# Discharging Capacitors

## 1) Capacitors

- Capacitors are simple circuit devices for storing charge and energy.
- They are extremely important and have many uses.
  - frequency tuning
  - energy for fast release
  - maybe one day powering your car.
- The equation describing their behavior is

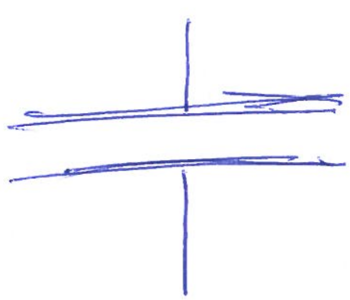
$$V = \frac{Q}{C}$$

Charge on the positive plate

potential drop across the capacitor plates

capacitance — a property that ideally depends only on the geometry of the capacitor and the dielectric it is embedded in.

Circuit Symbol

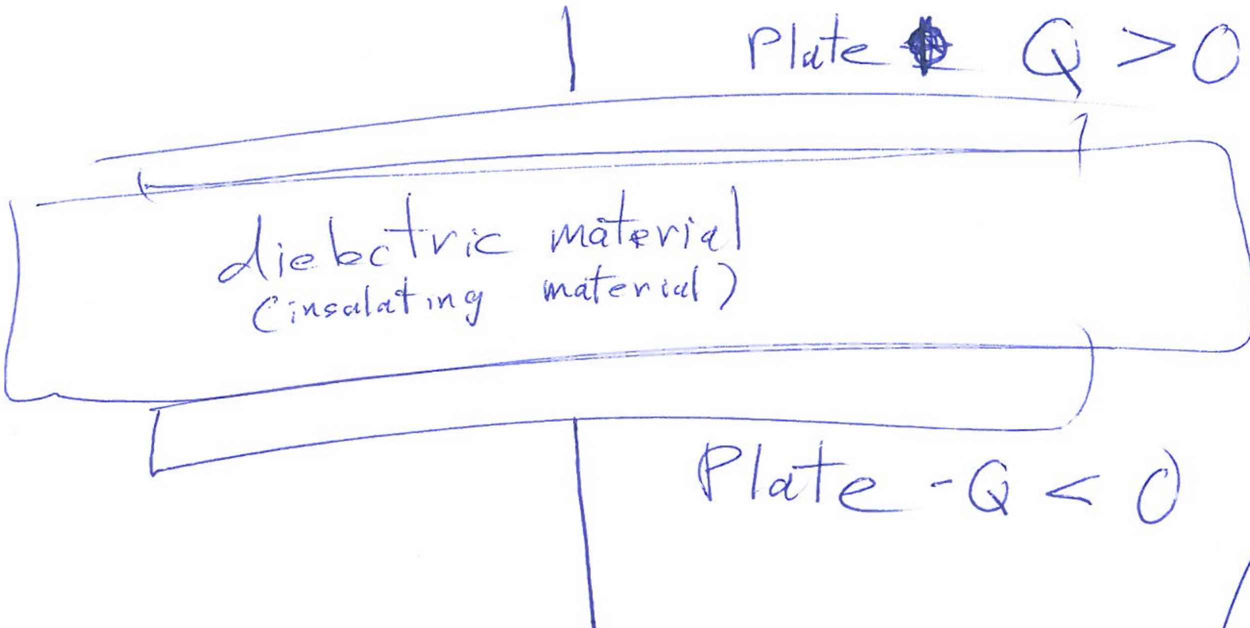


or $C = \frac{Q}{V}$

which gives the interpretation of $C$ as how much charge can be stored per volt.

Physically one has two metal plates

Don't have to be planar - They can be rolled up in a cylinder or other configurations




a charge $Q$ is put on the positive plate and a negative $-Q < 0$ is put on the negative plate.

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The <u>potential energy</u> stored in a capacitor is easily found

$$d\,PE = V\,dq$$

change in PE by moving $dq$ from $-$ve plate to $+$ve plate

$V = \frac{q}{C}$ is the potential difference at the time of transferring $dq$

$$PE = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}QV$$

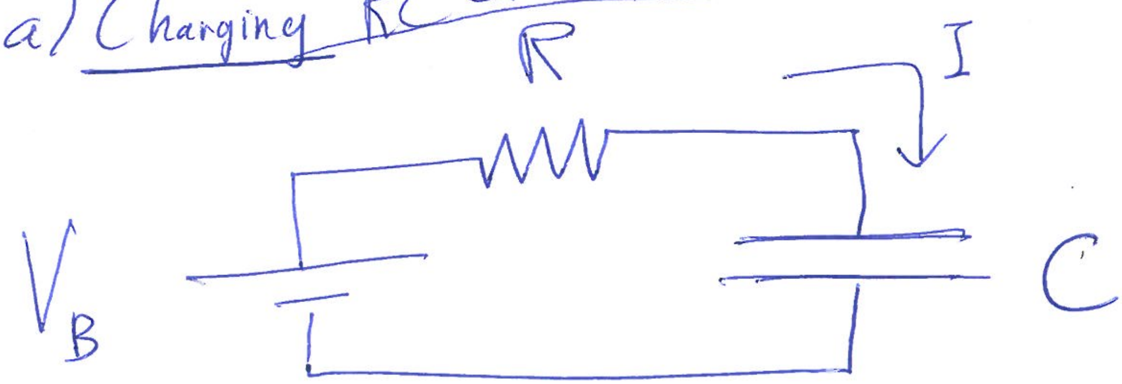
$$PE = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

## 2) RC Circuit

A simple RC circuit has a <u>resistor</u> and <u>capacitor</u> in series.

— One also includes a battery for charging.

a) <u>Charging RC Circuit</u>



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We apply Kirchhoff's Voltage Law

$$V_B = IR + \underbrace{\frac{Q}{C}}_{\text{from capacitor law}}$$

from ohm's law

Now we assume the capacitor ~~potential~~ charge ~~was entirely~~ is due to current flowing into the capacitor:

$$Q = \int_{0}^{t} I(t')\,dt' + \underbrace{Q_0}_{\text{time zero charge}}$$

We assume at time $t = 0$,

$$Q = 0$$

i.e., the capacitor starts uncharged.

$\therefore$ at $t = 0$, $V_B = I_0 R$

$I_0 = \frac{V_B}{R}$ is the time zero current.

We note that the current builds up the charge $Q$.

$$\therefore \quad Q = \int_0^t I(t')\,dt'$$

$$\therefore \quad \frac{dQ}{dt} = I$$

We can solve for $I$.
Experience tells me the easiest way is to differentiate the

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Kirchhoff's law equation to

$$0 = \frac{dI}{dt} R + \frac{I}{C}$$

A differential equation for I, not an algebraic equation.

which rearranges to

$$\frac{1}{I}\frac{dI}{dt} = -\frac{1}{RC}$$

unit [RC]

$= \frac{\text{Volt}}{\text{Coulomb/s}} \frac{\text{Coulomb}}{\text{Volt}}$

$=$ second and thus RC does have units of time.

We define $\tau \equiv RC$,

where $\tau$ is the time constant or <u>e-folding time</u> (for reasons that soon become evident).

Now $$\frac{dI}{I} = -\frac{dt}{\tau}$$

We integrate both sides to get

$$\ln(I')\Big|_{I_0}^{I} = -\frac{t}{\tau}$$

$$I/I_0 = e^{-t/\tau}$$

$$I = I_0 e^{-t/\tau}$$

*Recall* $\displaystyle Q = \int_0^t I(t')\,dt'$

$$Q = I_0(-\tau)e^{-t/\tau}\Big|_0^t$$

$$= I_0\tau(1-e^{-t/\tau})$$

$$= \frac{V_B}{R}RC(1-e^{-t/\tau})$$

$$= CV_B(1-e^{-t/\tau})$$

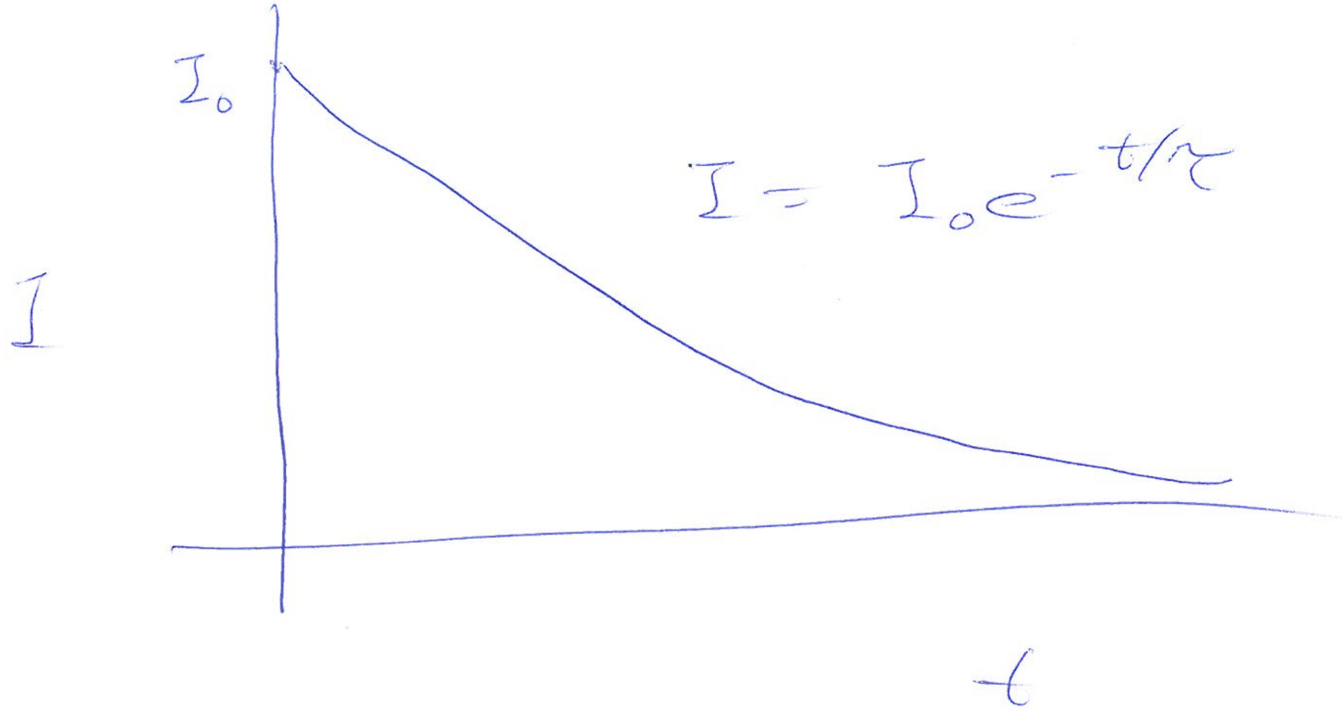
$$= Q_\infty(1-e^{-t/\tau})$$

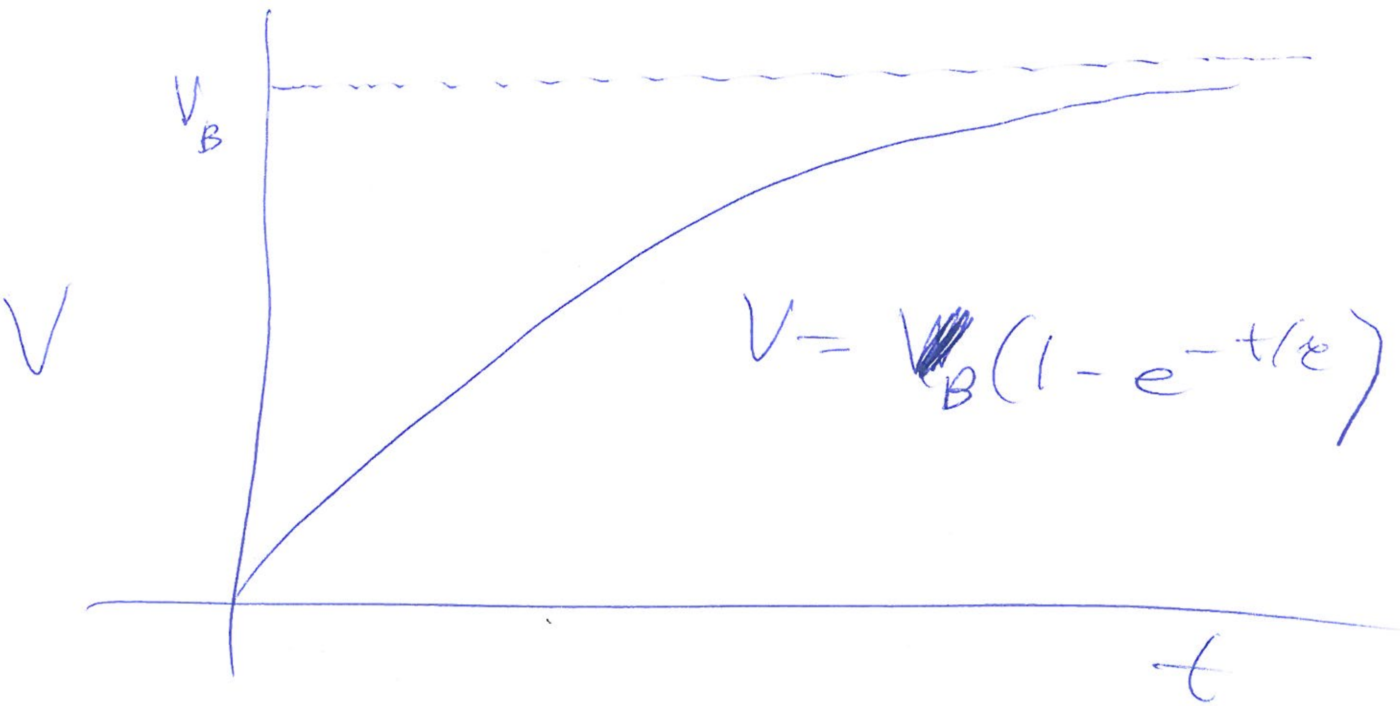
where we define $Q_\infty = CV_B$

*Now* $\displaystyle V = \frac{Q}{C} = V_B(1-e^{-t/\tau})$

$$V = V_B(1-e^{-t/\tau})$$

Our solutions graphically are




Formally $I$ goes to zero and $V$ goes to $V_B$ only at $t = \infty$.

But practically after
a few $\tau$'s typically
$I$ is so close to zero
and $V$ to $V_B$ that
the differences are
smaller than
error in the measurements
and/or
fluctuations in the system.

So for practical purposes
$I \to 0$
and $V \to V_B$
in a finite time.

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## b) Discharging the RC circuit

We remove the battery of the RC circuit when the capacitor is fully charged.

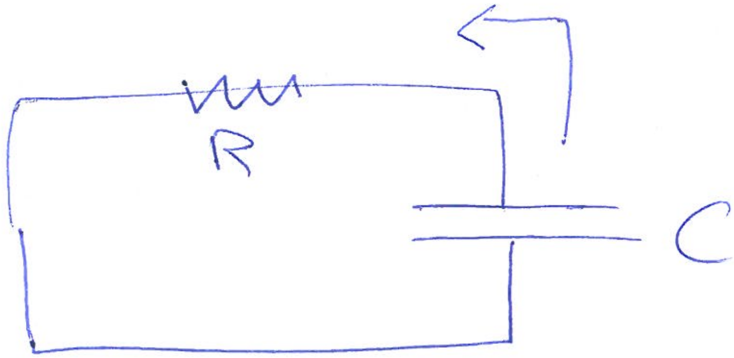
<u>For</u> the new calculation

$$Q_{0\,\text{new}} = Q_{\infty\,\text{old}}.$$

<u>Kirchhoff's Voltage</u> law applied gives after one has closed the circuit.

$$0 = IR + \frac{Q}{C}$$

No driver term this time.




$$I_0 = -\frac{Q_0}{RC} = -\frac{Q_0}{\tau}$$

We can still use

$$\frac{dQ}{dt} = I$$

as long as we understand a negative current as flowing clockwise and dischanging the capacitor.

Differentiating ~~again~~ the Kirchhoff Voltage law, we get

$$0 = \frac{dI}{dt} R + \frac{I}{C}$$ just as on p. 8

Thus the solutions for $I$ is the same

$$I = I_0 e^{-t/\tau}$$

$$I = -\frac{Q_0}{\tau} e^{-t/\tau}$$

We integrate to get Q

$$\int_{Q}^{t} I\,dt = -\frac{Q_0}{\tau}(-\tau)\,e^{-t/\tau}\Big|_0^t$$

$$Q - Q_0 = -Q_0\left(1 - e^{-t/\tau}\right)$$

$$Q = Q_0 e^{-t/\tau}$$

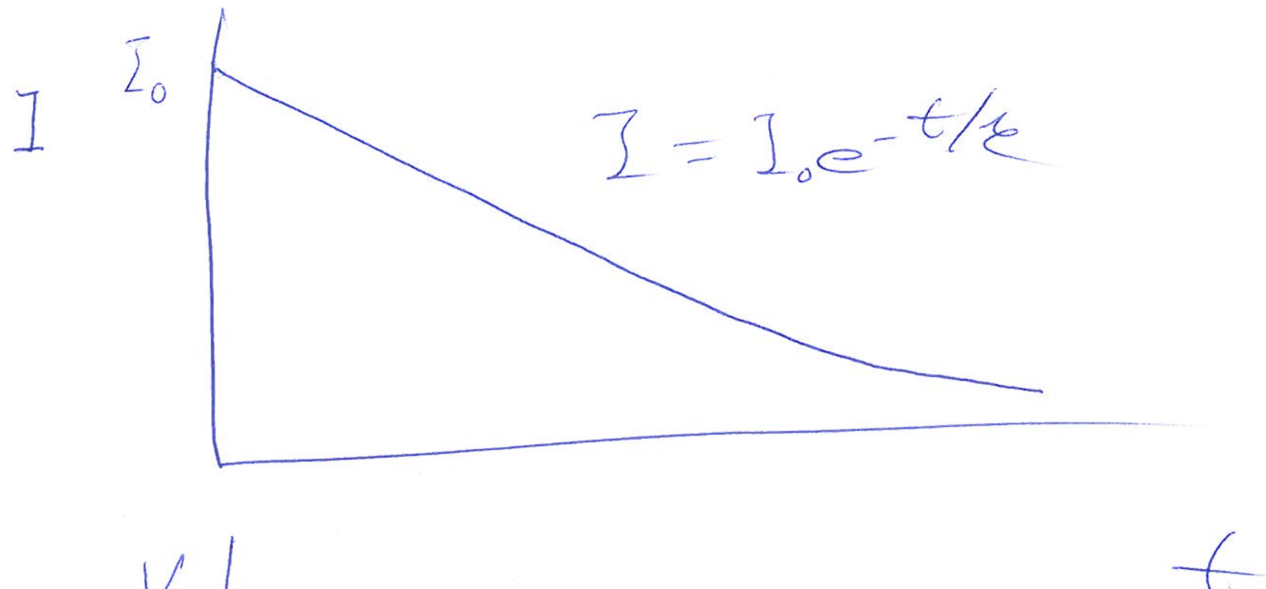
$$V = \frac{Q}{C} = \frac{Q_0}{C} e^{-t/\tau} = V_0 e^{-t/\tau}$$

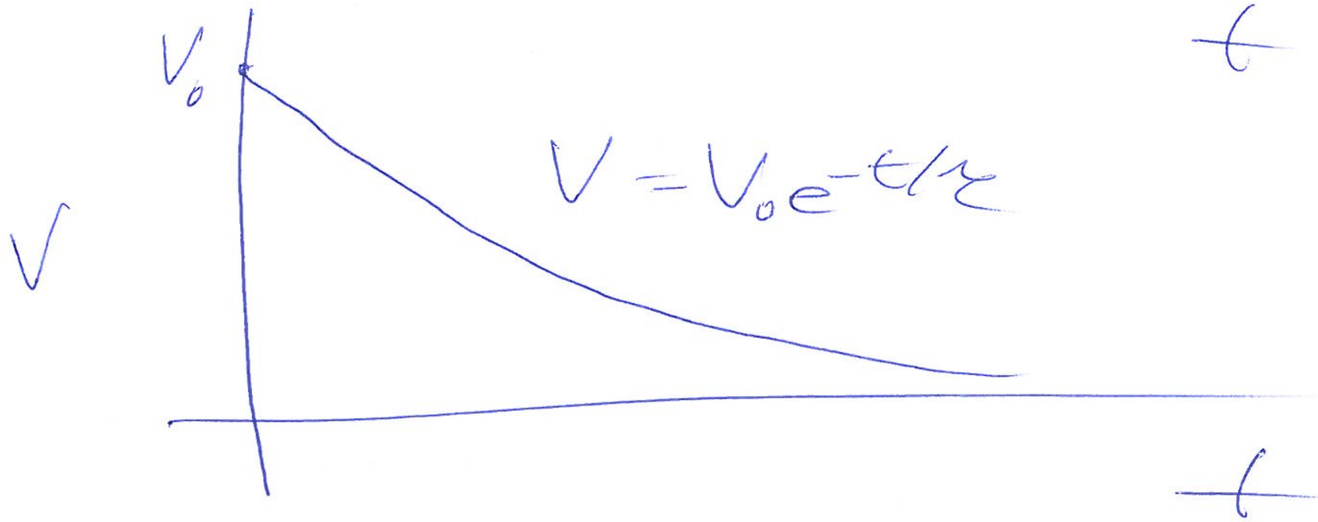
Changing our sign convention on $I$ ( $I_{new} = -I_{old}$ ) our solutions become

$$I = I_0 e^{-t/\tau}$$

$$V = \underbrace{V_0}_{=V_B \text{ if we charged up with a battery.}} e^{-t/\tau}$$

Graphically




Formally $I \to 0$, $V \to 0$
only at $t = \infty$.

But, in fact, their differences from ~~other~~ zero becomes less ~~than~~ error ~~and~~/or fluctuations within a few $\tau$'s typically.

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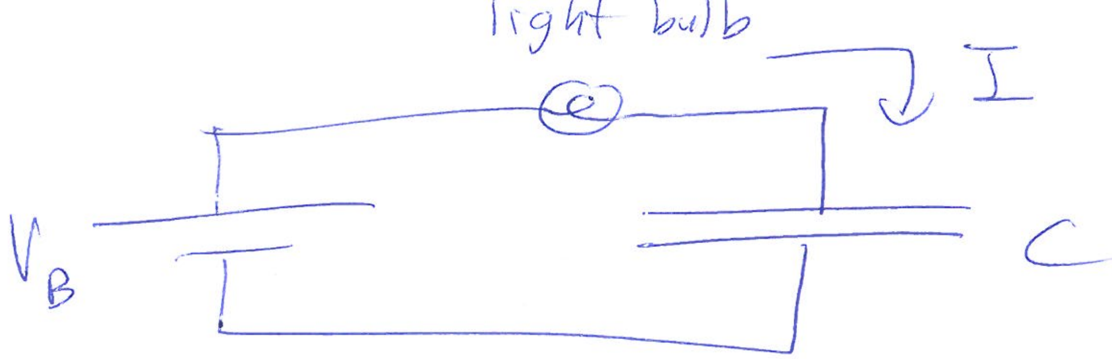
c) Charging a bulb - C circuit



What if we charge the capacitor using a light bulb rather than a resistor?

Well Kirchhoff's law gives

$$V_B = V_{bulb}(I) + \frac{Q}{C}$$

Final state $I=0$

i.e. $Q = CV_B$

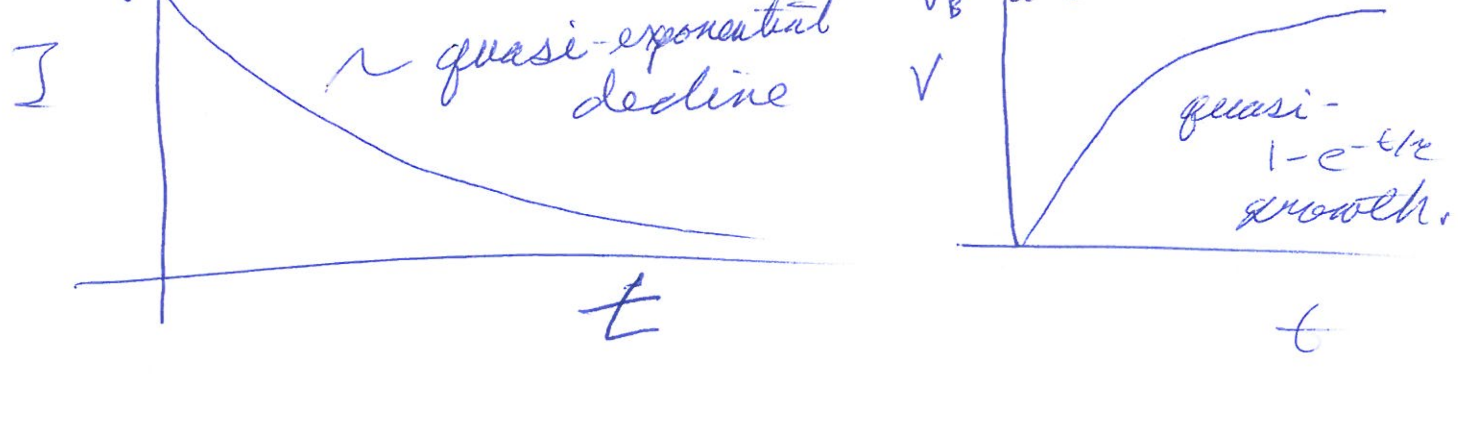
The bulb is <u>not</u> ohmic.

So $V_{bulb} \neq IR$

We cannot solve the differential equation without specifying $V_{bulb}(I)$

and even if we did an analytic solution may not be possible — Maybe only a numerical solution could be obtained.

**But** qualitatively we expect the behavior to be much like charging an RC circuit

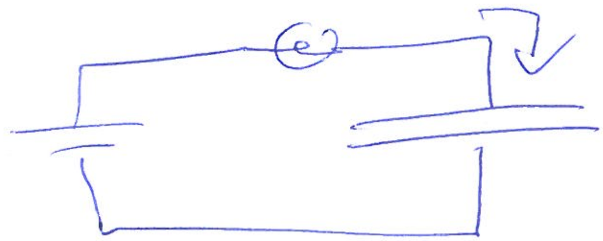


## 3) Testing Our Discharge Theory

$$V = V_0 e^{-t/\tau}$$

$$\tau = RC$$

We charge up our capacitor using a bulb-C circuit



The bulb is ~~just~~ for display. The capacitor would charge very quickly with no bulb.

Maybe too quickly

Initial current $I_0 = \frac{V_B}{R}$ (see p. 7)

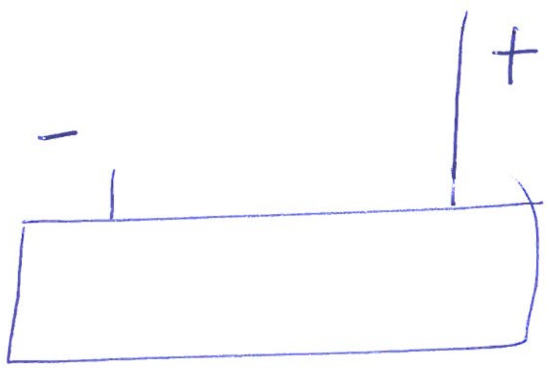
If R is very small, $I_0$ can become very large for a short time

— time scale for changing is $\tau = RC$ recall.

Our capacitors are modern tantalum electrolytic capacitors of $C = 1.0$ Farad

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

Unlike standard capacitors the electrolytic capacitors have a definite polarity




long prong must be attached to positive lead

Must be attached to negative lead (Wik)

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~~They actually~~

Ours actually seem to work in reverse mode —— But that is NOT recommended.

Actually 1 Farad is a huge capacitance for a small ~~lab/ capa~~ lab/circuit capacitances

- typical lab capacitors are of order

picofarads $= 10^{-12}$ F

or nanoFarads $= 10^{-9}$ F

or microfarads $= 10^{-6}$ F

1 Farad means the capacitor holds a Coulomb with 1 Volt.

- A <u>net charge</u> of a coulomb

is actually an enormous dangerous charge, but of course the capacitor is overall neutral  1 C on +ve plate

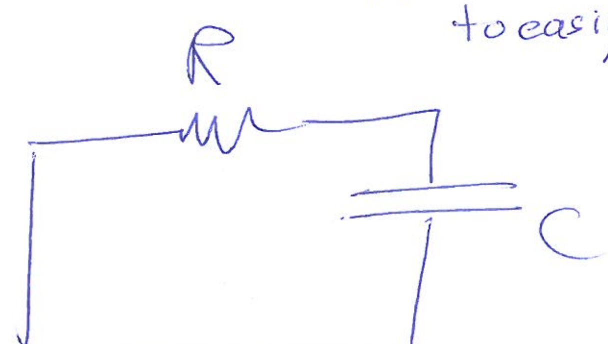
-1 C on the -ve plate

## Measurements

a) Charge up capacitor

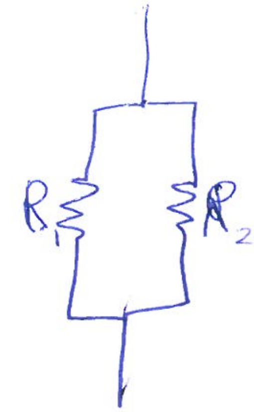

Use Galvanometers ~~as~~ ammeters to check current. NOT digital ammeters (their fuses blow to easily).

b) Discharge




- we don't use resistor boxes any more (they kept getting overheated) and ~~breaking~~ burning out

Use two resistors of $150\,\Omega$ in parallel.




What is their net resistance?

Measure $V_0$ at time zero

Table

| $t$ (s) | $V$ (V) | $\ln(V)$ |
|---|---|---|
| $0 \pm ?$ | $V_0 \pm ?$ | $\ln V_0$ |
| 10 | ⋮ | ⋮ |
| 20 | ⋮ | ⋮ |
| 30 | ⋮ | ⋮ |

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Use 10 second intervals and read ~~as~~ ~~long~~ for 3 minutes

18 data points altogether.

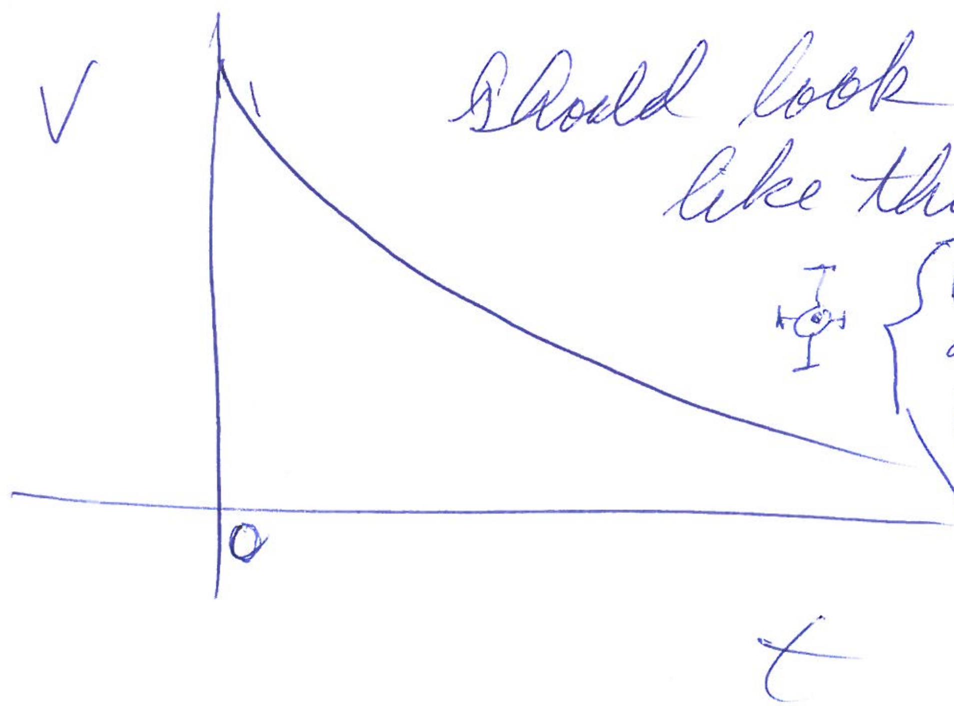
(Redundant data is good. Bad data points due to misreading etc., can be spotted and left out of consideration)

You must estimate a time error $\Delta t$ and a potential error $\Delta V$.

These should be the same for whole set.

~~Or~~ Use Graphical Analysis to tabulated data and plot V versus t

mark errors by hand on table of data




Should look like this.

Representative data point with error bars

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But a curve can only be judged to be qualitatively consistent. with theory by eye.

We want a quantitative check.

So we need a linearized theoretical equation

Our equation is

$$V = V_0 e^{-t/\tau}$$

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a) So what is the linearized version?

<u>Hint</u> You need natural logarithms.

Answer on your lab.

~~b) What is the~~

b) So what do you plot against what to test the linearized version?

This is graph 2

— Some points should have error bars if they are large enough.

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c) You will need to do best and ~~worst~~ fit lines on Graph 2 to get slope and error in slope. Enter on your lab report.

Is the data linear within error? Answer on your lab.

d) From the slope you should get a value for the capacitance

$$C \pm \underbrace{\Delta C}_{\text{error in value.}}$$

How does this compare to the manufacturer value of 1.0 F.

Do you get agreement within error?

Answer on your lab

30)

e) You will need errors for $\ln V$.

How you find those?

Old trick for small error propagation

Expand $f(x+\Delta x)$ in a Taylor's series about $x$.

$$f(x+\Delta x) = f(x) + \Delta x\, f'(x) + \frac{\Delta x^2}{2!} f''(x) + \ldots$$

If terms of order $O(\Delta x^2)$ are negligible

$$\Delta f = f(x+\Delta x) - f(x)$$

$$\cong \Delta x \, f'(x)$$

Recall error is always positive.

$$\therefore \Delta f = |f'(x)| \Delta x$$ is the 1st order error propagation formula

Examples

a) $$y = x^{\frac{1}{2}}$$

$$\Delta y = \frac{1}{2} |x^{-\frac{1}{2}}| \Delta x$$

b) $$y = x^n$$

$$\Delta y = n |x^{n-1}| \Delta x$$

c) $$y = e^{kx}$$

$$\Delta y = k e^{kx} \Delta x$$

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d) $y = \ln|x|$

$$\Delta y = \frac{\Delta x}{|x|}$$

The error in a natural logarithm of $x$ is relative error of $x$

(Which is quite cute.)