Chapter 33

— AC circuits
— Alternating Current

— Electric generators naturally tend to produce AC as we seen & electric motors to use it.

But actually conversion between AC & DC can be done and there is no strong reason just based generation & production mode to favor one over the other.

Transmission efficiency became the reason AC

33-2

won out for the power grid in the 1880s in the ~~W~~ War of the Currents

Westinghouse and Tesla pro AC & Edison pro DC

~~A~~ AC won — but never totally.

~~Tran~~ High Voltage transmission is ~~power~~ efficient in not losing energy in transmission (heating the wires)

- But that can be done with either AC or DC.

But AC can use transformers

to step-up
& step-down potential
easily.

this allows
high potential for transmission
and lower less dangerous
potential for generation and use.

<u>But</u> that was in the 1880s
since then step-up
& step-down DC
technology has developed

and high-voltage DC is
more ~~very~~ efficient ~~for~~ than AC for
very long-range transmission
and is used for remote communities
in Canada, Siberia, & Scandinavia.

33-4)

In fact a new advanced power grid for the US (and maybe all of North America) could be HVDC.

§33.1

Typical AC is <u>sinusoidal</u>

— easy to generate, is to transform.

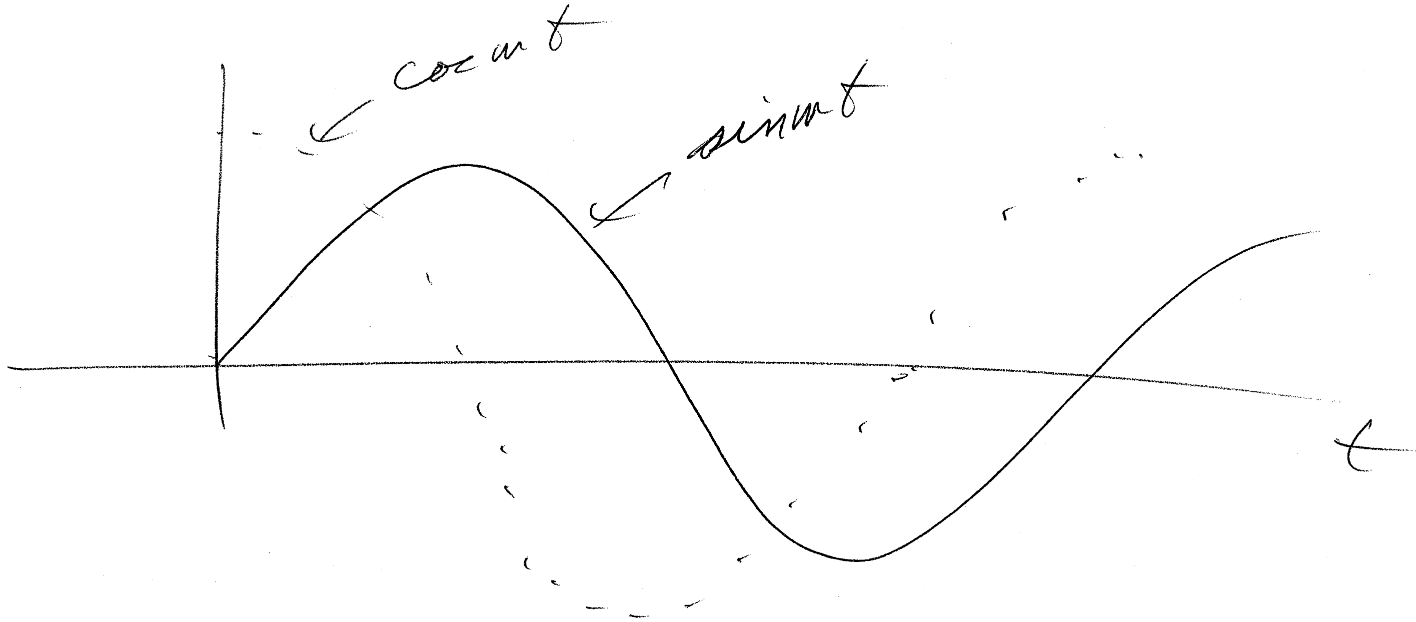
So an AC emf could be

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

or $\mathcal{E}_0 \cos \omega t$

↳ the two functions have the

same shape — one is just displaced from the other



Since in steady-state applications the time origin is unimportant, one can use either one.

Say $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

$\mathcal{E}_0$ is the amplitude.

ω is the angular frequency.

If $\omega \Delta t = 2\pi$, then the

33-6

~~cycle~~ function repeats.

$\therefore\ T = \frac{2\pi}{\omega}$ is ~~the~~ period.

and $f = \frac{1}{T} = \frac{\omega}{2\pi}$

is the frequency in cycles per unit time.

In ~~USA~~ A we use $f = 60\,Hz$

hertz $= \frac{1}{s}$

but some countries use $f = 50\,Hz$

(Japan was indecisive on this and uses both $50\,Hz$ & $60\,Hz$ (Wik))

— there are tradeoffs to using higher or lower frequencies, and probably no optimum choice.

Niagara Falls once (long ago) generate 25 Hz, but that was a bit low and caused light bulbs to noticeably flicker (WiK).

$\mathcal{E}_0$ is the amplitude

<u>but</u> this is not the <u>voltage</u> usually reported to characterize the potential.

Root-Mean-Square (RMS) potential is reported.

$$\mathcal{E}_{RMS} = \sqrt{\frac{1}{T}\int_t^{t+T} \mathcal{E}_0^2 \left\{\begin{matrix}\sin^2 \omega t' \\ \cos^2 \omega t'\end{matrix}\right\} dt'}$$

$x = \omega t' - \omega t$
$dx = \omega dt'$
$T\omega = 2\pi$

$$= \mathcal{E}_0 \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left\{\begin{matrix}\sin^2 x \\ \cos^2 x\end{matrix}\right\} dx}$$

$\boxed{33-8}$

$$= \mathcal{E}_0 \sqrt{\frac{1}{2\pi} \underbrace{\int_0^{2\pi} \frac{1}{2}(1 \mp \cos 2x)\,dx}_{\underbrace{\frac{1}{2}\left(x \mp \frac{\sin 2x}{2}\right)\Big|_0^{2\pi}}_{\pi}}}$$

$$\mathcal{E}_{RMS} = \frac{\mathcal{E}_0}{\sqrt{2}}$$

Northern & South Am. & Saudi Arabia & Japan — Most others use $\mathcal{E}_{RMS} = 240\,V$

WiK Mains Power Systems

In North America we use

$$\mathcal{E}_{RMS} = 120\ V$$

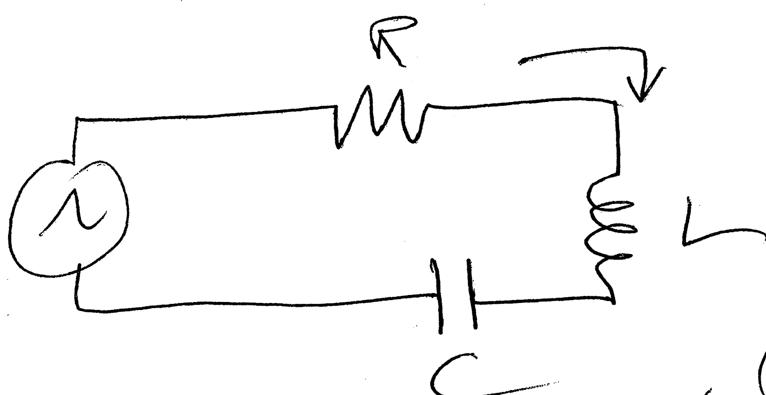
which implies $\mathcal{E}_0 = \sqrt{2}\,\mathcal{E}_{RMS}$

$$= 169.7056\ldots\ V$$

§ 33.5 RLC Circuit

with an AC driver

- we're skipping 3 sections and phasor diagrams (I don't like them because I know nothing about them — which is a treacherous argument for a pedagogue to make)



We use Kirchhoff Voltage law for one loop

- the order of RLC is not important in the analysis

Obvious this is part a

33-10

simple circuit with resistor, inductor, and capacitor but it is illustrative

Kirchhoff Voltage law

$$\underbrace{\mathcal{E}}_{} = \underbrace{IR}_{} + L\frac{dI}{dt} + \frac{q}{C}$$

emf rise

$\mathcal{E} = \mathcal{E}_0 \sin \omega t$

a driver

drop for $I > 0$

if $I < 0$, it's a negative drop

drop for $\frac{dI}{dt} > 0$

if $\frac{dI}{dt} < 0$ it's a negative drop

drop for $q > 0$

if $q < 0$ it's a negative drop.

This distinguishes the problem from our previous case.

→ the solutions we found there ~~th~~ have analogs here

— but those are <u>transient solutions</u>

~~for~~ → they disappear in time ~~[illegible]~~ — often so rapidly as to be negligible.

Say $I_{general} = I_{driven} + I_{transient}$

$$\mathcal{E} = \underline{I_{dri} R + L \frac{dI_{dri}}{dt} + \frac{q_{dri}}{C}}$$

$$+ \underbrace{I_{tr} R + L \frac{dI_{tr}}{dt} + \frac{q_{tr}}{C}}_{= 0}$$

because it is the solution with no driver.

If I_{driven} solves this ~~[illegible]~~ part

then $I_{general}$ is a solution

But since the transient disappears in time and such circuits are primarily of interest long after the transient is gone we only need to consider the driven solution or in DE jargon the inhomogeneous solution.

We do a solution by trial function since we have centuries of experience telling us what works.

First we really want I not q.
So let's differentiate once.

$$\mathcal{E}_0 \sin \omega t = IR + LI' + \frac{q}{C}$$

$$\mathcal{E}_0 \omega \cos \omega t = I'R + LI'' + \frac{I}{C}$$

The trial solution is

$$I = I_0 \sin(\omega t - \phi)$$

I_0 is amplitude of the current

ϕ is the phase shift from the driver.

Both of these are determined by $\mathcal{E}_0$, R, L, C.

33-14

Here we just give the results — the mathematics of the solution is a bit too involved.

$$I = I_0 \sin(\omega t - \phi)$$

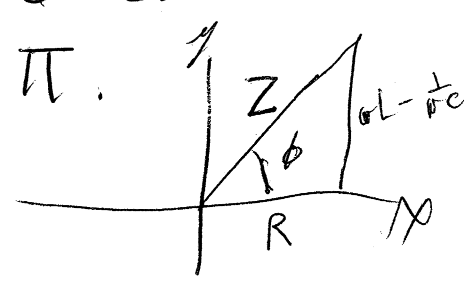
$$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

impedance

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

phase shift

where since $R > 0$, there is no possible additive factor of π.



Optional Complex Number Solution

Replace our trial solution by a complex solution

$$\mathbf{I} = I_0 e^{i(\omega t + \alpha)}$$

where we can restrict I_0 to be pure real. The α parameter gives us enough freedom for a consistent solution.

I_0 and α are two parameters ~~that~~ whose values are set by the solution.

33-16)

Using ~~Euler~~ Euler's formula (Art-264)

$$e^{ix} = \cos x + i \sin x$$

where i is the imaginary unit, we find

$$\underset{\text{real part}}{Re[I]} = I_0 \cos(\omega t + \alpha)$$

$$\underset{\text{imaginary part}}{Im[I]} = I_0 \sin(\omega t + \alpha)$$

In complex numbers ~~jargon~~ "real" and "imaginary" are jargon terms ~~for~~ for the two components of a complex number — both are really real.

The original DE is

$$\mathcal{E}_0 \omega \cos \omega t = I'R + LI'' + \frac{I}{C}$$

The analog complex DE is

$$\mathcal{E}_0 \omega e^{i\omega t} = I'R + LI'' + \frac{I}{C}$$

The real part of the trial solution solves the original DE. This is the solution we want

Substituting the trial $I = I_0 e^{i(\omega t + \alpha)}$ into the complex DE gives

$$\mathcal{E}_0 \omega = R I_0 i\omega e^{i\alpha} + L(-\omega^2) I_0 e^{i\alpha} + \frac{I_0 e^{i\alpha}}{C}$$

canceling the common $e^{i\omega t}$ factor.

The solution for $I_0 e^{i\alpha}$ is

$$I_0 e^{i\alpha} = \frac{\mathcal{E}_0 \omega}{i\omega R - L\omega^2 + \frac{1}{C}}$$

$$= \frac{\mathcal{E}_0}{iR - L\omega + \frac{1}{\omega C}}$$

33-18

$$I_0 e^{i\alpha} = \frac{\mathcal{E}_0\left[-iR - \omega L + \frac{1}{\omega C}\right]}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$I_0 = \sqrt{\left(I_0 e^{i\alpha}\right)\left(I_0 e^{i\alpha}\right)^*}$$

complex conjugation

$$= \sqrt{\frac{\mathcal{E}_0^2\left(R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right)}{\left(R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right)^2}}$$

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

where $Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

is defined as impedance

$X_L = \omega L$ is inductive reactance

$X_C = \frac{1}{\omega C}$ is capacitive reactance.

Now $e^{i\alpha} = \cos\alpha + i\sin\alpha$

$$\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-R}{-\omega L + \frac{1}{\omega C}}$$

$$\cancel{\alpha = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$

$$\alpha = \tan^{-1}\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right) + n\pi$$

Needed because of the ambiguity of the inverse tangent function.

where $n = 0$ for $\omega L - \frac{1}{\omega C} > 0$

and $n = 1$ for $\omega L - \frac{1}{\omega C} < 0$

33-20

~~Now the~~ real solution which is ~~the~~ solution of the real part of the ~~D~~ complex DE is ~~the~~ solution of ~~the~~ original ~~sol~~ DE;

~~the~~ real solution is ~~the~~ real solution.

$$I = I_0 \cos(\omega t + \alpha)$$

but our Emf was in terms of the sine function

$$\mathcal{E} = \mathcal{E}_0 \sin(\omega t)$$

and so it is convenient

to write I in terms of sine for easy comparison.

$$\cancel{\sin}\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} = \cos x$$

and $\cos\left(x+\frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = -\sin x$

$$\therefore\ I = I_0 \sin(\omega t + \underbrace{\alpha + \tfrac{\pi}{2}}_{\equiv -\phi})$$

Now $\tan(-\phi) = \tan\left(\alpha+\frac{\pi}{2}\right)$

$$= \frac{\cos\alpha}{(-\sin\alpha)}$$

$$= -\cot\alpha$$

$$= -\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

33-22

where since $R > 0$
there is no additive $n\pi$
constant needed.

So to summarize.

$$I = I_0 \sin(\omega t - \phi)$$

$$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

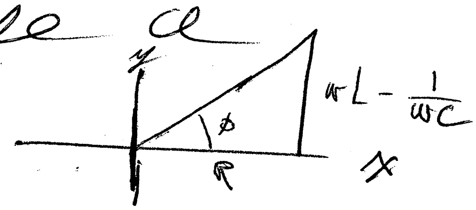
$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Now that wasn't so bad.

What does the solution mean?

Well the current is also sinusoidal with the same ~~curren~~ frequency as the driver.

But the inductor and capacitor cause a phase shift



$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$ which is frequency ~~independen~~ dependent.

If $\omega \to 0$, $\phi \to 0$ but then $\mathcal{E}$ is constant $\mathcal{E}_0$ and $I = I_0 = 0$ since the ~~capacit~~

$\boxed{33-24}$

capacitor is just all charged up

$\hookrightarrow \quad Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$\hookrightarrow \quad \infty$

$, \quad I_0 = \frac{\mathcal{E}_0}{Z_0} = 0.$

Inductance L, inductive reactance ωL

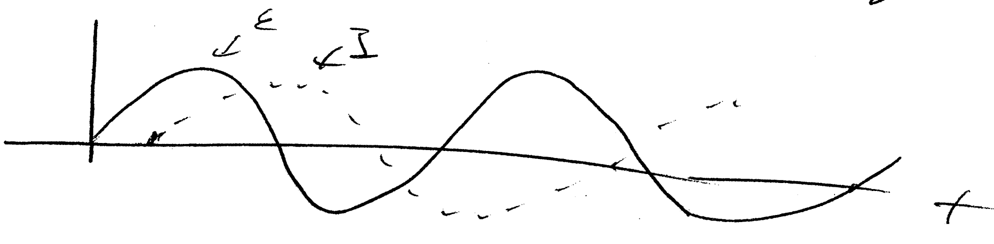
Capacitance C, capacitive reactance $\frac{1}{\omega C}$

We ~~try to~~ can shift in opposite senses:

(a) $\omega L > \frac{1}{\omega C}$

$\phi > 0$

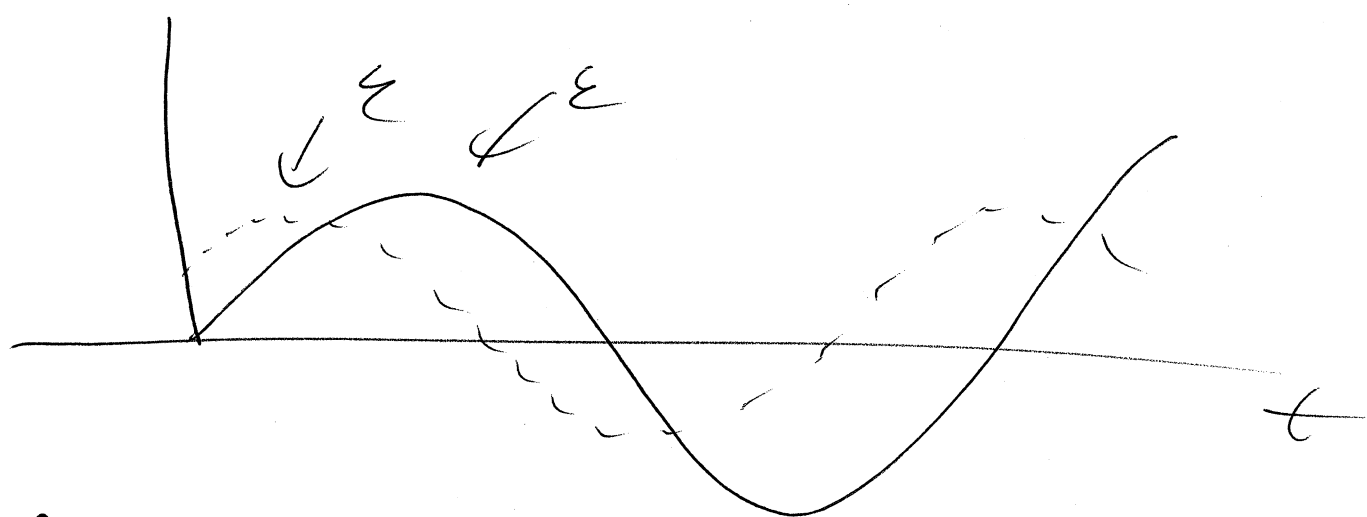
and I shifts rightward



(b) $\omega L < \frac{1}{\omega C}$

and $\phi < 0$ and I

shifts leftward



§ 33.6 Power in AC circuits

— As we saw in the simple driven RLC circuit there can be a shift between current & voltage sinusoids. And such shifts are general in AC circuits, 33-26) not just the simple driven RLC circuit

This has an affect on the power output or input.

Recall

$P = IV$

power output or input.

drop or rise in potential across a device

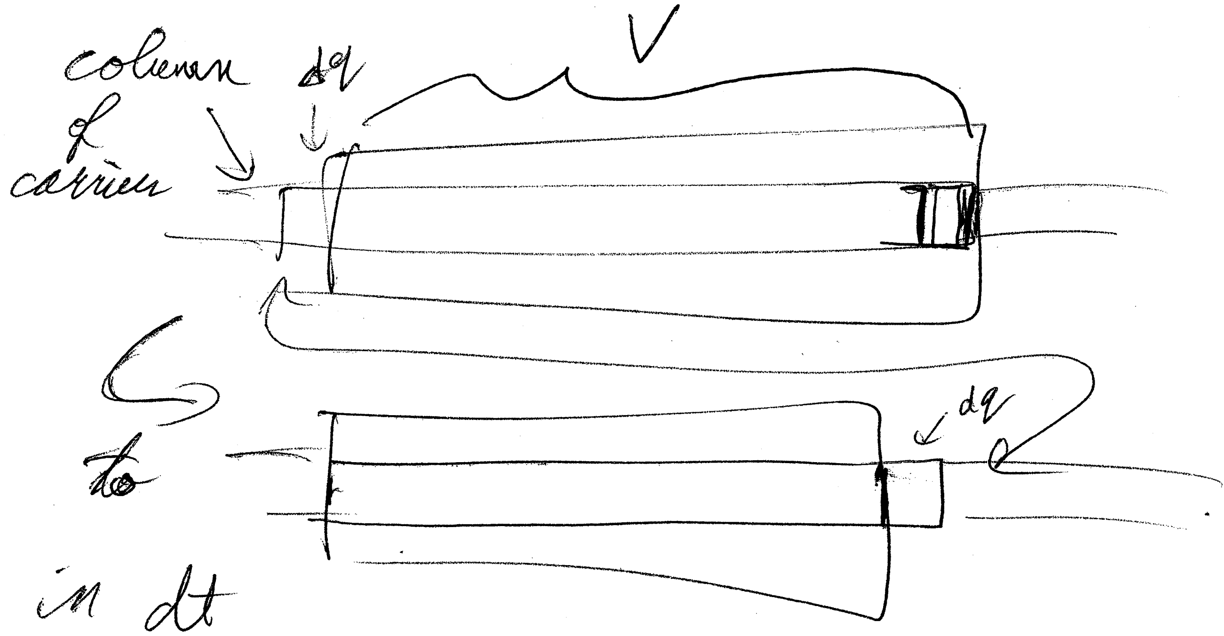
— This expression is true even for time-varying ~~system~~ systems.

Sat at an instant the potential difference is V and current I.

In dt the individual carriers don't move much perhaps, but ~~Pdt~~

$$dE = P\,dt = IV\,dt = dq\,V$$

is still ~~the~~ energy deposited



~~The~~ It's the same PE energy change as if you moved the carriers from one end to another in dt.

Now in dt, V could change from

V to $V + dV$

But $dE = dq(V + dV) \simeq dq\,V$ to 1st order

32-28

in ~~small~~ changes
and in the derivative
limit all higher order
changes vanish.

So $P = IV$ is valid
for varying systems.

Say $V = V_0 \sin(\omega t)$
and $I = I_0 \sin(\omega t - \phi)$

$$P = I_0 V_0 \sin(\omega t) \sin(\omega t - \phi)$$

but it's usually time averaged
power ~~that~~ is of interest.

So $P_{ave} = \frac{1}{T} \int_{t}^{t+T} I_0 V_0 \sin(\omega t') \sin(\omega t' - \phi)\, dt'$

Trig identity

$\sin(\omega t - \phi)$

$$= \underbrace{\sin \omega t \cos\phi}_{} - \cos \omega t \sin \phi$$

This term leads to $\frac{1}{2}$ factor as we've shown before

$$\frac{1}{T}\int_t^{t+T} \begin{Bmatrix} \sin^2 \omega t' \\ \cos^2 \omega t' \end{Bmatrix} dt'$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \begin{Bmatrix} \sin^2 x \\ \cos^2 x \end{Bmatrix} dt = \frac{1}{2}$$

using $x = \omega(t' - t)$

when $\omega T = 2\pi$

This term leads to zero

$$\frac{1}{T}\int_t^{t+T} \sin \omega t \cos \omega t \, dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin x \cos x \, dx = \frac{1}{2\pi} \left. \frac{\sin^2 x}{2} \right|_0^{2\pi} = 0$$

33-~~3~~0

So $P_{ave} = I_0 V_0 \left(\frac{1}{2}\right) \cos\phi$

$= \frac{I_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} \cos\phi$

$= I_{RMS} V_{RMS} \cos\phi$

Recall $V_{RMS} = \sqrt{\frac{1}{T}\int_t^{t+T} V_0^2 \sin^2\omega t \, dt}$

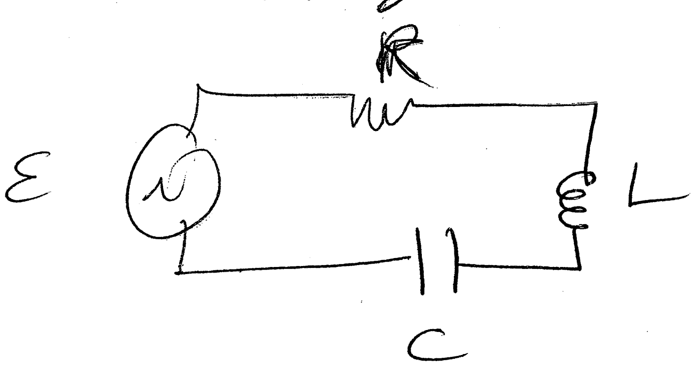
root mean square

$= \frac{V_0}{\sqrt{2}}$

If $\phi = 0$,

$P_{ave} = I_{RMS} V_{RMS}$

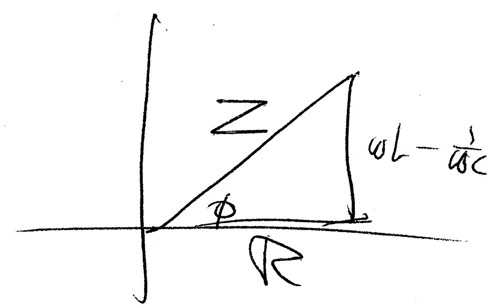
Now to apply to our driven RLC circuit



$$P_{\text{ave out of emf}} = I_{RMS}\, \mathcal{E}_{RMS} \cos\phi$$

$$= \frac{\mathcal{E}_0^2}{Z} \frac{\cos\phi}{2}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$



$$\cos\phi = \frac{R}{Z}$$

and $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

the impedance.

$$P_{\text{out of emf into circuit}} = \frac{\mathcal{E}_0^2}{2Z^2} R$$

$$P_{\text{ave of resistor into heat}} = I_{RMS}\, V_{RMS} \cos\phi_{\text{resistor}}$$

which in fact is zero.

33-32

Now ohm's law applies at each instant.

$$V = IR$$ is the potential across the resistor.

$$\therefore \quad V_{RMS} = I_{RMS} R$$

$$= \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{Z} R$$

and $\phi_{resistor} = 0$

$$\therefore \quad P_{\substack{ave \\ \text{out of resistor}}} = \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{Z} \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{Z} R$$

$$= \frac{\mathcal{E}_0^2}{2Z^2} R = P_{\substack{ave \\ \text{out of emf.}}}$$

and so we've verified

that energy is conserved.

This was really built into system by Kirchhoff's laws and the ~~laws~~ rules of PE and Ohm's laws.

What of the capacitor and inductor?

Well energy is being ~~constantly~~ put into them and taken out again in a period fashion.

— it goes into creating E-fields & B-fields recall.

Consider inductor

$$V = L \frac{dI}{dt}$$

$$= L\, I_0 \omega \cos(\omega t - \phi)$$

33-34

So $P_{ave} = \frac{1}{T}\int_{t}^{T+t} I_0 \sin(\omega t' - \phi) * L\omega\cos(\omega t' - \phi)\,dt'$

$= 0$

— no net power gain or loss on average

Capacitor.

Well $V = \frac{q}{C}$

where $q = \int_{0}^{t} I_0 \sin(\omega t - \phi)\,dt$

~~$= I_0 \sin \omega t$~~

$= \underbrace{-\frac{I_0}{\omega}\cos(\omega t - \phi)} + \underbrace{C_0}_{\text{a time zero constant}}$

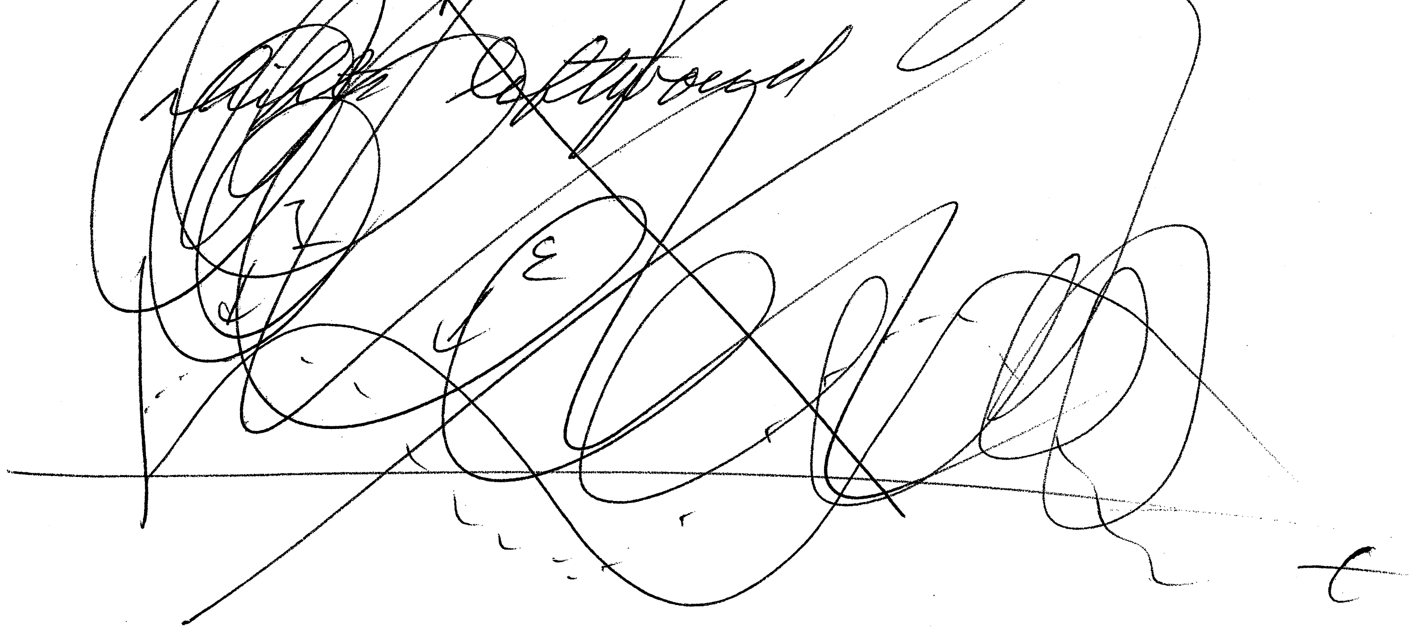
So P_{ave} for capacitor

$= \frac{1}{T}\int_{t}^{t+T} I_0 \sin(\omega t' - \phi)\left[\quad\right]dt$

$= 0$

~~$\omega L < \frac{1}{\omega C}$~~

~~and $\phi < 0$ and~~



§ 33.7 Resonance

in Driven RLC circuit

The really interesting practical interest of RLC circuits — at least at our worm-like level is resonance.

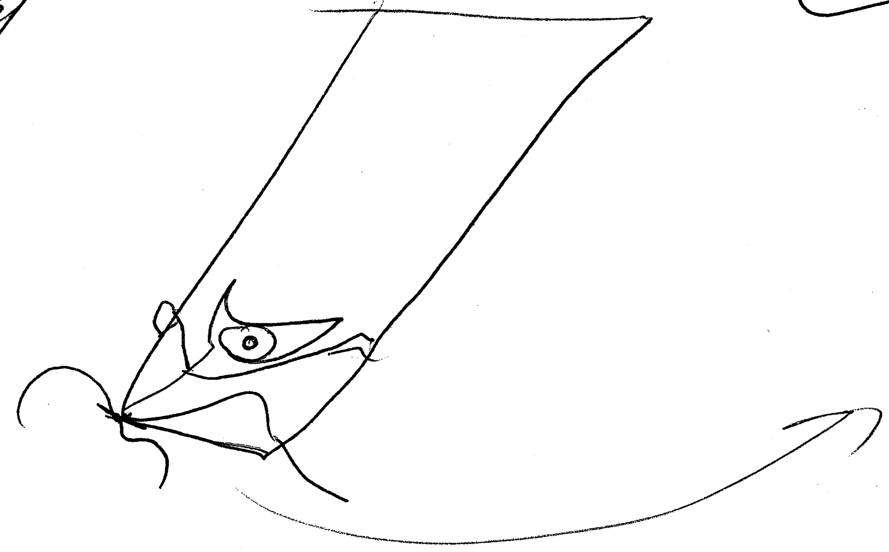
33-~~3~~6

Many physical systems
have oscillatory behavior
and the frequencies
at which the oscillations
can grow large — sometimes
very large are the resonance
frequencies.

In fact everyone in this
class knew how to
drive an oscillator at
a resonance before they
knew the name of ~~torque~~.

— the good old playground swing.

~~swing~~



Really close to being a simple pendulum with resonance frequency

$$\omega = \sqrt{\frac{g}{\ell}} \quad (SJ-432)$$

$$f = 2\pi\omega = 2\pi\sqrt{\frac{g}{\ell}}$$

$$\cancel{f} = \frac{1}{2\pi}\sqrt{\frac{\ell}{g}}$$

ℓ

mg

the frequency on undriven pendulum — natural frequency

pendulum are simple in that they have only one resonance for a given ℓ

— You can drive a pendulum at any frequency; but.

33-38

then you constantly
have to be adding
& subtracting mechanical
energy in a complex way
to get big oscillations.

— If you drive it
on resonance, you
at little kicks of energy
just at the resonance
frequency —— and
the oscillations tend to
grow without bound — of
course, the ~~[illegible]~~ chains or ropes
eventually lose tautness and that
changes things and the fear
factor sets in too.

Forces and owner reacting with frame + frame and ground?
↳ complex but still ultimately moving by pushing/pulling off something

All traditional musical instruments rely on mechanical and/or sound resonance.

— atoms and molecules have QM resonances at which ~~they~~ absorb & emit electromagnetic radiation.

Our driven RLC circuit solution exhibits a resonance.

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

If $\omega L - \frac{1}{\omega C} = 0$,

or $\omega_{res} = \frac{1}{\sqrt{LC}}$

33-~~4~~0

also $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$
$= 0$ and so $\mathcal{E}$ and I are in phase.

I_0 is largest for a given $\mathcal{E}_0$

Similarly the power dissipated in the resistor (See p. 33-32.

$$P_{ave} = I^2 R = \frac{1}{2} \frac{\mathcal{E}_0^2 R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

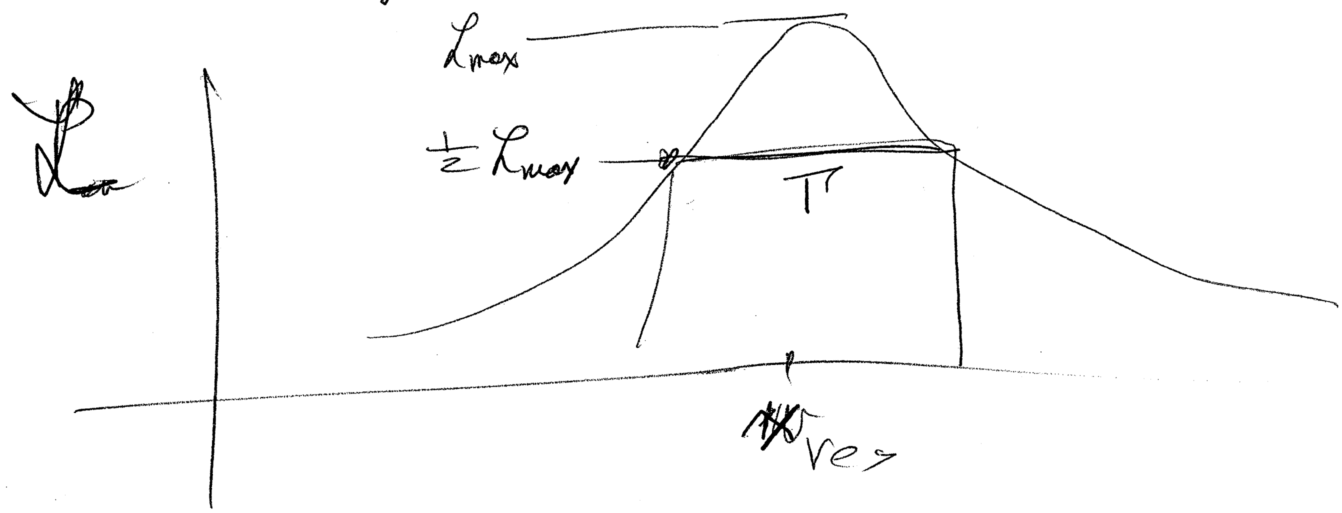
maximizes for $\omega_{res} = \frac{1}{\sqrt{LC}}$

From SJ-909
we recall $\omega = \frac{1}{\sqrt{LC}}$
is the frequency
of pure LC circuit
— it's the "natural" frequency of the ~~ckt~~ RLC circuit.

This shape is quasi-Lorentzian

$f(x) = \frac{1}{\pi} \frac{\Gamma/2}{(\frac{\Gamma}{2})^2 + x^2}$ is ~~natural~~ Lorentzian (BeV-53)

— Just the name for this kind of function which often turns ~~of~~ up in resonance phenomena.



Γ is the full-width half maximum
FWHM (Bev-51)

So $\frac{\Gamma}{2} = R$

or $\Gamma = 2R$

— so R is half of FWHM

But our function is only an exact Lorentzian for $x \equiv \omega_L - \omega_c$ (not ω)

33-42

Driven RLC circuits are used in tuning.

— e.g., radio, at least it can be.

— a radio antenna responds to all frequencies of radio emission.

How does one select one's favorite country & western station ("She done me wrong..")

— all signal are run through an RLC ~~sep~~ circuit.

— they are superimposed drivers.

— a variable capacitor
C

allows one to tune
for the radio frequency
one wants.

— So all other frequencies
are relatively suppressed
and the selected frequency
is used to drive amplifiers
& speakers. (SJ-939)

There are probably trickier
ways of selecting frequencies
with modern electronics, but
the RLC circuit is the
conventional method.

33-44)

§33.8 Transformer & Power Transmission

High voltage is better for transmission.

~~Say you had emf $\mathcal{E}$ and some intentional load R. (assumed pure resistor for simplicity) — but also wires R_{wire}.~~

Say you had a circuit where you had an emf source of $\mathcal{E}$

that output power $P = \mathcal{E} I$

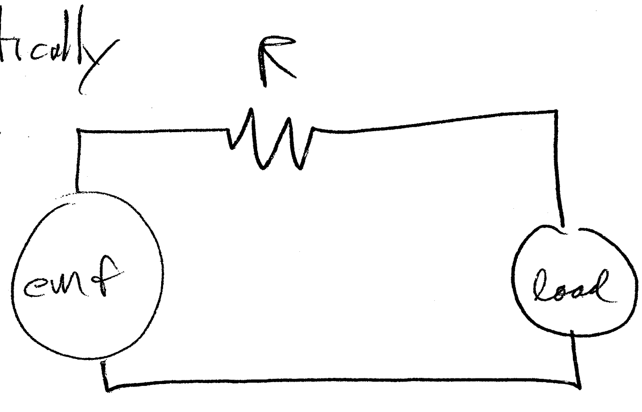
and you used that source to power a load with power $P_e = V_e I$

<u>but</u>

there was also a resistance in the ~~transmission~~ wire R, and so a loss of energy ~~to~~ waste heat ~~that~~ is $P_{wire} = V_{wire} I$

$= I^2 R$ using Ohm's law

Schematically ~~the~~ circuit is



Our variables are

$P, \mathcal{E}, I, \quad P_{wire}, R$

and P_e, V_e

33-46

and we have some relationships.

Say we take P, $\mathcal{E}$, R

as our independent parameters.

the things we can control and adjust.

$$I = P/\mathcal{E}$$

$$P_{wire} = I^2 R = \left(\frac{P}{\mathcal{E}}\right)^2 R$$

$$P_e = P - P_{wire}$$

$$= P - \left(\frac{P}{\mathcal{E}}\right)^2 R$$

Note $\frac{dP_e}{dP} = 1 - 2\frac{P}{\mathcal{E}^2} R \overset{?}{>} 0$

$P < \frac{1}{2}\frac{\mathcal{E}^2}{R}$

So P_e increases with P up to $P = \frac{1}{2}\frac{\mathcal{E}}{R}$

and

$$V_e = \frac{P_e}{I} = \frac{P_e}{P} \mathcal{E}$$

$$= \mathcal{E} - \left(\frac{P}{\mathcal{E}}\right) R$$

Say we don't care what $\mathcal{E}$ or V_e we use. We can adjust those (using transformers to be uneasy).

If for fixed, P we want smaller P_{wire} (wasted energy)

larger P_e (useful energy)

33-48

then we can do
two things:

1) make R smaller
(obvious)
2) make E bigger
(less obvious)

— and, of course,
people try to do both
but there are practical
limits somehow.

— But high potential transmission
is advantageous.

true for AC & DC.

So what ~~there~~ can
be done is step up potential
from ~~safe or~~ practical creation
potential to high potential

transmission

then step down potential
for practical potential use.

Historically the first
practical way to do this
was with AC and transformers,

and so AC won the
war of currents in the 1880s

Westinghouse & Tesla
beating Edison.

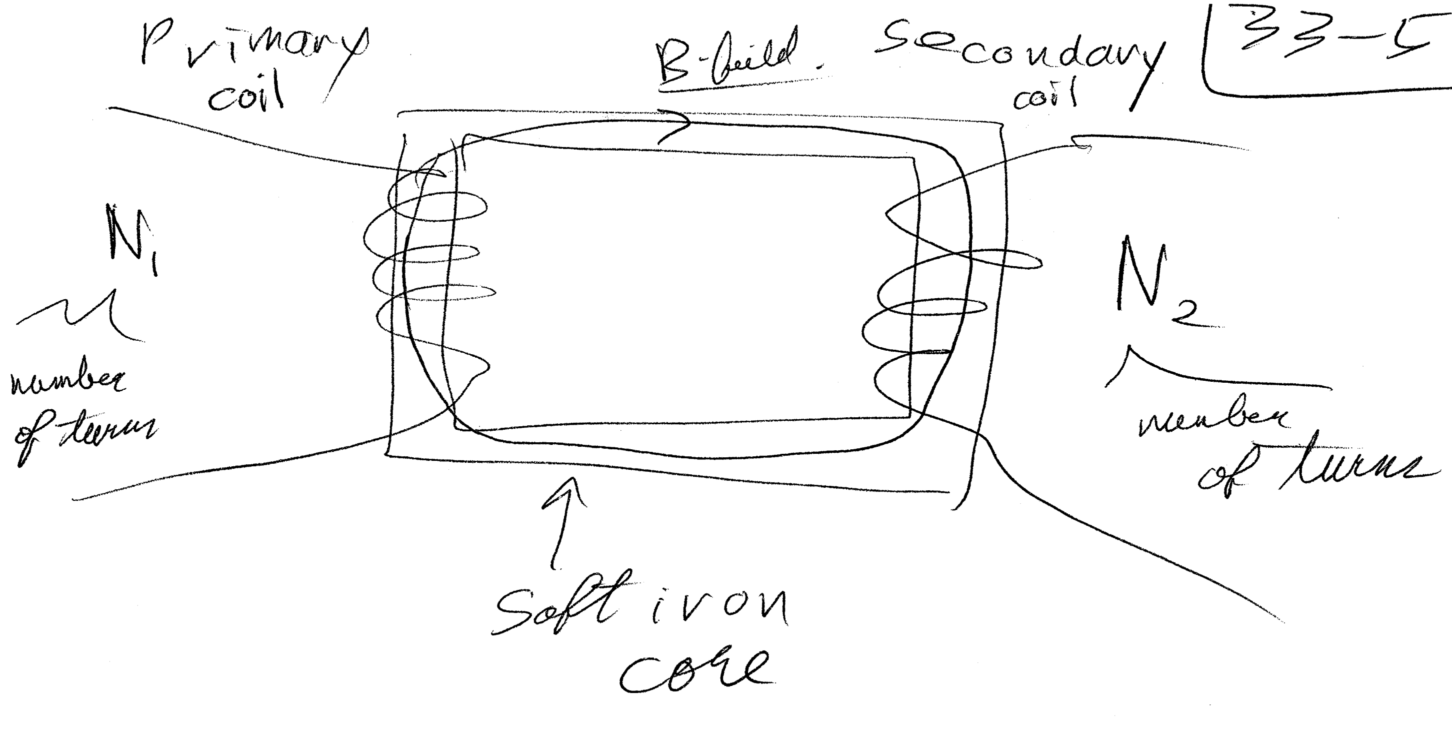
33-50

- But as I mentioned before, it was never a complete victory

and HVDC (high voltage DC) has been used advantageously for very long transmission and might be the way of the future for new advanced grids.

Transformers

The essence is simple



Ideally all the magnetic flux linked by one coil is linked by ~~the~~ other.

the soft iron core greatly enhances ~~the~~ magnetic flux and channels it from ~~one~~ the primary to the secondary

~~the field generated by a coil causes ~~the~~ an alignment the core greatly increases B.~~ But the channelling is the key point

Both effects are actually important to transformer operation.

But modern transformers can transfer energy with $\gtrsim 98\%$ efficiency (CTM-1004)

~~3~~3-52

Potential drop across primary

drop any risky negative signs.

$$V_1 = \mathcal{E}_1 = N_1 \frac{d\Phi_1}{dt}$$

induced emf in 1

number of turns in 1,

Φ_1 magnetic flux in 1 turn.

For secondary, the same

$$V_2 = \mathcal{E}_2 = N_2 \frac{d\Phi_2}{dt}$$

Assuming ideal flux linkage between two coils

$$\Phi_1 = \Phi_2 = \Phi$$

$$\therefore \frac{\mathcal{E}_1}{N_1} = \frac{d\Phi}{dt} = \frac{\mathcal{E}_2}{N_2}$$

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}$$

or $\mathcal{E}_2 = \frac{N_1}{N_2}\mathcal{E}_1$

one of the ideal transformer equations.

This one applies
at each instant
but one can take
an RMS average of it and
get the RMS form

$$\frac{\mathcal{E}_{1\,RMS}}{N_1} = \frac{\mathcal{E}_{2\,RMS}}{N_2}$$

~~Now~~

But these are induced
emfs.
With no changing current
they are both zero.

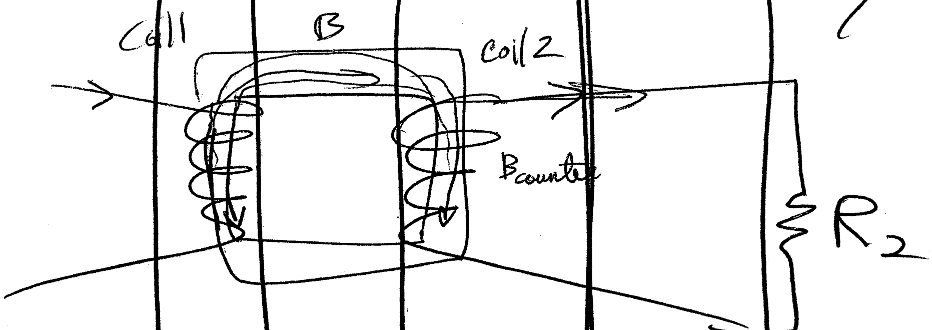
33-54

This is why transformers work with AC where the current is changing all the time.

Let's assume I_1 is sinusoidal since $\mathcal{E}_1 = \frac{d\Phi_1}{dt} \propto \frac{dI_1}{dt}$

$\mathcal{E}_1$ must be sinusoidal as well

Now for the tricky bit (totally omitted by text; 33-940 Just skip it part)



Consider a resistive load

$\phi_{tran} = \phi_{pri} + (-\phi'_{cnt})$

$= \phi_{in} + \phi_{count}$

$\mathcal{E}_2 = I_2 R_2$

and $\mathcal{E}_2$ and I_2 are in phase since $\mathcal{E}_2$ drives I_2

Now for a bit I can't prove —— (TM-1005 gives an explanation but either it's defective or I am)

$$N_1 I_1 = N_2 I_2$$

at least ideally for sinusoidal currents + emfs.

The two currents are in phase — or 180° out of phase depending on how you look at the system — since the currents are in different circuits, it's a matter of perspective.

and these currents are in phase with their coils's emfs (ideally)

33-56

So we have the ideal transformer equations

① $$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}$$

and ② $$N_1 I_1 = N_2 I_2$$

which hold ideally at every instant for ~~AC~~ sinusoidal $\mathcal{E}_1$ which is the driver. with all $\mathcal{E}_1, \mathcal{E}_2, I_1, I_2$ in phase.

① × ② gives ~~energy co~~ = ③

consistent with energy conservation

$$\underbrace{P_1 = \mathcal{E}_1 I_1}_{\text{Power out of coil 1}} = \underbrace{\mathcal{E}_2 I_2}_{\text{power into coil 2}} = P_2$$

One can time average for in phase sinusoids to get

$$P_{1\,avg} = \mathcal{E}_{1rms} I_{1rms} = \mathcal{E}_{2rms} I_{2rms} = P_{2\,avg}$$

— one can drop the rms subscripts

if one knows what one is talking about

(I generally like to drop cluttering subscripts if one knows what is meant from context.)

So we have three ideal transformer equations.

real transformers aren't quite ideal, but as mentioned above $\gtrsim 98\%$ energy transfer is achievable with good ones (TM-1004).

One source of loss is eddy currents (Wik) in the iron core

I assume the metal layered somehow to make it a poor conductor — ~~perhaps~~.

?

but the core can be made of laminated soft iron to make it less conducting (TM-1004 note)

33-58)

Because of

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}$$

or $\mathcal{E}_2 = \frac{N_2}{N_1} \mathcal{E}_1$,

the ratio $\frac{N_2}{N_1}$ ~~can~~ allows

step-up $\frac{N_2}{N_1} > 1$

or

step-down $\frac{N_2}{N_1} < 1$

transformers

(or even step-level $\frac{N_2}{N_1} = 1$ transformers

which may have no practical use)

Stephen

Example 33.7

Power station needs to deliver 20 MW to a city 1.0 km

its generators create

$\mathcal{E}_1 = 22\text{ kV}$

I've no idea how realistic these values are.

and then this is stepped-up to 230 kV for transmission.

$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

$= \frac{230}{22}$

≈ 10.5

step-up

The wire resistance is $2\,\Omega$.

$$P_1 = \mathcal{E}_1 I_1 = \mathcal{E}_2 I_2 = P_2$$

$$P_{lost} = I_2^2 R = \left(\frac{P_1}{\mathcal{E}_2}\right)^2 R$$

33-60

$$= \left(\frac{20 \times 10^6}{230 \times 10^3}\right)^2 \cdot 2$$

$$\simeq \left(10^2\right)^2 \cdot 2$$

$$= 2 \times 10^4 \text{ W}$$

So compared to 20 MW $= 2 \times 10^7$ W

this is ~~pretty~~ small.

But what if there were no step-up.

$$P_{loss} = \left(\frac{P_1}{\mathcal{E}_1}\right)^2 R$$

$$= \left(\frac{20 \times 10^6}{22 \times 10^3}\right)^2 \times 2$$

$$= 2 \times 10^6 = 2 \text{ MW}$$

which is a significant loss

33-61

about 10% of
the total transmission.

So high-voltage transmission
is good.