Intro to Circuits

1) Intro

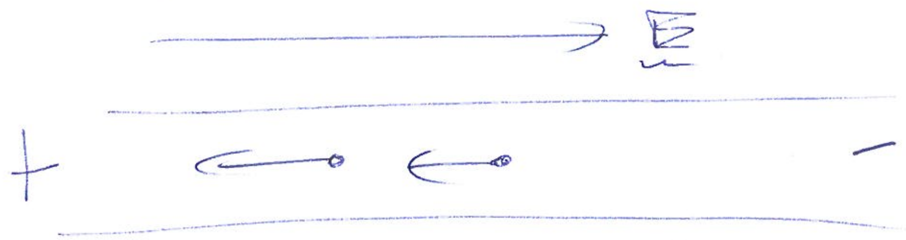
In a circuit, you have a closed system of flowing charge. 

— Charge flows around about in the circuit, but never gets in or out.

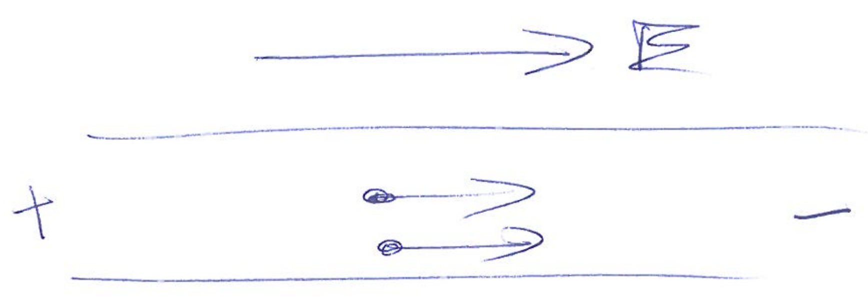
In DC circuits (direct current circuits) the flow in most common modes of operation is everywhere steady-state

In AC (alternating current) circuits, the flow in most common modes of operation is periodic or steady-state on average.

In most parts of most circuits, the conductors are metals and the ~~carries~~ charge carriers are electrons, which are negative.



negative carriers



positive carriers

In fact, in ~~most~~ for most effects, one can NOT tell if we have −ve carriers going to the left or +ve carriers going to the right.

It was NOT known ~~on~~ before <u>1879</u>, that metal carriers were negative (discovery then of magnetic <u>Hall effect</u> distinguished the sign of the carriers)

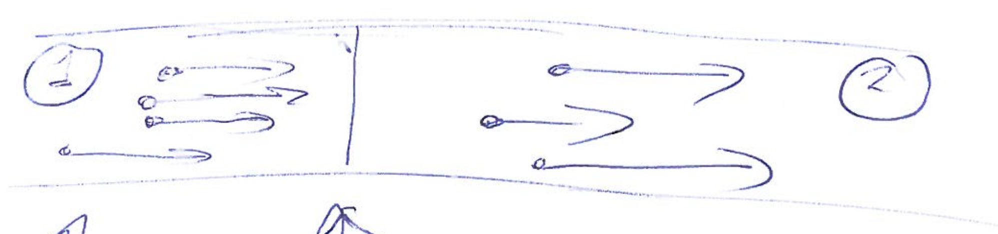
By then the <u>convention</u> had been long established to assume +ve carriers going from +ve to -ve

and that <u>convention</u> has held.

So <u>conventional current</u> flows from +ve to -ve

1) Ben Franklin actually named positive and negative and he guessed wrong since it would have been convenient to call the most mobile carriers electrons as positive.

2) Definition of Current



conducting path

some imagined boundary

~~Say Δq goes past boundary in time Δt~~

~~$I = \frac{\Delta q}{\Delta t}$ to the right.~~

Say Δq goes to the right across the boundary in time Δt.

$$I_{Ave} \equiv \frac{\Delta q}{\Delta t}$$

$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

MKS unit of current C/s $= \underbrace{1\text{ A}}_{\text{ampere}}$

often shortened to amp.

$\Delta q > 0$, flows right, $I > 0$
$\Delta q > 0$, flows left, $I < 0$
$\Delta q < 0$, flows left, $I > 0$
$\Delta q < 0$, flow right, $I < 0$

These sign conventions make sense.

Say Q is the net charge in section 2

$I > 0$ means Q is increasing

$I < 0$ means Q is decreasing

6

In simple DC circuits without ~~con~~ capacitors or inductors,

we think of current in every branch as steady-state usually.

i.e., I in each branch is not changing.

But of course, it must change when the circuit is changed.

— e.g., ① closing an open circuit

to allow flow in an ~~open~~ circuit

② opening a closed circuit

— to stop flow in an circuit.

These changes seem to happen instantaveously.

Question Why when you turn on a light, does the light come on instantaneously ~~seemley~~ seemingly?

Doesn't it take time for the electrons to get from switch to light.

ANS. The electrons are already in the wire and light.

Anologous to water in a pipe. Open a tap and water flows "instantly". The signal to go into motion propogates at about speed of sound in water.

An electric field signal starts them in motion and that signal travels at roughly the speed of light

$$c = 3 \times 10^8 \, m/s$$

$$= 1 \, ft/ns$$

3) Kirchhoff's Laws

Kirchhoff's laws are essential to understanding circuit behavior.

They are NOT fundamental laws, they are derived from more basic principles, but historical reasons they are usually called "laws" rather than "rules".

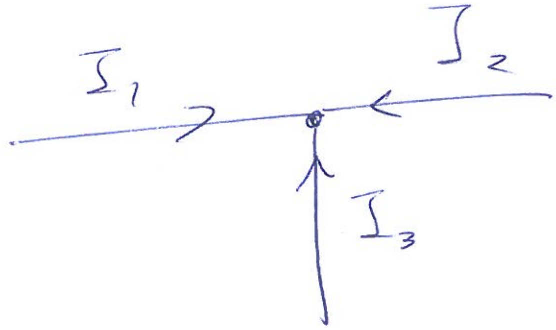
There are two of them.

a) Kirchhoff's current law

AKA the node law or the junction law

$$\sum_i I_i = 0$$

sum of currents into a node or junction equals zero



Or $\sum_i I_{i\,\text{inflow}} = \sum_i I_{i\,\text{outflow}}$

Given a no-charge build-up condition { Not necessarily steady-state.

~~Given steady state conditions~~ ~~(no charge build ups)~~

and conservation of charge

the current law must hold.

What if you have charge build ups?

Net charge build-ups are strongly resisted by the Coulomb force.

~~so~~ small fluctuating ones happen all the time, but usually too small to notice.

— e.g., the ~~end~~ end of a line wire will have a small build up.

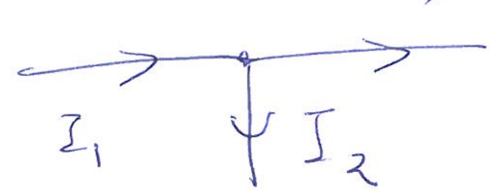
At the highest level of accuracy, you might need to worry about them.

<u>But</u> for most circuit work, they are insignificant.

~~In fact, the current,~~

<u>Examples</u>

i)



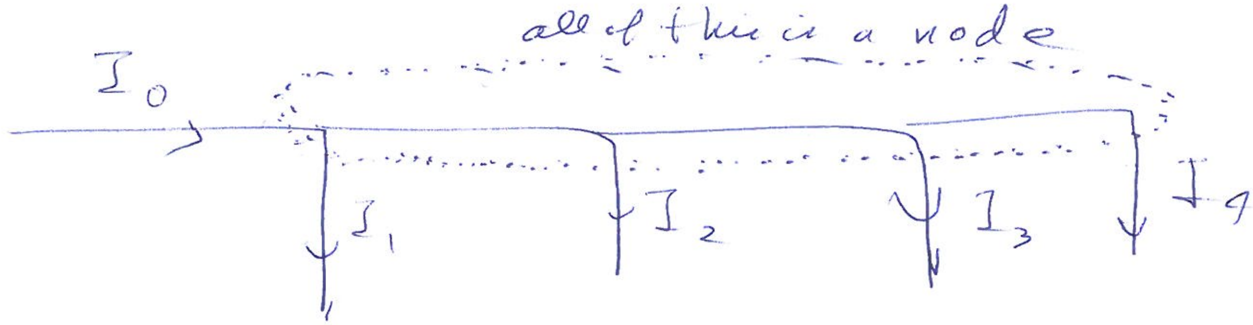
$$I_1 = 1\ A$$

$$I_2 = 2\ A$$

$$I_1 = I_2 + I_3$$

$$I_3 = I_1 - I_2$$
$$= 1 - 2 = -1\ A$$

If you choose the wrong direction for a current, you just get a negative answer which is fine.



$$I_1 = I_2 = I_3 = I_4 = 1A$$

$$I_0 = I_1 + I_2 + I_3 + I_4$$

$$= 4\ A$$

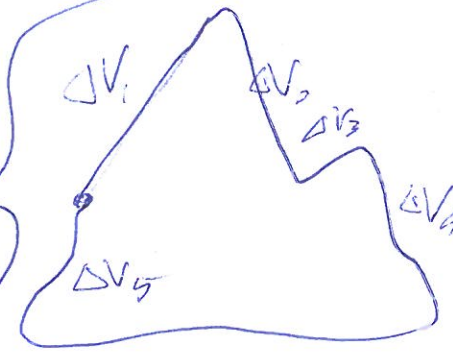
b) Kirchhoff's Voltage Law

(AKA the loop law)

Units of potential and emf are Volts (symbol V roman)

$V = J/C$

$$\sum_i \Delta V_i = 0$$



The changes in potential going around a closed loop sum to zero

rises and drops, but around a closed loop one always returns to the same elevation. Very analogous to potential.

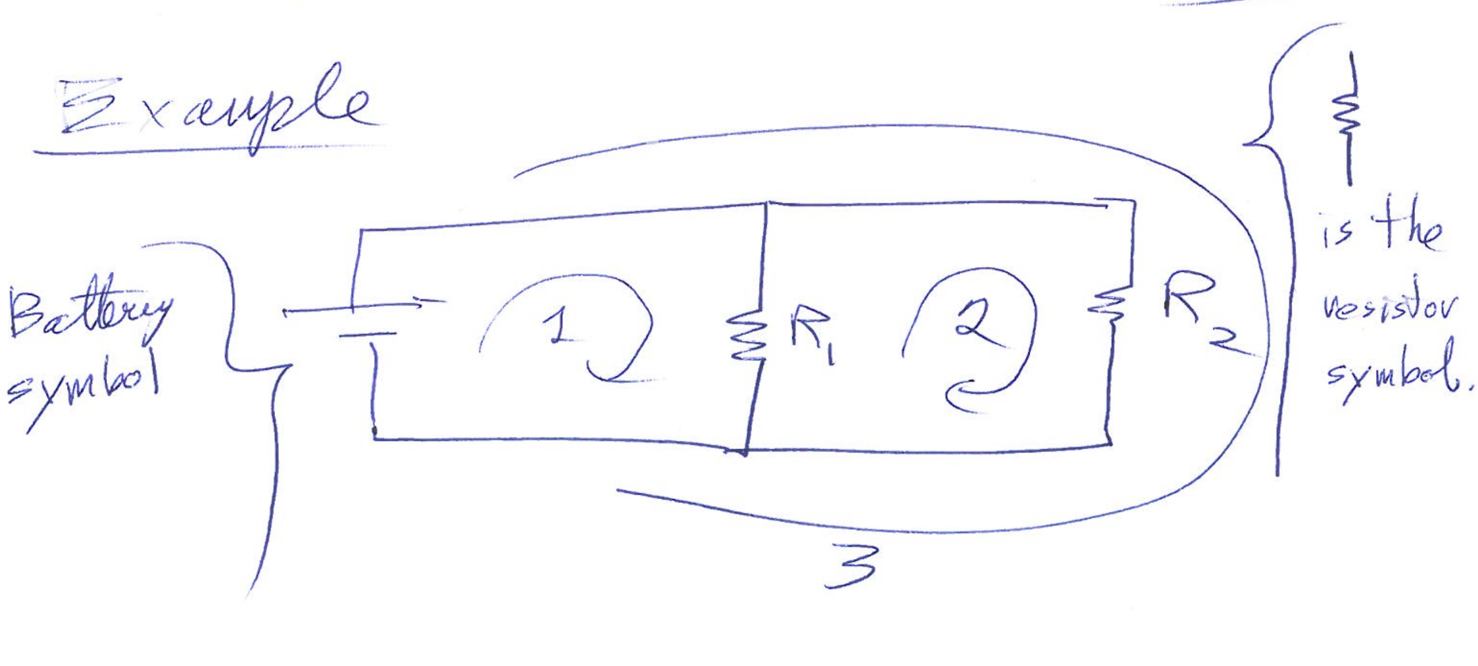
or

$$\sum_i \Delta V_{rises} = \sum_i \Delta V_{drops}$$

It's more general to say "sum of <u>emf</u>'s" rather than "sum of potentials" since the law works when potentials strictly speaking don't exist as in a current loop caused by a <u>Faraday law induced E-field</u>.

<u>But</u> it's common to say potential and <u>emfs</u> take a bit of discussion that comes later.

<u>Example</u>



loop 1 $$\underbrace{V_{Ba}}_{\text{rise across battery}} = \underbrace{V_1}_{\text{drop across resistor 1}}$$

loop 2 $$\underbrace{V_1}_{\text{rise across resistor 1}} = \underbrace{V_2}_{\text{drop across resistor 2}}$$

loop 3 $$\underbrace{V_{Ba}}_{\text{rise across battery}} = \underbrace{V_2}_{\text{drop across resistor 2}}$$

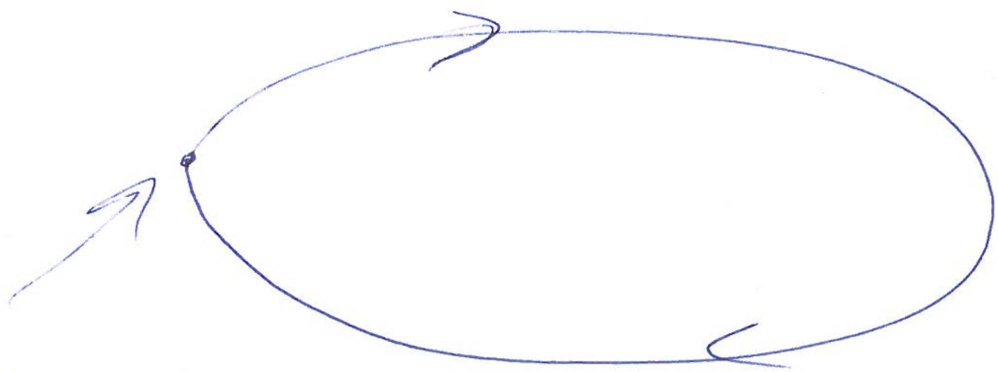
Proof of Kirchhoff's Voltage Law

A classical proof and electrons in circuits are quantum objects — but I think it can be made valid with some argument that eludes me.

Assume steady state.

Recall the work-kinetic-energy theorem

$$\underbrace{\Delta KE}_{\text{Change in Kinetic energy}} = \underbrace{W}_{\text{work done on object.}}$$



Now take a charge (conventional positive charge for simplicity) ~~an electron~~ arround a closed loop.

You can imagine doing this in an instant in time.

Since the flow is steady state

$\Delta KE = 0$ for a closed loop.

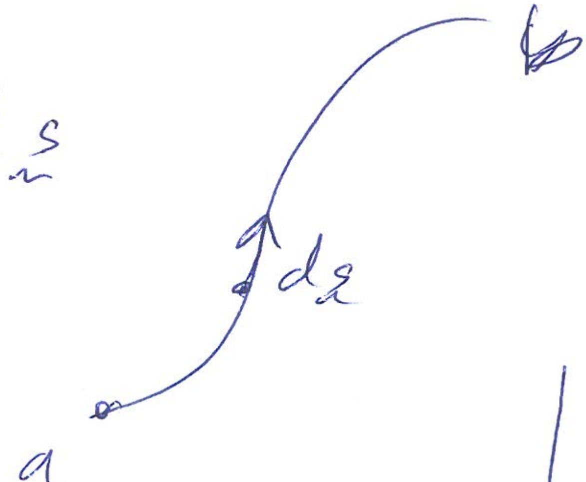
and $W = \sum_i q_i \mathcal{E}_i$

$\mathcal{E}_i$ are the <u>emfs</u>

work done per unit charge by what ever set of ~~charges~~ forces encountered by charge.

formally emfs (electromotive forces although they are NOT forces) are defined at an instant in time

$$\mathcal{E} = \int_a^b \underbrace{\underset{\sim}{f}}_{\text{force per unit charge}} \cdot \, d\underset{\sim}{s}$$



Since $\Delta KE = 0$

$$0 = W = \sum_i q\mathcal{E}_i$$

$$\therefore \quad \sum_i \mathcal{E}_i = 0$$

Usually (but not always)

$$\mathcal{E}_i = -V_i$$

$$\therefore \quad \sum \mathcal{E}_i = 0$$

implies $\sum_i V_i = 0$

What if there is no <u>steady-state</u>?

Again one can think of going around in an instant in time.

Well $\Delta KE \neq 0$ then.

<u>But</u> in most circuit cases,

the nature of the system is such that ΔKE is very, very tiny compared to significant emfs.

So $\sum_i \mathcal{E}_i \triangleq 0$ to excellent approximation

and then $\sum_i V_i = 0$

in a single branch

Lots of KE will just not get into the electrons actually.

There are exceptions of course like charged particles in a cyclotron, but those are NOT ordinary circuits.

Flows of energy into electrons and outflows balance. The electrons are in a sense glued to each other by ~~their~~ ~~the need to~~ constraint of near neutrality. Their density stays pretty constant. There can't be any rapid speed-up and slow-downs. The situation is again like water under pressure, in a pipe. The water is forced to respond collectively all over. Similarly with charge carriers where ~~speed of~~ near speed of light signaling keeps them in lockstep.

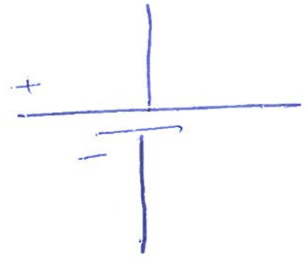
4) Using Kirchhoff's laws

Given Kirchhoff's laws and some formulae for the behavior of <u>circuit devices</u> you can solve for circuit unknowns (e.g., currents, potential drops, etc.) given sufficient knowns.

<u>Devices</u>

a) <u>Battery</u>

The ideal battery maintains the same potential between its terminals no matter what current flows through the battery.

So if V_{Ba} 

is the rise in potential across the battery, it is constant.

Real batteries actually have V_{Ba} decrease a little as current ~~flow~~ increases.

b) IV devices

Such devices have

$$I = I(V)$$

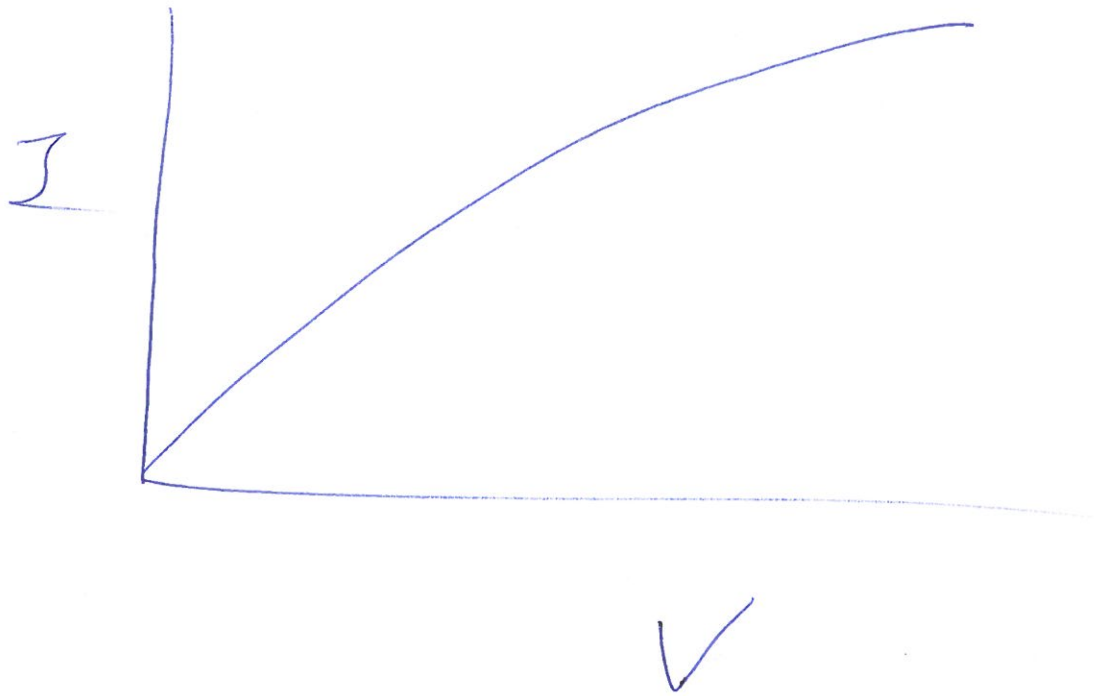
current through

current through is a function of potential drop across.

This can be inverted to $V = V(I)$ usually.

Many such devices have IV curves that look like so

e.g., light bulbs.



The resistance of an IV device is defined to be

$$R = \frac{V}{I} = \frac{V}{I(V)}$$

Not constant in general

Units of R are Volt/coulomb = $V/C = \Omega$

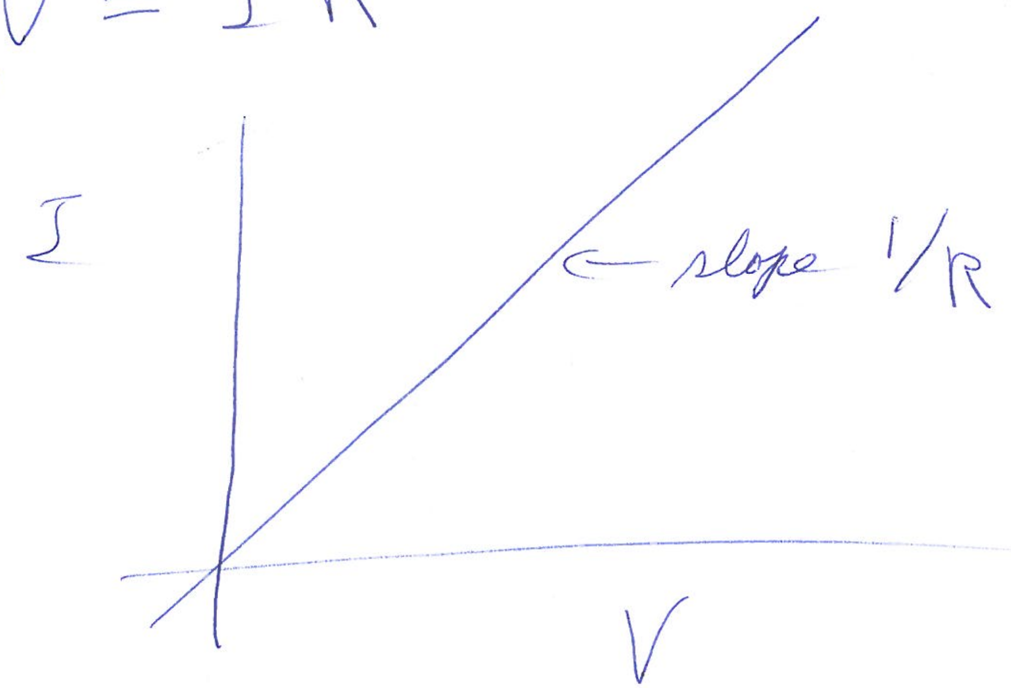
special symbol capital Greek omega, the Ohm.

c) Ohmic Devices

These are IV devices where R is a constant.

$\therefore \quad V = IR$

Ohm's law.



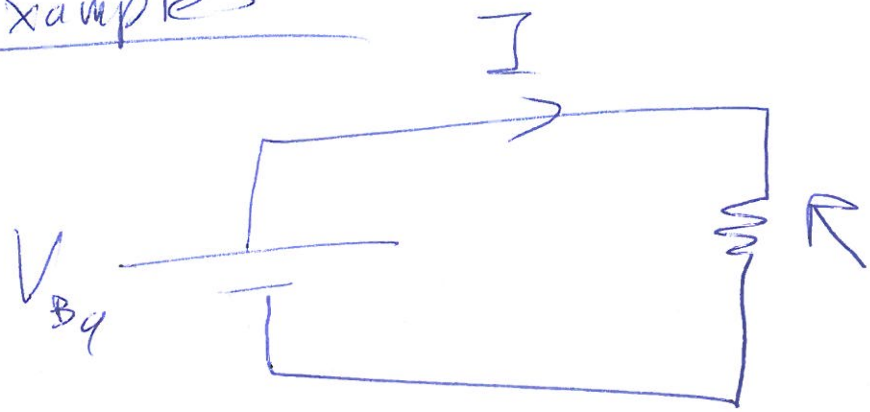
Usually such devices are resistors.

symbol 

d) Others like capacitors and inductors that will come up later. (or earlier)

Examples

a)



Solve for current I.

$$\underbrace{V_{Ba}}_{\text{Kirchoff's Voltage law}} = \underbrace{V_R}_{\text{drop across resistor}} = \underbrace{IR}_{\text{Ohm's law}}$$

$$I = V_{Ba}/R$$

b)

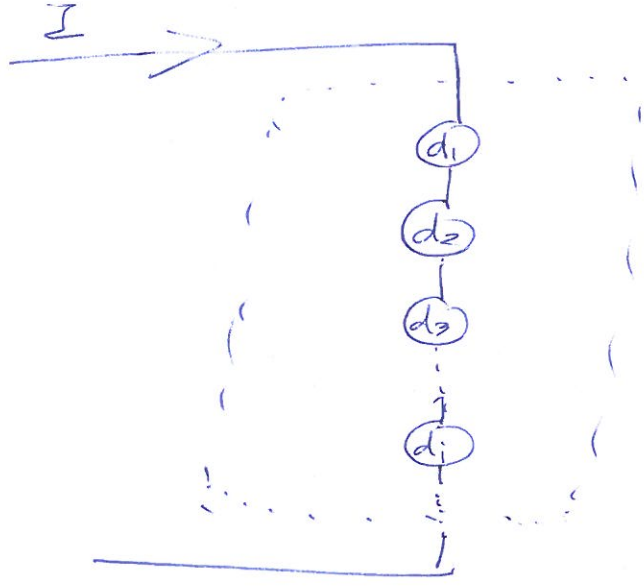
Solve for I_0, I_1, I_2.

$$V_{Ba} = I_1 R_1 = I_2 R_2$$

$$I_1 = \frac{V_{Ba}}{R_1}, \quad I_2 = \frac{V_{Ba}}{R_2}$$

$$I_0 = I_1 + I_2 = V_{Ba}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

c)



d_i is an IV device

$$R_i = \frac{V_i}{I_i(V_i)}$$

What is R for the set of devices in series?

By definition

$$R = \frac{V_{\text{drop across } \cancel{\text{all device}}\ \text{the set}}}{I_{\text{current through the set}}}$$

$$= \frac{\sum_i V_i}{I} = \frac{\sum_i V_i(I)}{I}$$

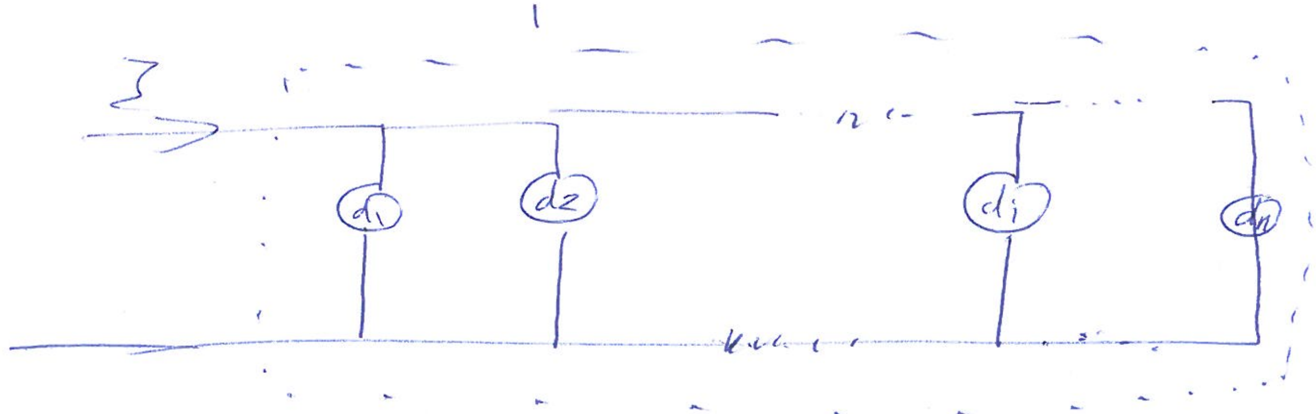
which is the same for all devices.

If the devices are all ohmic

$$V_i = I R_i$$

and $$R = \sum_i R_i$$

d)



What is R for the set of IV devices in parallel?

$$R = \frac{V_{\text{drop across}}}{I_{\text{through}}} = \frac{V}{\sum_i I_i(V)}$$

where ~~i is any device~~ since all V_i's are the same

If the devices are all ohmic:

$$I_i = V_i / R_i = V / R_i$$

$$\therefore \quad R = \frac{V}{\sum_i V/R_i} = \frac{1}{\sum_i (1/R_i)}$$

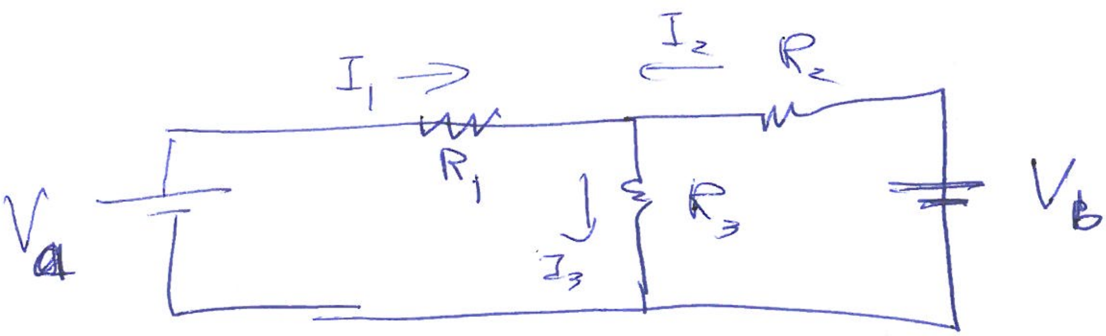
$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$

Note $\quad \frac{1}{R} \geq \mathrm{Max}\left(\frac{1}{R_i}\right)$

and so

$$R \leq \mathrm{Min}(R_i)$$

e)



Solve for I_1, I_2, I_3.

OK, it's very tough.

But exploiting symmetry makes it tractable.

Three equations and three unknowns. A solution can be found.

$$\begin{cases} I_3 = I_1 + I_2 \\ I_1 R_1 + I_3 R_3 = V_a \\ I_2 R_2 + I_3 R_3 = V_b \end{cases}$$

— eliminating I_3 preserves symmetry

$$I_1 R_1 + (I_1 + I_2) R_3 = V_a$$

$$I_2 R_2 + (I_1 + I_2) R_3 = V_b$$

$$I_1 (R_1 + R_3) + I_2 R_3 = V_a$$

$$I_2 (R_2 + R_3) + I_1 R_3 = V_b$$

$$I_2 = \frac{V_b - I_1 R_3}{R_2 + R_3}$$

$$\therefore \quad I_1 (R_1 + R_3) + \left(\frac{V_b - I_1 R_3}{R_2 + R_3} \right) R_3 = V_a$$

$$I_1 \left[R_1 + R_3 - \frac{R_3^2}{R_2 + R_3} \right] = V_a - \frac{V_b R_3}{R_2 + R_3}$$

$$I_1 \left[\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3} \right] = \cancel{V_a R_2 + (V_a - V_b) R_3} \quad V_a - \frac{V_b R_3}{R_2 + R_3}$$

$$I_1 = \frac{V_a (R_2 + R_3) - V_b R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_1 = \frac{V_a(R_2+R_3) - V_b R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

by symmetry (I, and I₂ are symmetrical in circuit and so just interchange 1 ↔ 2 indexes

$$I_2 = \frac{V_b(R_1+R_3) - V_a R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$\cancel{I_3 = (V_a+V_b)(}$$

$$I_3 = \frac{(V_a+V_b)R_3 + V_aR_2 + V_bR_1 - (V_a+V_b)R_3}{\quad\cdots\quad}$$

$$= \frac{V_aR_2 + V_bR_1}{R_1R_2 + R_1R_3 + R_2R_3}$$

<u>Special Cases</u>

$R_3 \to \infty \qquad I_1 = \frac{V_a - V_b}{R_1+R_2}, \quad I_2 = \frac{V_b - V_a}{R_1+R_2}, \quad I_3 = 0$

$R_3 \to 0, \quad I_1 = V_a/R_1, \quad I_2 = V_b/R_2, \quad I_3 = \frac{V_a}{R_1} + \frac{V_b}{R_2}$

which are all what one would expect.

5) Power

In general power is energy transferred per unit time.

$$P = \lim_{\Delta t \to 0} \frac{\Delta E}{\Delta t} = \frac{dE}{dt}$$

If $\Delta E = \Delta q V$,

$$P = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} V = \frac{dq}{dt} V$$

$P = IV$ a very general formula

The power transferred is the current through a device times the

the potential rise/drop across it

What are the sign convention?
Well whatever you like as long as you know what is happening.

Example
In a battery

$$P = I V_{rise}$$

is the power into circuit

and also the power out of battery chemical store.

Power in resistors

$$P = I_{\text{current through}} V_{\text{drop across}}$$

or $P = IV$

but using Ohm's law $V = IR$

$$P = IV = \underbrace{I^2 R = \frac{V^2}{R}}_{}$$

only for resistors

— well actually they will be valid for any IV device, but since R is NOT constant, they are NOT too useful in those cases.

34)

Energy conservation

P_{in} for whole circuit $=$ P_{out} for whole circuit

assuming negligible KE for electrons.

— We built this in with Kirchhoff's laws actually and there seems no simple alternative proof.

The Voltage law really builds in energy conservation for quasi-steady state.

6) Practicalities

In this lab one assumes the ~~the~~ bulbs are OHMIC (even though they are NOT).