0$]

12)

$$\vec{B}_{1\,at\,2} = \frac{\mu_0 I_1}{2\pi r}\hat{\theta}$$

$$d\vec{F}_{1\,on\,2} = I_2\, d\vec{s}_2 \times \frac{\mu_0 I_1}{2\pi r}\hat{\theta}$$

$$\vec{F}_{1\,on\,2} = \frac{\mu_0 I_1 I_2 l}{2\pi r}\left(\mp \hat{r}\right)$$

for length l

$$f_{1\,on\,2} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

force magnitude per unit length.

$-\hat{r}$ for I_2 out of page

$+\hat{r}$ for I_2 into page

or

$-\hat{r}$ for I_2 and I_1 in opposite directions

$+\hat{r}$ for I_2 and I_1 in the same direction.

So do parallel currents attract or repel?

Do antiparallel currents attract or repel?

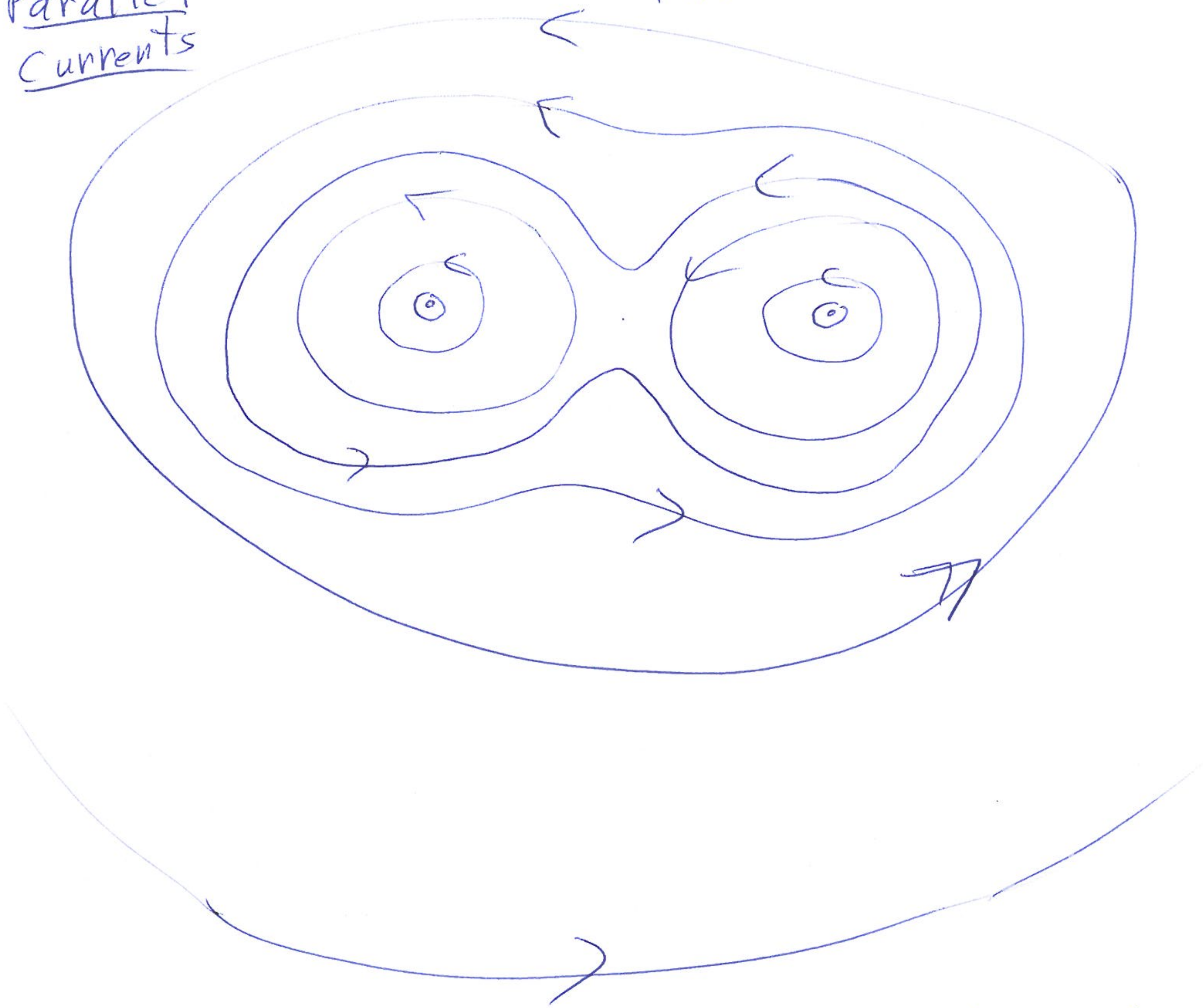
5) Net Field of Two infinite ~~aligned~~ parallel wires

To calculate the force between too wires you don't need to know this net field,

Which is good because it's complex.

Recall $\underset{\sim}{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$
for an infinite thin wire

Parallel Currents



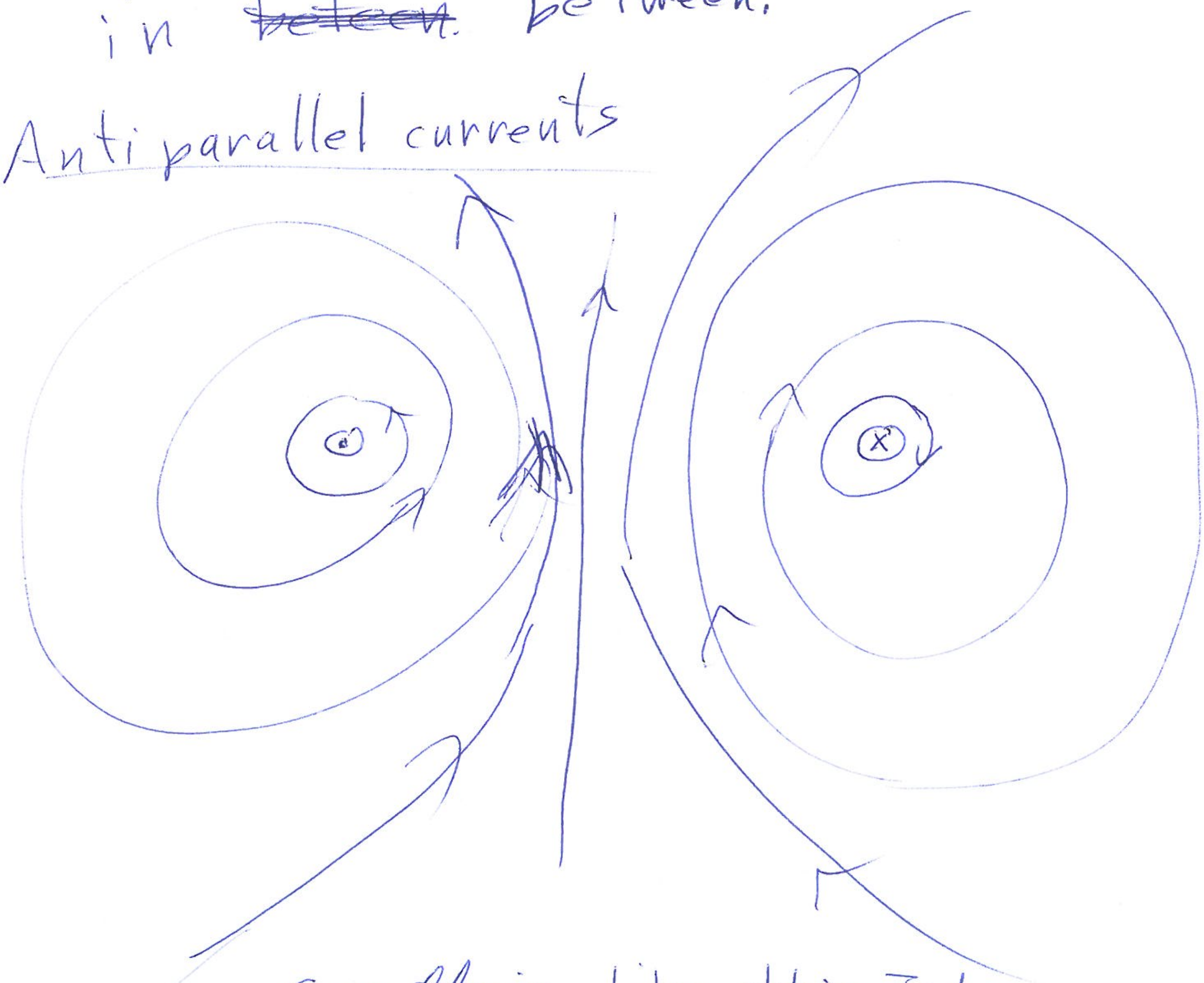
Close to each one, their individual field dominate's

Far away, they must become like one infinite wire.

In between near
and far field limits,
one can interpolate.

There must be a point
of zero field somewhere
in ~~beteen.~~ between.

Anti parallel currents



Something like this I'd guess

In far field limit, there seems
no net current

and so $B_{far\ field}$ must fall off faster the $\frac{1}{r}$ where r is measured from a point between the wires.

6) Practicalities

We are using power sources that can be crank up to high voltage and current.

So try to keep

$V \lesssim 5\,V$

$I \lesssim 10\,A$

and don't touch live (i.e., current carry or high potential conductors with your hands).

We are just being ultrasafe. Just good practice.

Use a pencil or pen to damp oscillations of the <u>current balance</u>.

Turn knobs of source down when turning off source for any length of time.
(Just so when you turn it ~~do~~ on you arn't surprised)

With $V \lesssim 5V$, there is probably no danger of a shock.

<u>But</u> if you break

a large current ~~there w~~ quickly, there will be an induced EMF

$$\mathcal{E} = L \frac{dI}{dt}$$

$\frac{dI}{dt}$ gets large, $\mathcal{E}$ gets large

that can cause arcing at the point of break.

~~Not too dangerous~~ but surprising.

L is the inductance of the current loop — determined by geometry

Also with the large current, wires can get <u>hot</u>.

Task 1

a good way to hold wires

Use the right hand rule to predict direction of the magnetic force.

Task 2
Task 3
} the repulsion and attraction forces are rather small

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$ force per unit length

$$= \frac{(4\pi \times 10^{-7}) I_1 I_2}{2\pi r}$$

$$= \frac{2}{r} \times 10^{-7} I_1 I_2$$

r in meters.

So even $r = 10^{-3}$ m

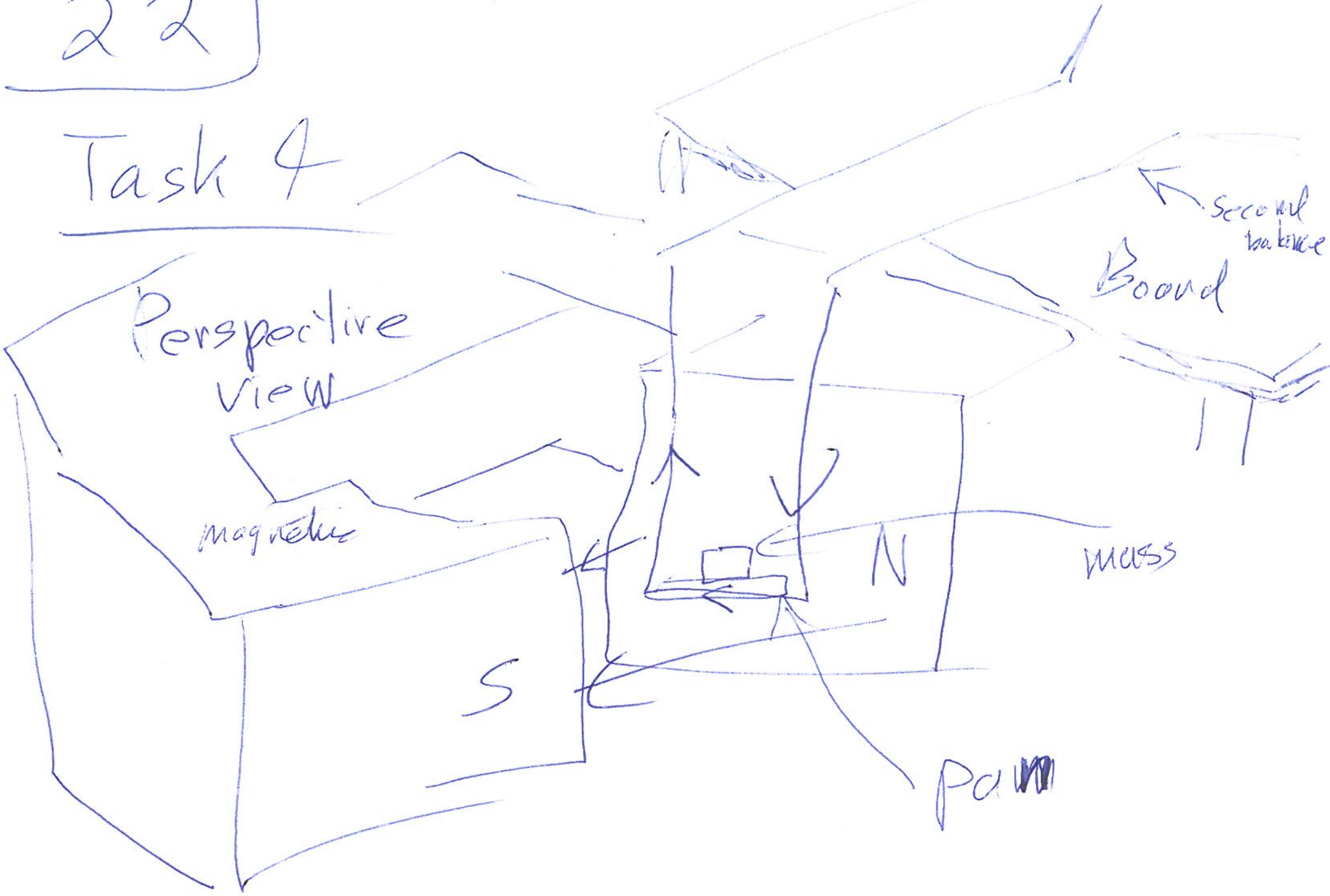
= 1 mm

won't give a big force.

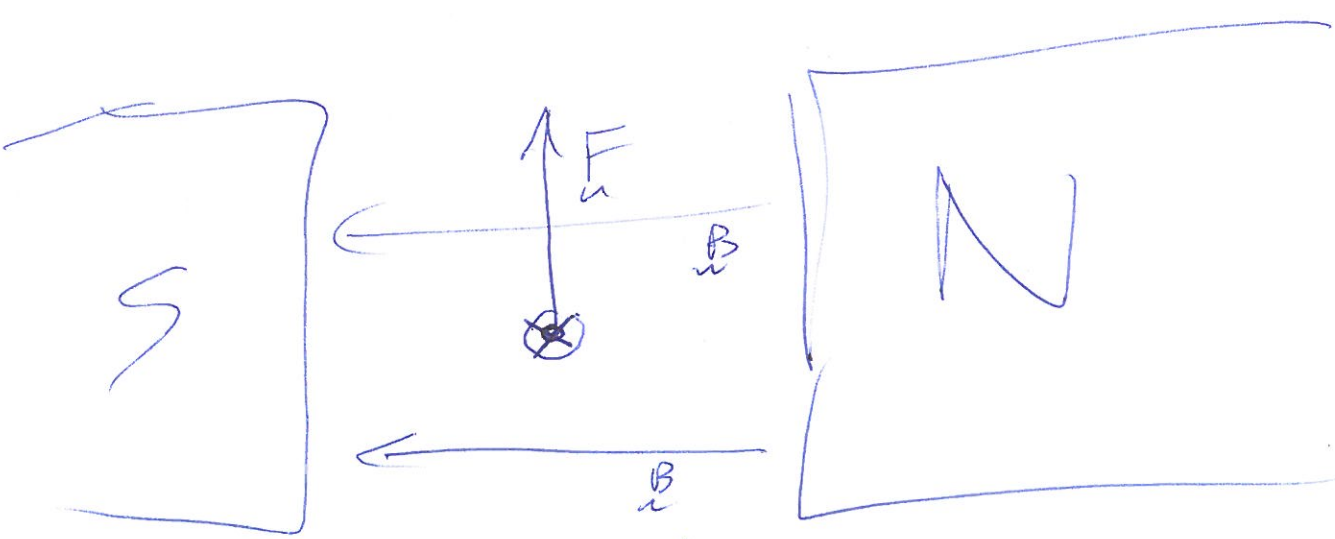
The balance will just bob a little with switching current off and on.

Task 4

Perspective view

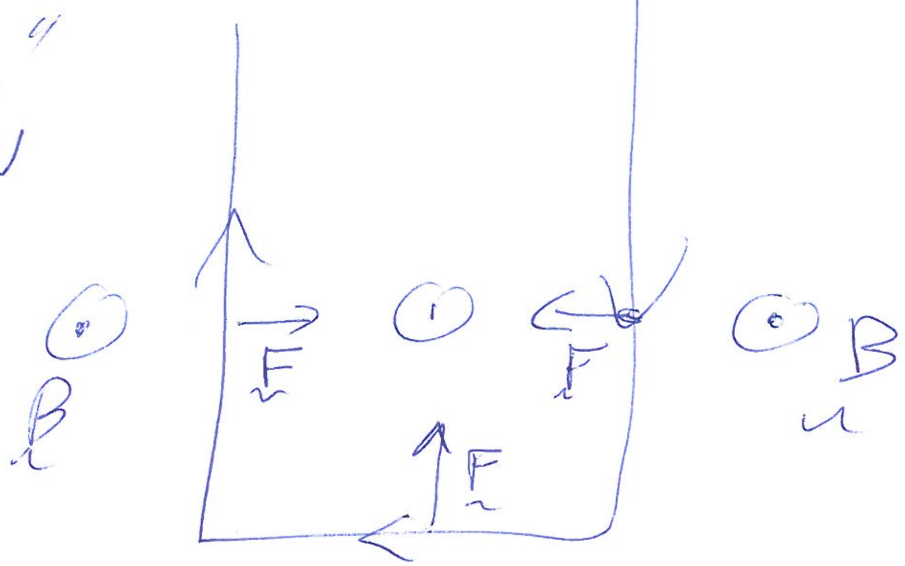


Side view



current out of page.

"Front" View



The side forces cancel and in any case the balance isn't free to move horizontally. It's constrained.

Bottom

$$F = \int I \, d\underset{\sim}{s} \times \underset{\sim}{B}$$

$$= I s B$$

s: length of horizontal current segment ~~carry~~

24)

If the second balance is balanced with No ~~B field an~~ B-force or extra mass, and ~~then~~ with extra mass and B-force it is balanced again.

then

$$\left| F_{B\,\text{up}} \right| = \left| F_{g\,\text{down}} \right| .$$

on an extra mass.