Chapter 24

Gauss's law (for Electrostatic cases)

- Gauss's law in it's integral form is a conceptually very interesting law with a few very useful applications.
- In its differential equation form (which we only touch on maybe), it's very powerful, but that's mostly beyond our scope.

Gauss's law is actually

24-2) equivalent
~~to~~ Coulomb's law

each implies the
the other
So it's not really
a new law of nature,
but rather a new way
of looking at an already
studied law.

§24.1 Electric Flux

— I skip most of
the electric field line
discussion

— I've never found them (field lines) of ~~great~~ mental use in ~~this~~ particular context (other contexts yes.)

— the word "flux" suggests something is ***flowing***

→ it's not — or doesn't have to be

— it's not for our developments since we're limiting ourselves to electrostatic cases for the moment.

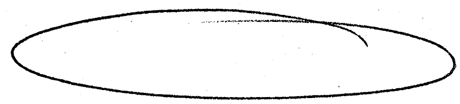
— the E-field's arn't changing.

24-4) ~~The ter car~~
Electromagnetism (EM)
was developed in sort
of parallel with fluid
dynamics — some of
the same 19th folks worked
on both and a lot
of the math they
needed was the same

→ So EM terminology
got to sound sort of
fluidy.

Consider ~~a~~ a differential
bit of surface ~~are~~ area

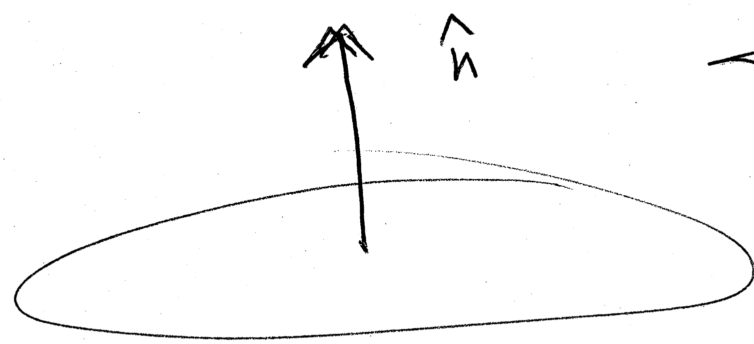
dA



Maybe this is the surface of something, maybe just empty space — We are being general

We will "vectorize" dA

to $d\underset{\sim}{A} = dA\,\hat{n}$



$\hat{n}$ is a unit vector normal to dA. Since dA is differentially small, it is perfectly flat and there is no problem defining a <u>normal</u>

What is the <u>sense</u> of $\hat{n}$?
(In math "sense" is one of two directions a vector may point)
(Ba-1102)

24-6

The usual convention is "outward".

- if $d\underset{\sim}{A}$ is part of a closed surface, then outward is clear.
- if not, then it's something one usually has to define for convenience in the system of interest. (convenience in math or in mental picturing)

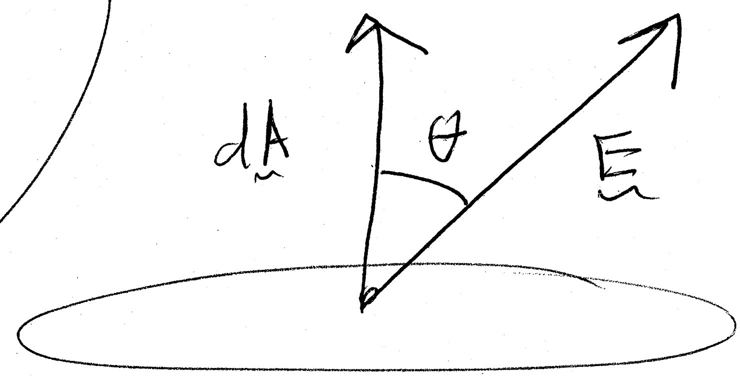
The electric flux thru $d\underset{\sim}{A}$ is defined by

$$d\Phi \equiv \underset{\sim}{E} \cdot d\underset{\sim}{A}$$

$= E\,dA\cos\theta$

dot product and so Φ is a scalar

Capital Greek Phi as in Φιλ~~α~~δελφια



In fluid dynamics a similar integral arises with current density (current per unit area) This is ~~[illegible]~~ a "flux" and the term carried over to EM

$\underset{\sim}{E}$ = the electric field at the position of ~~dA~~ dA

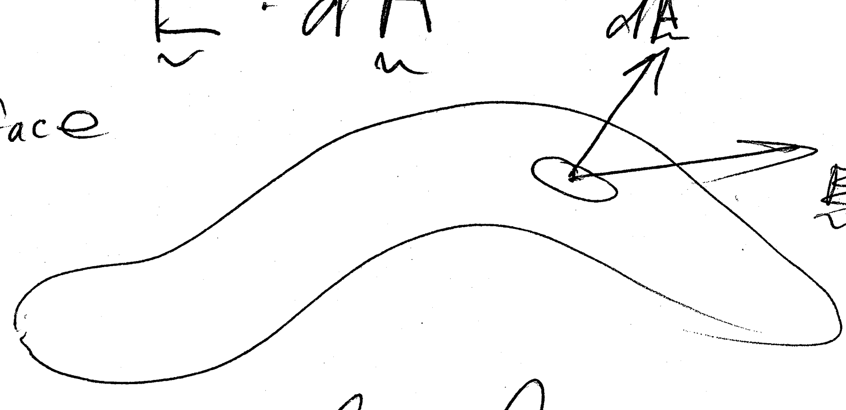
Since dA is differentially small $\underset{\sim}{E}$ is constant over dA.

<u>Units</u> of Φ are $\frac{N}{C}\,m^2$

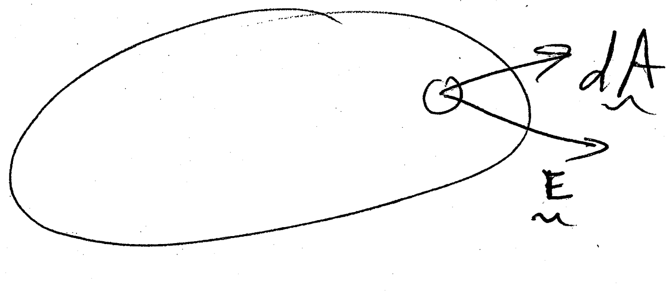
but since one only ever uses MKS units one seldom has to write them explicitly.

For a finite surface one must to an <u>integral</u> which in general might be hard, but we'll usually do easy special cases.

$$\Phi = \int_{\text{surface}} \underset{\sim}{E} \cdot d\underset{\sim}{A}$$



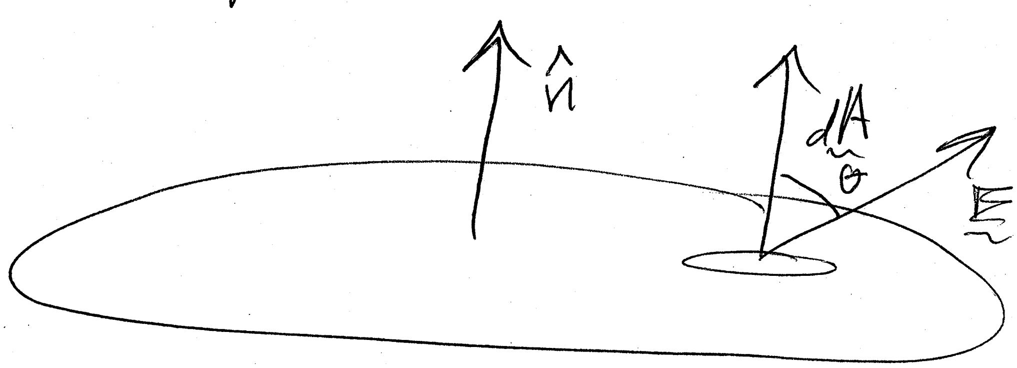
If the surface is closed,



one adorns the integral symbol with a little circle i.e. $\Phi = \oint \underset{\sim}{E} \cdot d\underset{\sim}{A}$

Example 1

$\underset{\sim}{E}$ is constant over a planar area A



θ is same for all part of surface and so is E

In this case

$$\Phi = \int_{\text{surface}} \underset{\sim}{E} \cdot dA$$

$$= E A \cos\theta$$

~~Note if $\theta = 0$,~~

special cases.

EA	$\theta = 0°$
$-EA$	$\theta = 180°$
0	$\theta = 90°$

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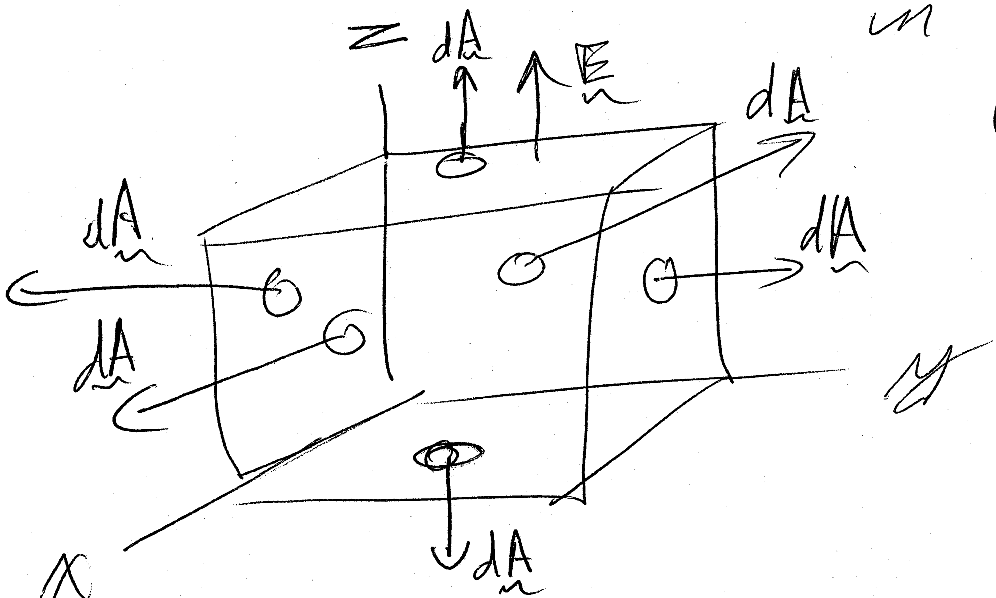
Example 2

$\underset{\sim}{E}$ uniform in space
in ~~and~~ points in z-direction.
and box with top and bottom area A_{TB}



$$\Phi = \oint_{Box} \underset{\sim}{E} \cdot d\underset{\sim}{A}$$

The vertical sides contribute zero since $\underset{\sim}{E} \cdot d\underset{\sim}{A} = 0$ since $\theta = 90°$

The $\Phi = EA_{TB}\cos\theta_T + EA_{TB}\cos\theta_B$
$= EA_{TB}(1 + (-1)) = 0$

The net flux thru the box is zero.

§ 24.2 Gauss's Law

Now for Gauss's law itself

K.F. Gauss (1777-1855) was one ~~of~~ the ~~wot~~ history's greatest mathematicians and also theoretical astronomer too.

[Up to about Gauss's time mathematicians and theoretical astronomers were often the same people →

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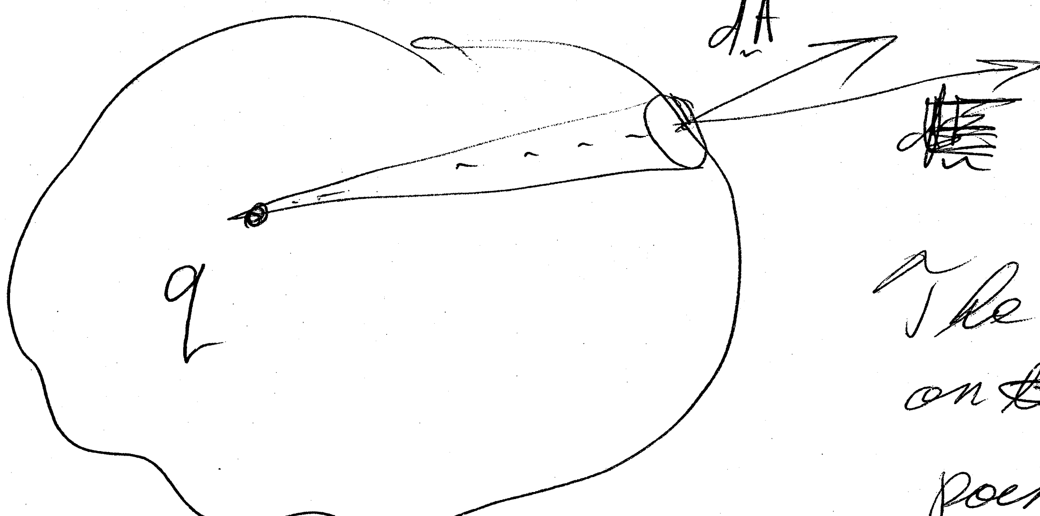
Newton, Ptolemy,
Lagrange, Laplace

— after about Gauss
math got to specialized
and tricky for astronomers
(were not that bright you know)

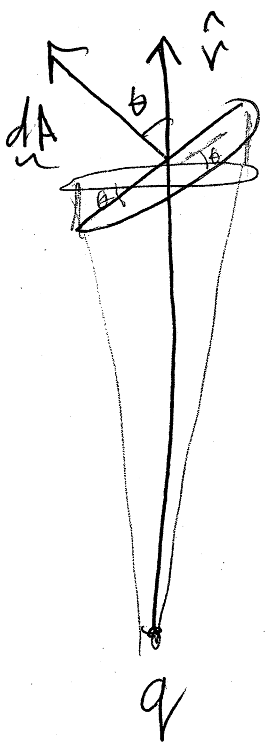
Consider a point charge
and ~~draw~~ put a general
closed surface around it



The $\underset{\sim}{E}$-field
on ~~the~~ $d\underset{\sim}{A}$
points straight
away from q
(or toward if $q<0$)

$$d\Phi = \underset{\sim}{E} \cdot dA$$

$$= \frac{kq}{r^2}\,\hat{r} \cdot d\underset{\sim}{A}$$



close up look

$$\hat{r} \cdot d\underset{\sim}{A} = dA \cos\theta$$

since $|\hat{r}| = 1$

$$= dA_{perpendicular}$$

Now define Solid Angle Ω

which Serway never does

(it's a scandal)

Capita Greek omega is the standard symbol.

(also used for ohms but that's a later usage

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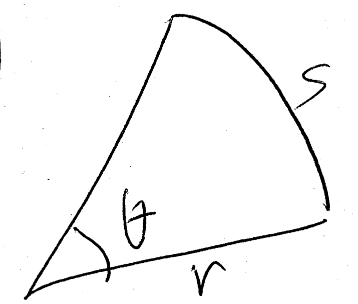
$$d\Omega = \frac{\hat{r} \cdot d\underset{\sim}{A}}{r^2}$$

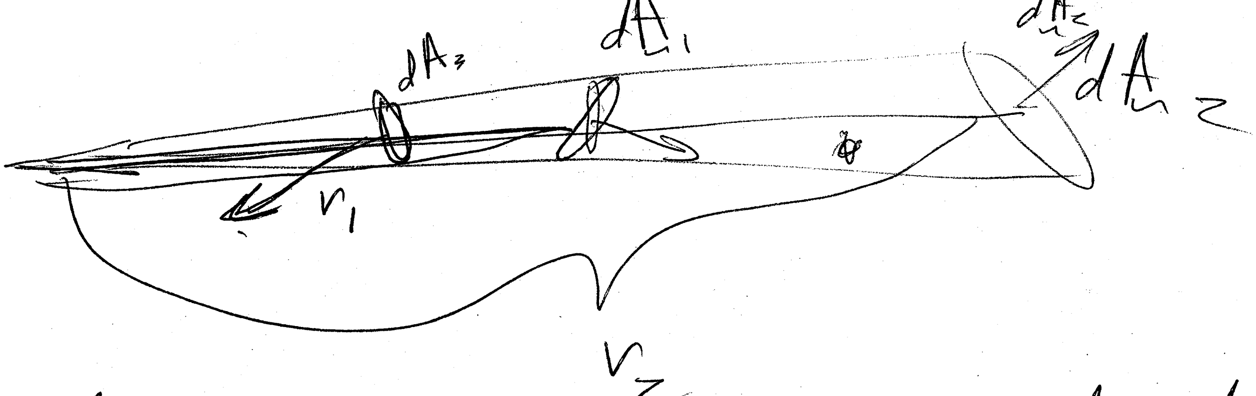
Solid angle is the 2-d analog of angle in radians

$$\theta = \frac{s}{r}$$



The solid angle subtended by any $d\underset{\sim}{A}$ that slices thru the cone is the same (except ~~maybe~~ for sign)

Has to be differentially narrow.

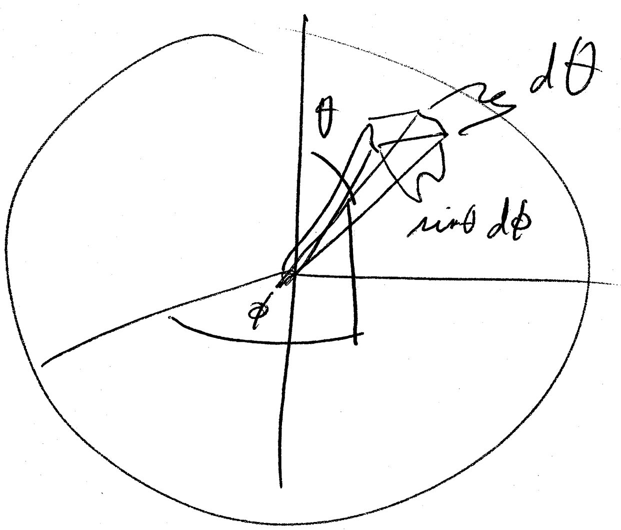


This is just an aspect of 3-d Euclidean space we live in.

Any surface area scales up as the square of length

In spherical polar coordinates

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

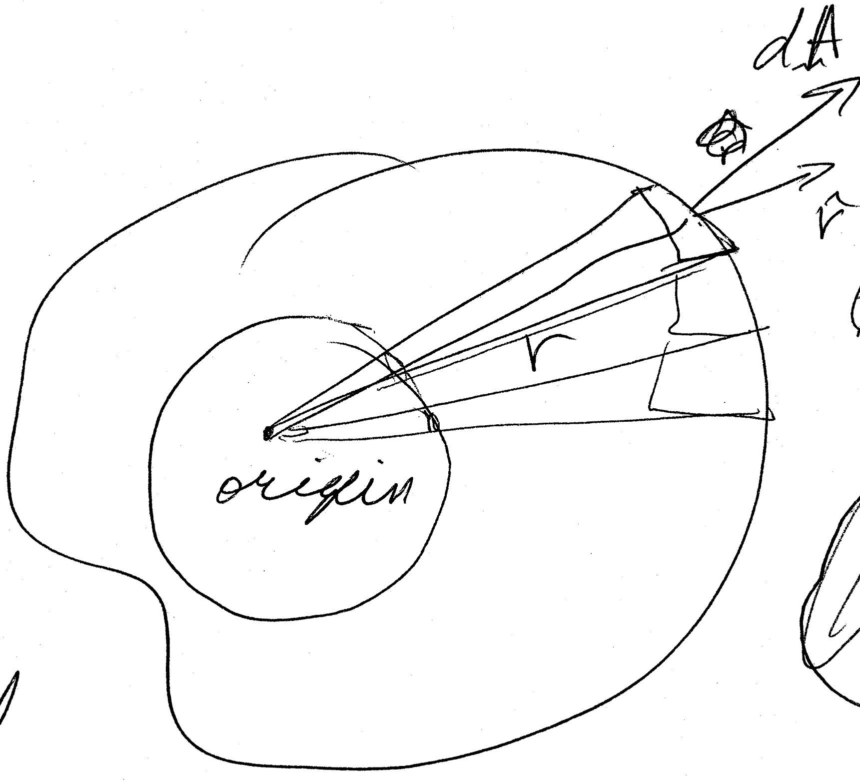


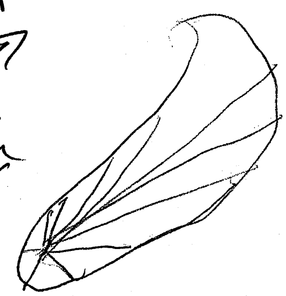
— one can think of all the bits of area on our ~~enclos~~ closed surface as "rectangles" in $d\theta$, $d\phi$ if it's a mental aid.

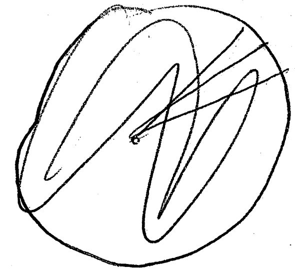
Area of a sphere of radius r
$= 4\pi r^2$

The solid angle ~~subt~~ subtended by a sphere is $\Omega = \frac{4\pi r^2}{r^2}$
$= 4\pi$

$\boxed{24\text{-}16}$



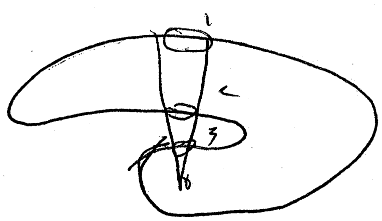




In fact ~~the~~ solid angle subtended by any closed surface

$$\Omega = \int \frac{\hat{r} \cdot dA}{r^2}$$

$d\Omega_1 + d\Omega_2 + d\Omega_3 = d\Omega_1$



Even funny shapes

$$= 4\pi$$

~~Th~~ Now

$$d\Phi = E \cdot dA \quad \text{constant}$$

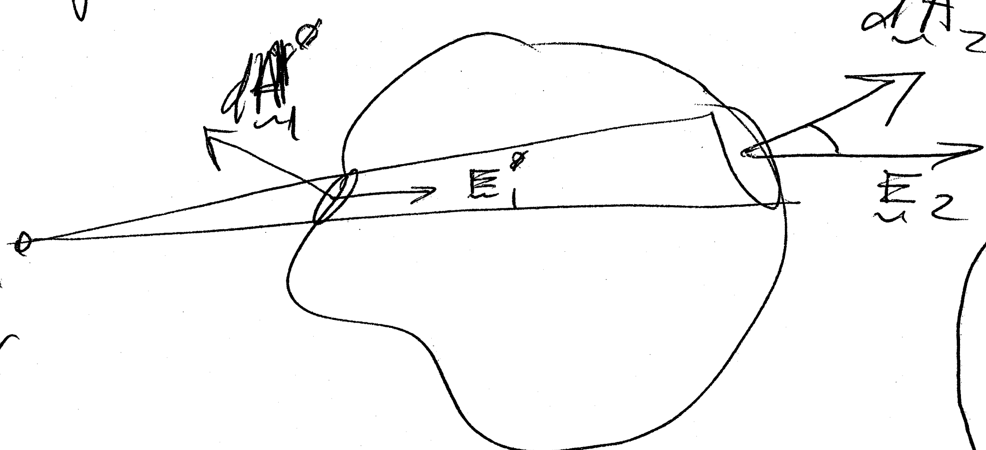
$$= \frac{kq}{r^2}\,\hat{r} \cdot dA = kq\, d\Omega$$

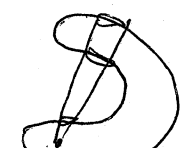
$$\Phi = 4\pi kq$$ for ~~an enclosed~~

closed surface around any point charge.

Note this result is a consequence of the inverse-square law nature of the Coulomb force.

What is Φ for a closed surface where the point charge is outside





tricky enclosed cases too

~~Well~~ $\cancel{d\Phi_1 + d\Phi_2 = \frac{kq\,\hat{r}_1 \cdot dA_1}{r_1^2} + \frac{kq}{r_2^2}\hat{r}_2 \, dA_2}$

Well

$$d\Phi_1 + d\Phi_2 = \frac{kq}{r_1^2}\hat{r}\cdot d\underset{\sim}{A}_1 + \frac{kq}{r_2^2}\hat{r}\cdot d\underset{\sim}{A}_2$$

$$= kq\,(d\Omega_1) + kq\, d\Omega_2$$

minus since the $d\underset{\sim}{A}$ vector doesn't point "out".

but $d\Omega_1 = -d\Omega_2$

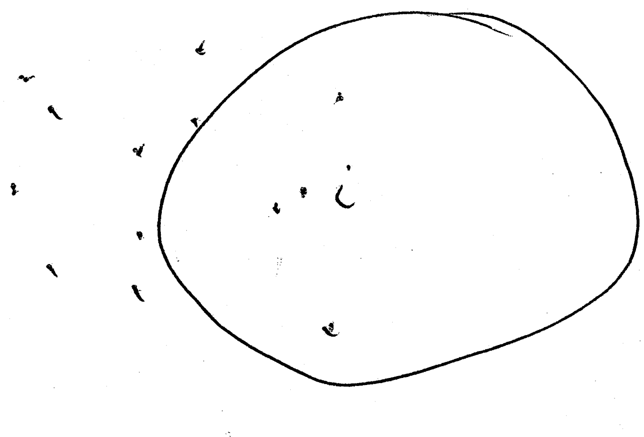
and so $d\Phi_1 + d\Phi_2 = 0$

There is a pairwise cancellation of all flux.

$\therefore \quad \Phi = 0$ for a point charge outside the closed surface.

Again a special consequence of the inverse-square law nature of Coulomb's law.

Now Consider a system of point charges — some in and some outside of a closed surface



$$\Phi_i = \begin{cases} 4\pi k q_i & \text{if } q_i \text{ inside} \\ 0 & \text{if } q_i \text{ outside} \end{cases}$$

Nothing forbids us summing up all the ~~the~~ point source fluxes

$$\Phi = \sum_i \Phi_i = \sum_i \begin{Bmatrix} 4\pi k q_i \\ 0 \end{Bmatrix}_{out}^{in}$$

$$\Phi = 4\pi k q_{enclosed}$$

$q_{enclosed}$ is the total enclosed charge.

Now recall Coulomb's constant is related to the alternative vacuum permitivity (or electric constant) ε_0 by

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \frac{C}{N}$$

24-20)

and $\cancel{\Phi = \oint \mathbf{E}\cdot d\mathbf{A}}$

~~the electr~~

$$\Phi = \sum_i \Phi_i = \sum_i \oint \mathbf{E}_i \cdot d\mathbf{A}$$

$$= \oint \left(\sum \mathbf{E}_i\right) d\mathbf{A} = \oint \underbrace{\mathbf{E}}_{\text{The total electric field.}} \cdot d\mathbf{A}$$

$$\therefore \quad \Phi = 4\pi k q_{encl}$$

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{encl}}{\varepsilon_0}$$

which is the usual form for ~~the~~ Gauss's law (in integral form)

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \frac{q_{encl}}{\varepsilon_0}$$

~~or~~ and q_{encl} can be written just q if one knows what one means.

Note this is general for any charge distribution, any ~~enclose~~ ~~cold~~ closed surface → Could be a physical surface or just one you imagine.

And we derived it just from

24-22)

Coulomb's law and geometry.

Really the inverse-square law matters was the key point.

$d\Phi = \underset{\sim}{E} \cdot dA$

$= \frac{kq}{r^2} \hat{r} \cdot dA$

$= kq\, d\Omega$

all distance dependence killed. Wouldn't happen if $\underset{\sim}{E} = \frac{kq}{r^p} \hat{r}$ and ~~Bnd~~ $p \neq 2$.

§24.3

Applications of Gauss's Law

It can be used to find the $\underset{\sim}{E}$-field ~~analts~~ ~~cases~~ analytically and exactly in

3 cases of very high symmetry

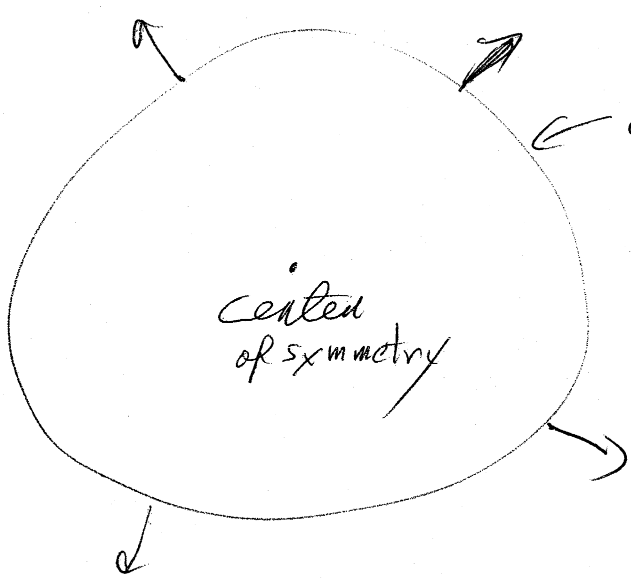
— the highest we have in a sense in a 3-d Euclidean world:

spherical, cylindrical, planar

1) Spherical symmetry

– one has any charge distribution ~~that has~~ spherical symmetry and ~~draw~~ one imagines a spherical surface concentric with the center of symmetry

a gaussian surface in the jargon (Serway-678)



← gaussian spherical shell.

By symmetry, the electric field must be radial everywhere on the surface + have constant magnitude

24-24

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$\cancel{E \oint 4\pi r^2 dr}$$

$$E \oint r^2 d\Omega$$

$$= E\, 4\pi r^2$$

$$\therefore \quad E = \frac{q_{encl}}{4\pi \varepsilon_0 r^2} = \frac{k\, q_{encl}}{r^2}$$

or $$\underset{\sim}{E} = \frac{k\, q_{encl}}{r^2}\hat{r}$$

Just like for a point charge.

Special case

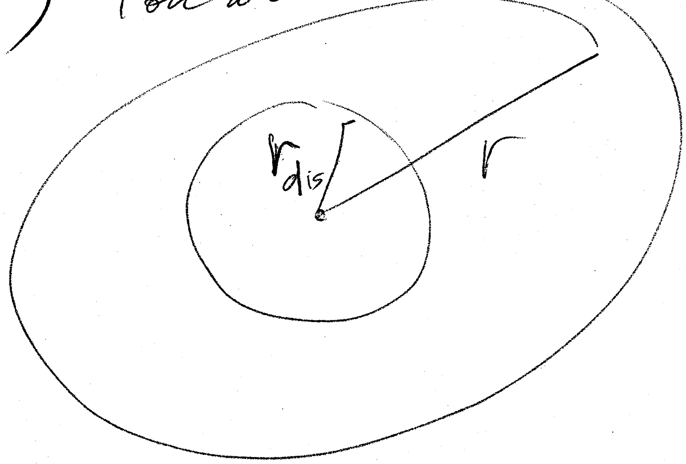
a) q_{encl} is just a point charge q

$$\underset{\sim}{E} = \frac{kq}{r^2}\hat{r}$$ and we've recovered the Coulomb's law form.

So Coulomb's law
implies Gauss's law
and vice versa
— which you take as primary
is a matter of choice.

In Maxwell's formulation
of EM, Gauss's law
in it's differential eqn. form
is taken as primary.

b) You are outside the region of charge
altogether,



Then the
charge distribution
seems from an
electrostatic point
of view just like
a point charge with
all charge at the
center.

~~practically~~ if it
were a solid ball
with rigidly fixed charge that
~~how~~ it could be

24-26

in calculations.

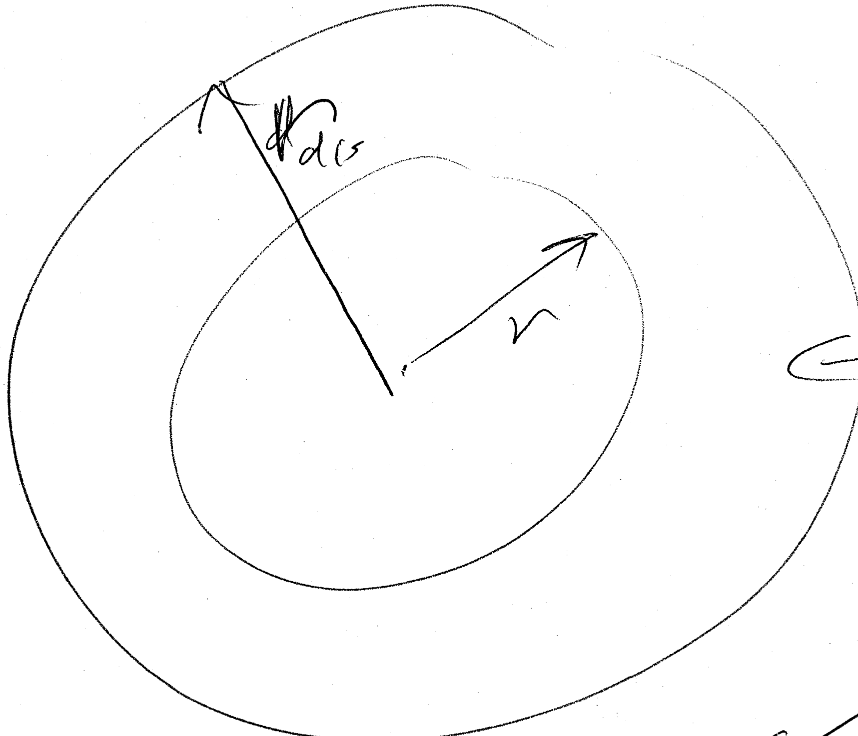
Note Gravity because it has an inverse-square law force, also has a Gauss's law.

with Gauss's law one almost instantly proves that a ~~spherical~~ spherically symmetric mass acts as point Mass if you are outside

— this makes treating planets & stars (which are nearly spherically symmetric easy.

— Newton with more primitive calculus ~~tools~~ had to work really hard to prove this result which was essential to his explanation of the solar system.

c) Inside the distribution



$$\underset{\sim}{E} = \frac{k\, q_{enc}}{r^2}\,\hat{r}$$

all the charge outside has no effect inside.

an amazing consequence of having an inverse-square ~~sq~~ law force.

So if $q_{encl} = 0$,

$\underset{\sim}{E} = 0$ and there is no E-field ~~inside~~ at r no matter

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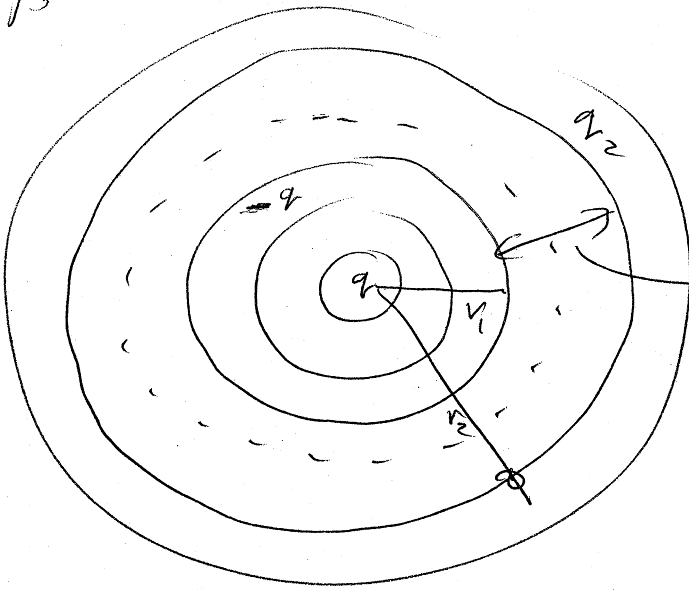
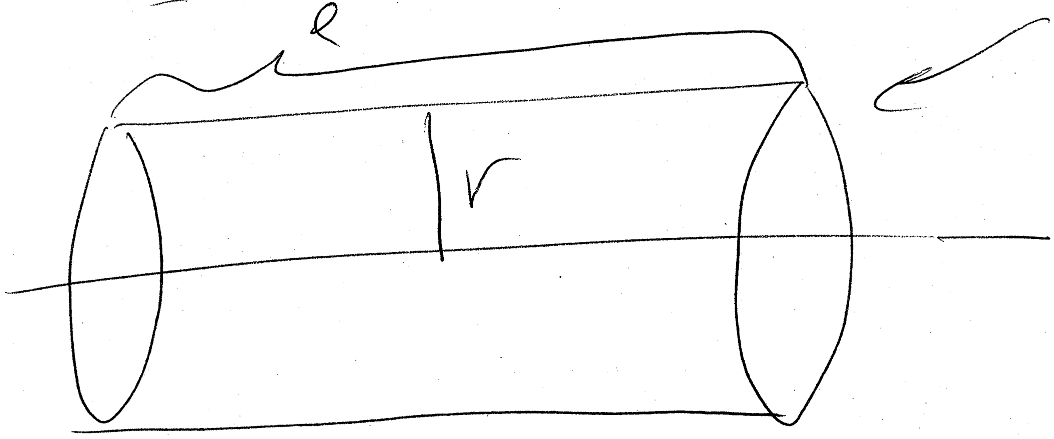
what it is further out
or further in

e.g.



in this region
$\tilde{E} = 0$
even though
$\tilde{E} \neq 0$ for $r < r_1$
or $r > r_2$

2) Cylindrical Symmetry



Gaussian cylinder concentric on axes of symmetry

$\tilde{E}$ must be radial only
(radial in cylindrical coordinates)

$\therefore \int_{\text{ends}} \underset{\sim}{E} \cdot dA = \emptyset$

since $\underset{\sim}{E}$ & $d\underset{\sim}{A}$ are always perpendicular on the ends.

On the side where $\underset{\sim}{E}$ can only be ~~constant~~ magnitude & radial

$$\int_{\text{side}} \underset{\sim}{E} \cdot d\underset{\sim}{A} = E \int_{\text{side}} dA = E\,(2\pi r\,l)$$

$$\therefore \oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E\,(2\pi r l) = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$\underset{\sim}{E} = \frac{q_{\text{encl}}/l}{2\pi r \varepsilon_0} = \frac{\lambda}{2\pi \varepsilon_0 r}\,\hat{r}$$

where $\lambda \equiv \dfrac{q_{\text{encl}}}{l}$ is the linear charge density

~~E~~ E doesn't fall off at ~~dist~~ great distance as $\frac{1}{r^2}$ ~~as~~ like a localizable charge distribution — but the charge distribution is NOT localizable. It extends to infinity along the axis.

24-30

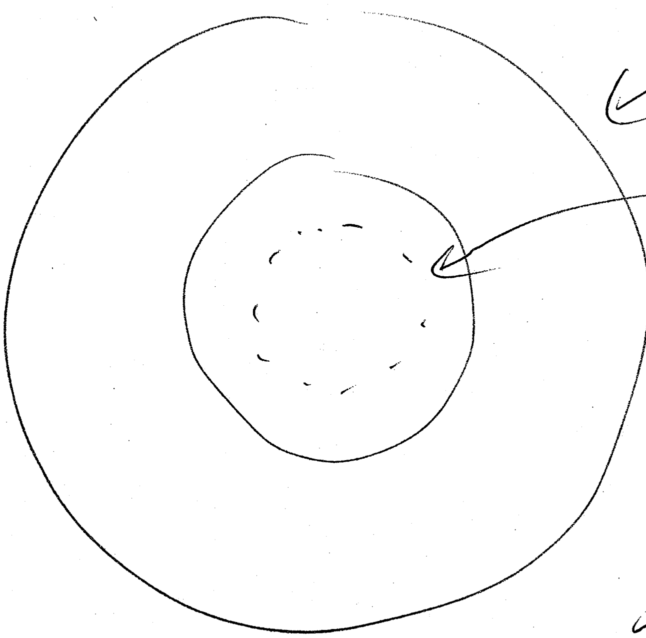
There are no infinite cylindrical charge distributions, of course. But close to a finite one, our result applies approximately.

Special cases

a) line of charge — the thin rod

$$\underline{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} \text{ and as } r \to 0, \quad \underline{E} \to \infty.$$

b)



Cross-section view.

no charge in cavity.

$\therefore \underline{E} = 0.$

I read someplace that Ben Franklin knew this result. He charged a metal tankard (they drank from them customarily in those days — and tried not to crack their teeth on those hard, massive things)

and noted that
paper bits floating in the
tankard (on water
one guesses unless
he intended to drink
the experiment) were
unaffected by the electric
force —— they are of
outside as we know from
the comb demo.

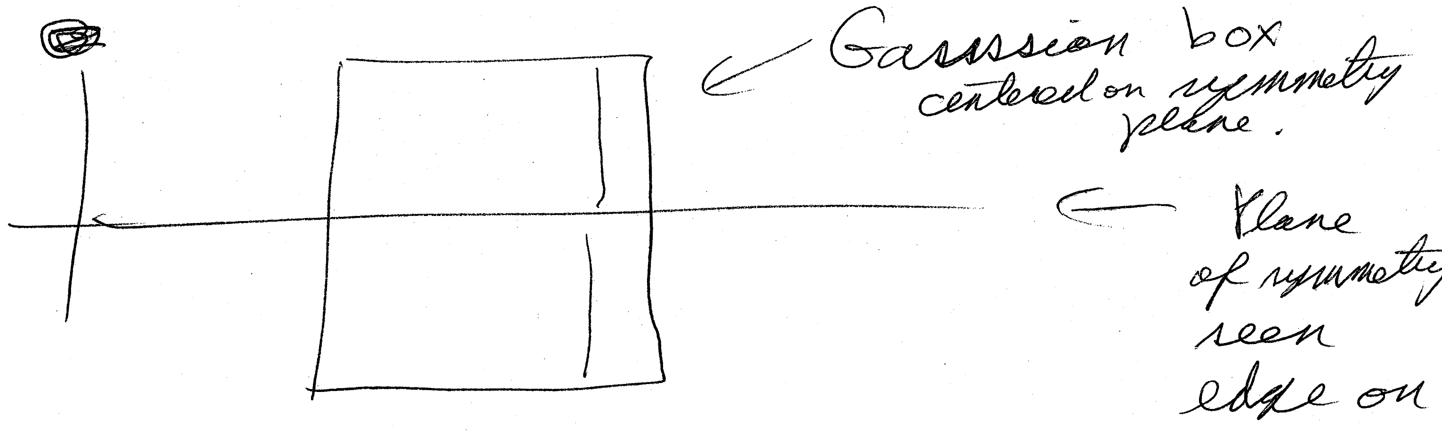
From this observation
he maybe deduced the
inverse-square law nature
of the electric force
"Maybe" because I recollect
or find the source of the

24-32) story now and I may have garbled it.

Ben wasn't a great mathematician, but he may have known of the result pre-Gauss.

Or maybe he made the observation, but only later folks recognized it's importance.

3) Planar Symmetry



$\underset{\sim}{E}$ can ~~dey~~ only be perpendicular to the plane of symmetry and point <u>outward</u> from it.

$$\int_{\text{Sides}} \underset{\sim}{E} \cdot d\underset{\sim}{A} = \emptyset$$

$$\int_{\substack{\text{top} \\ \text{\& bottom}}} \underset{\sim}{E} \cdot d\underset{\sim}{A} = \underset{\sim}{E} \cdot \underset{\sim}{A}_{top} + \underset{\sim}{E} \cdot \underset{\sim}{A}_{Bottom}$$

$$= 2EA_{TB}$$

since $A_{top} = A_{Bottom}$

~~Let~~

$$\therefore \quad 2EA_{TB} = \frac{q_{encl}}{\varepsilon_0}$$

$\frac{q_{encl}}{A} = \sigma_{encl}$ area charge density

E depends on $\hat{r}$, but NOT on r

$$\underset{\sim}{E} = \frac{\sigma_{encl}}{2\varepsilon_0}\hat{r}$$

the radius unit vector from the plane

points up ~~on~~ ~~top~~ above plane

– down on ~~the bottom~~ below plane.

24-34

It's constant ~~thru~~ all space.

Again at great distance

$\underline{E}$ does not fall off as $\frac{1}{r^2}$

~~since~~ but that OK since

~~the~~ charge distribution is

not ~~finite~~ localizable.

Of course there are no infinite planar charge distributions. But close to a finite one, our analytic result applies ~~approx~~ approximately.

Special cases

a) A thin plane

of charge density σ

$$\underline{E} = \frac{\sigma}{2\varepsilon_0}\,\hat{r}$$

$\uparrow \underline{E}$

$\downarrow \underline{E}$

A constant thru all of space except for the direction change across the plane

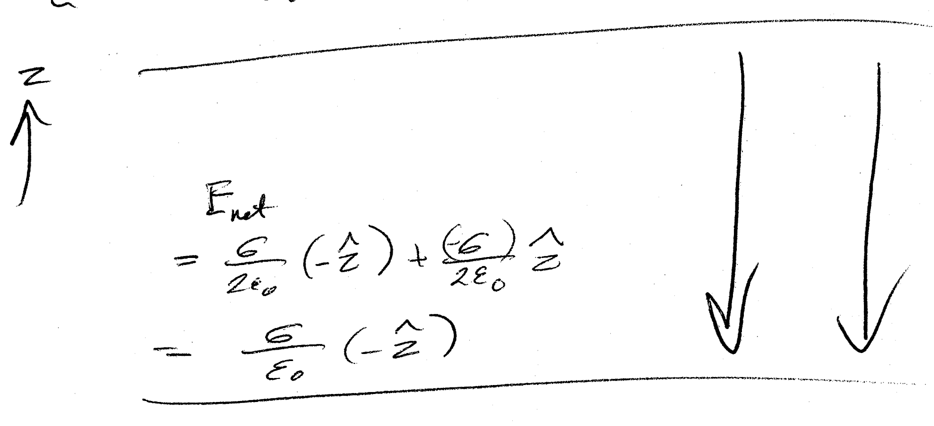
b) We can actually break pure planar symmetry making use of ~~the~~ superposition principle.

24-35

Say we had two ~~plates~~ thin planes of equal & opposite charge. There are 3 regions

$$\underset{\sim}{E}_{net} = \frac{\sigma}{2\varepsilon_0}\hat{z} + \frac{(-\sigma)}{2\varepsilon_0}\hat{z} = \emptyset$$

This with σ



$$\underset{\sim}{E}_{net} = \frac{\sigma}{2\varepsilon_0}(-\hat{z}) + \frac{(-\sigma)}{2\varepsilon_0}\hat{z} = \frac{\sigma}{\varepsilon_0}(-\hat{z})$$

This with $-\sigma$

$$\underset{\sim}{E}_{net} = \frac{\sigma}{2\varepsilon_0}(-\hat{z}) + \frac{(-\sigma)}{2\varepsilon_0}(-\hat{z})$$

$$= \emptyset$$

So outside $\underset{\sim}{E} = \emptyset$

Inside it is uniform with magnitude $\frac{\sigma}{\varepsilon_0}$

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and it points from the positive to negative plane.

This case actually approximates the case of a ~~plate~~ parallel-plate capacitor. (see Serway 724 in Ch. 26)

— experimentally ~~it's very~~ useful to have a way to construct uniform $\underline{E}$-fields.

— Capacitors have vast uses in technology, of course. Well beyond me.

§ 24.4

Conductors in Electrostatic equilibrium

This means cases where the charge is <u>not moving</u>.

Also a stable equilibrium.

— small perturbations don't cause the state to change.

— and small perturbations from thermal jostling of atoms & electrons are always present.

— Our considerations are for good conductors — which

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means we are really thinking (but not exclusively) of metals (e.g., Copper, Aluminum, silver and other good conducting metals)

We aren't thinking of superconductors which have weird special properties.

Properties

1) E-field is zero everywhere inside whether the conductor is solid or has cavities or what applied fields are present outside. or what net charge it has

$E=0$

Special Remark

i) This means macroscopic E-field. At the atomic level there are E-fields all over bonding atoms and electrons but averaged over a macroscopic ~~area~~ volume

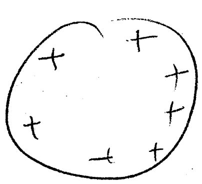
~~all~~ $\underline{E} = 0$ inside.

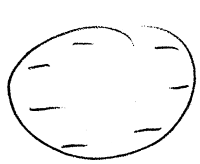
ii) An applied field can in principle penetrate a metal, but it would have to be so strong as to pull all the free charge out of region.

in an electro-static case.

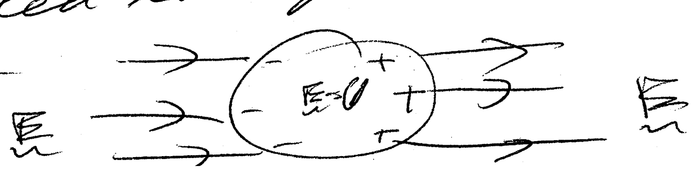
→ practically almost never a possibility although I suppose there must be extreme cases.

2) If an isolated conductor has <u>net charge</u>, it is on the surface.





— even a <u>neutral conductor</u> can have charge separation and become polarized in an applied field. But the separated charge is also on the surface

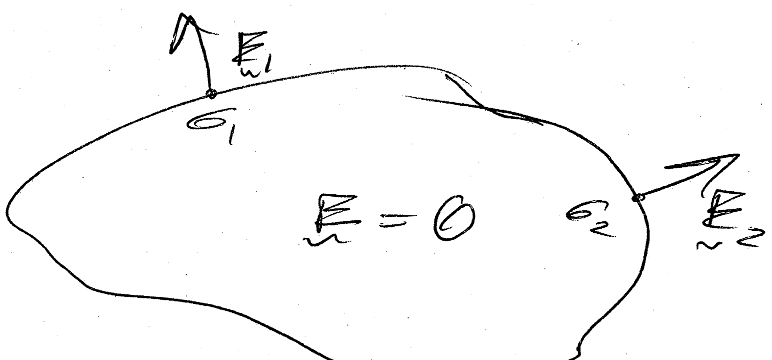


24-40

3) The E-field *just* outside the conductor has magnitude $\underset{\sim}{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

where σ is the local charge density which in general varies

Maybe only a tiny extent outward — or maybe a long way



$\underset{\sim}{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

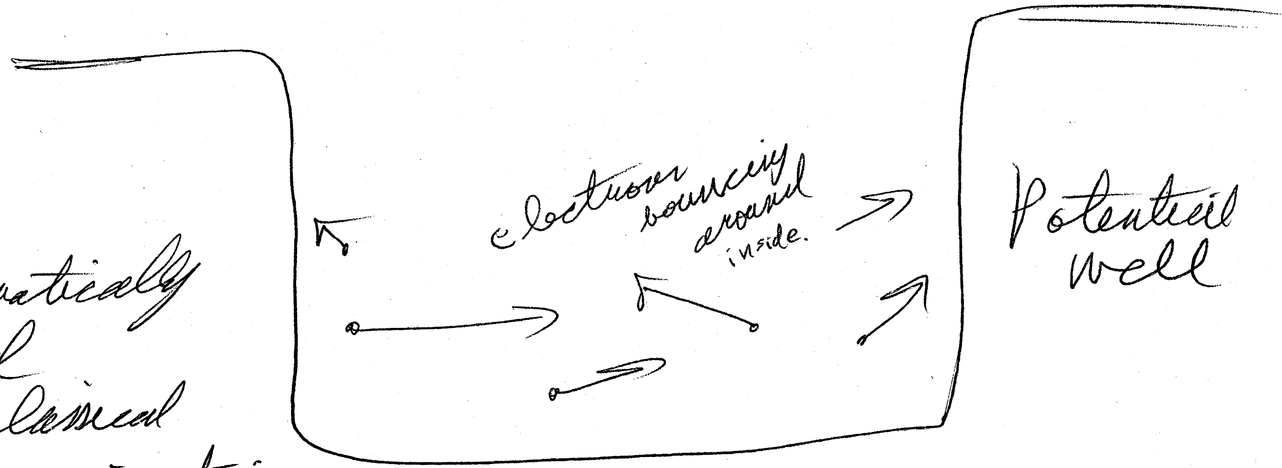
The E-field is normal to the surface.

Note in (2) & (3) we are idealizing the surface a thin impenetrable barrier.

— Actually at the micro level surfaces are rather complex, generally rough, generally impure.

→ But there is a steep ~~po~~ potential energy barrier near the surface that holds the electrons in

PE



Schematically and in classical approximation.

— actually a strong enough applied field can pull electrons ~~off a~~ out of a metal

or a large excess of -ve charge can push them out (Field emission WP-568)

(or a large excess of +ve charge (absence of electrons) pull electrons out of surrounding medium.

— heating the metal gives the electrons ~~KE~~ more KE

24-42)

and that helps electrons escape.

— hot cathodes are technologically important.

in old-fashioned tube TVs for example.

$\underset{\sim}{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

4) On irregular conductors the surface charge density and surface $\underset{\sim}{E}$-field tends to be highest on regions of ~~a~~ smallest radius of curvature. — i.e., the most pointy parts like corners or edges.

This makes corners & edges most dangerous for electrical ~~arcs~~ ~~sparks~~ discharges in the air.

Note "tends"

— there is no absolute rule here and the tendency can be overruled

24 ~~4~~3

by strange geometry
or strange applied
external fields
(WP-603)

We'll give a proof ~~oth~~ of this ~~tendency~~ in Ch. 25, Serway-708

Proofs (of 1, 2, 3)

1)

~~1) Say you have a neutral~~

1) Say ~~that there is~~ macrosopic a non-zero electric field in a conductor.

— since there is free charge, it will flow and one will have currents. (macroscopic currents).

— In fact as currents flow energy is lost by electrical resistance to heat (which doesn't happen in perfect superconductors

heat includes random microscopic KE and ~~is~~ ~~Hea~~ has no macroscopic currents

and unless there is
some "driver"
supplying ~~energy~~ the
charges must come
to rest.

So $\underset{\sim}{E} \neq 0$ is inconsistent
with our premise of
an electrostatic equilibrium.

2) Where is any excess
~~of~~ separated charge..
due to an external field?

say $\underset{\sim}{E}_{\text{ext applied}} = 0$
and $Q_{\text{excess}} = 0$.

— macroscopically the metal
is overall neutral including surface.

The strong attraction
of negative & positive
charge enforces neutrality
at the macro level
- QM and thermal perturbations
prevent neutrality at
the micro level.

Now what if one had excess
charge, but $E_{applied\,ext} = 0$

the excess charge repels
itself.

the individual charges
(electrons or absences
of electrons which are
sometimes called holes
but usually only in case of
semi-conductors WP-1130
(Wik))

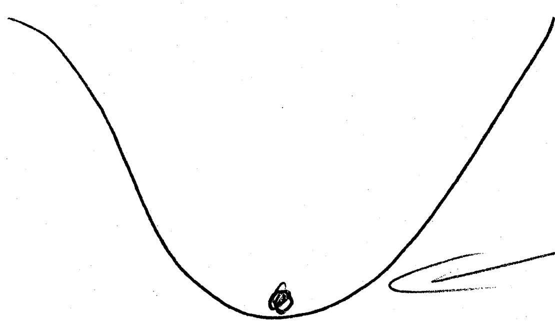
try to get as far from
each other as possible

24-46

and they keep moving until their mutal repulsion force is canceled by a surface force ~~that keeps them from escaping the metal.~~

The surface force is actually an electrostatic force too — but we won't go into that — and just say it is a force that keeps the charge from escaping (not absolutely. Recall that if the repulsion is strong enough electrons can escape or be pulled in from the outer medium.)

— the excess charge is pressed to the wall and dissipates any excess KE as heat and settles into a stable static equilibrium



Charge at bottom of potential well.

Now consider a neutral or charged conductor

If an external E-field is applied, at first it penetrates the conductor and causes currents.

A constant one, not a time varying one

↳ the flowing stops when the charge in the conductor

has arranged itself so as to cancel ~~all the~~ the external field and put the conductor into electrostatic equilibrium.

[Perpetual flow is ruled out by dissipation of energy to heat]

Any $E \neq 0$ would cause charge to flow

Now this means $E = 0$ inside the conductor.

At first one might wonder if this is possible with

i.e. $E = 0$

macroscopic charge separation in the interior of the conductor.

24 - 80

24-52

It's arranged to cancel the external applied field.

Two tricky questions arise.

1) Can a strong enough **field** pull all the free charge out of a region and penetrate it?

As we already mentioned on

p. 24-39.

In principle yes, but in most ordinary situations it never happens.

Recall 1g of copper contains (Wik)

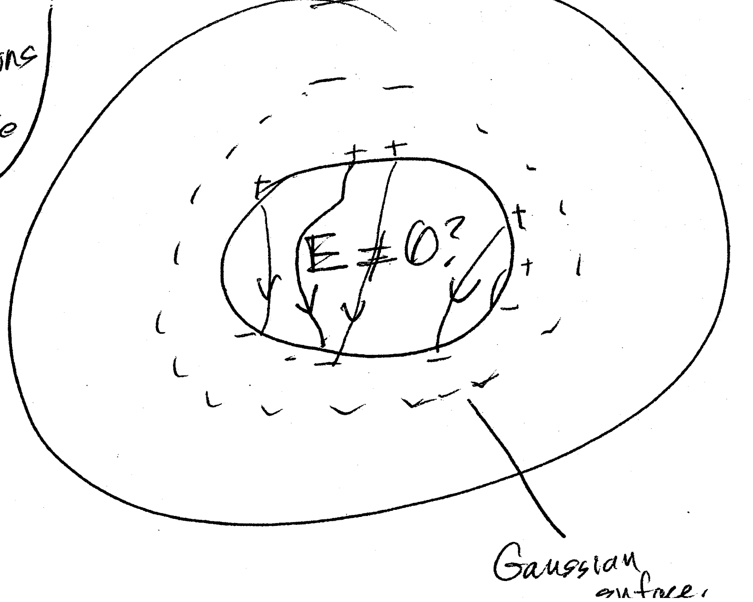
$$n \cong \frac{1g \cdot Z}{63.5 \times 1.6 \times 10^{-24} g} * 1.6 \times 10^{-19} \frac{C}{e}$$

$$= \frac{10^{5}}{30} C$$

$$\cong 3 \times 10^{3} C$$

Recall 1C is a huge net charge and trying to pull all free electrons out of 1g of copper would in most applications be impossible

2) What about cavities?

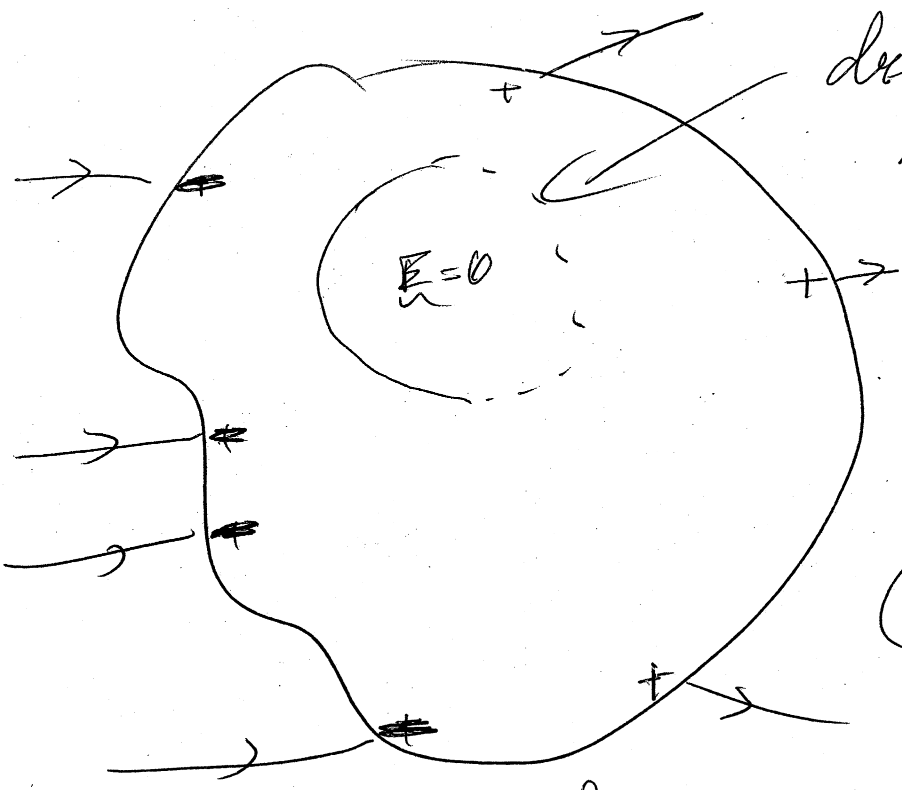


Our argument says $\underline{E} = 0$ in the solid where charge can flow. But in a cavity there is no charge and so $\underline{E} \neq 0$ is not ruled out.

Frankly this doesn't seem too likely

→ some sort of fine balance would be needed.

In fact, there can be no macroscopic clumps of ~~in the~~ charge in the solid interior.



draw a Gaussian surface anywhere in the solid conductor

Since $\underset{\sim}{E} = 0$

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = q_{encl}/\varepsilon_0$$

$$\rightarrow 0 = q_{encl}.$$

Separated ~~lumps of~~ charge can be on the surface and generally is

Our Gauss's law ~~arg~~ argument tells us there can be no separated charge in the solid where $E = 0$ everywhere but a Gaussian surface around a cavity only tells us there is no net charge inside.

There can be separated charge on the walls of the cavity as long as it sums to zero and thus an E-field inside

~~CP 2007~~

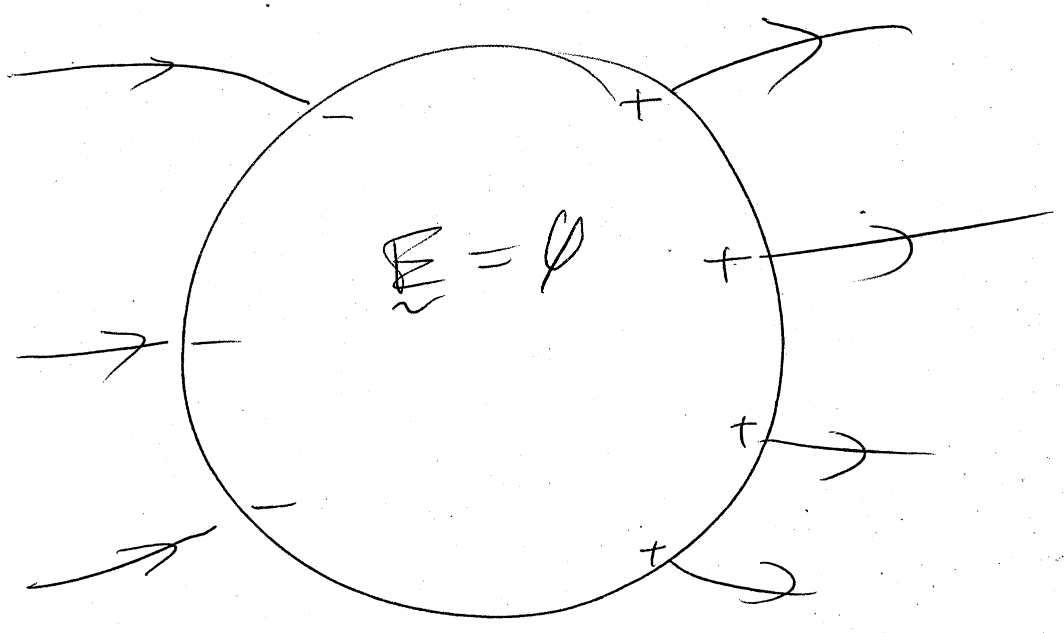
So our ~~a~~ arguments allows so far.

24-54

But we can go further.

Consider a solid
with no cavities
in electrostatic equilibrium.

— there may be a net charge
on the solid and an
external field



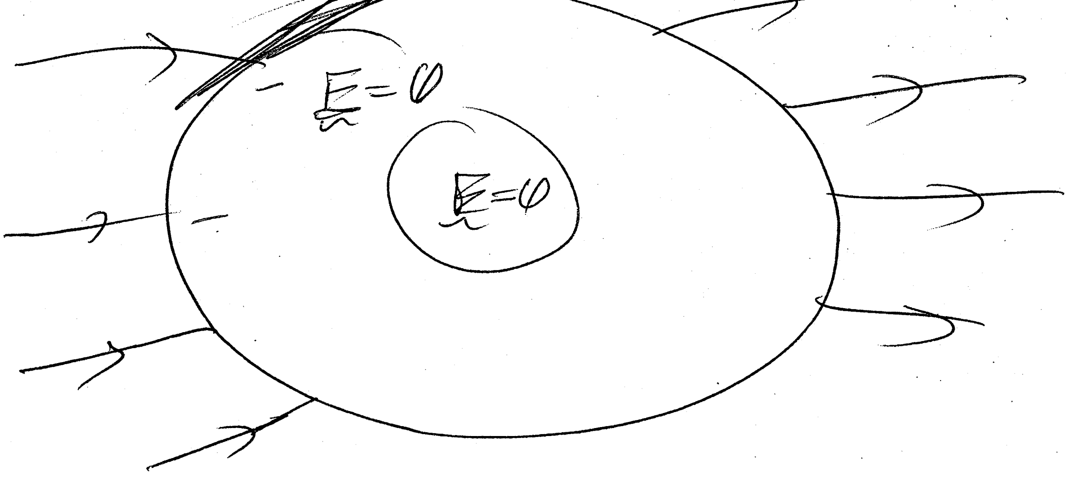
— all separated and net
charge is on the surface
and $\underset{\sim}{E} = 0$ everywhere
inside.

Now magically remove a clump of interior conductor to create a cavity.

- there was no $\underline{E}$-field ~~or~~ there before, there should be none after removal.

→ with no $\underline{E}$-field, no charge can flow around the cavity surface.

<u>Ergo</u> after removal, from an $\underline{E}$-field and charge distribution nothing should change.



24-56

So for any set up with a cavity a solution exists where $\underset{\sim}{E} = 0$ in the cavity and there is no charge separation on the cavity walls.

The only question is, is that the *unique* solution.

The answer is *yes*

It is the unique solution. But the math to prove it is just a bit beyond our scope. I'll give an optional proof in the Ch 25 *notes*. (Surface must be an equipotential.)

Serway -709 gives a "proof" that $\underset{\sim}{E} = 0$ in cavities but I think they've muffed it. — I'll explain why then.

— actually the overall solution for an electrostatic conductor is unique.

[I also give an optional proof of this in Ch 25]

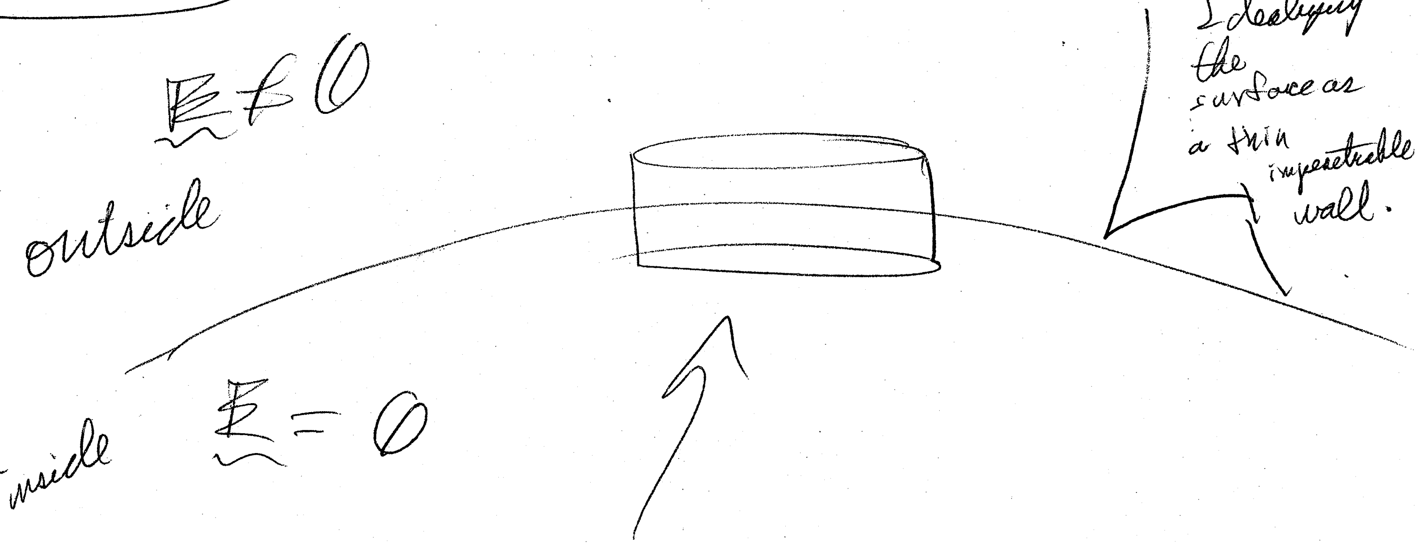
3) The last thing to prove is ~~that~~

$$\underset{\sim}{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$$

where σ is the surface charge density and $\hat{n}$ is normal to the surface.

This is proven using Gauss's law.

24-58)



tiny Gaussian surface just straddling the conductor surface.

It is small enough that σ is constant inside.

$\underset{\sim}{E}_{\text{Bottom}} = 0$ since inside conductor

$\underset{\sim}{E}_{\text{sides}}$ must be normal to the ~~[illegible]~~ surface or else there would be a force on the surface charge and it would flow and that ~~would be~~ is ruled out by the fact that we have already reached electrostatic equilibrium.

$$\therefore \int \underset{\sim}{E}_{bottom} \cdot d\underset{\sim}{A}_{bottom} = 0$$

$$\int \underset{\sim}{E}_{sides} \cdot d\underset{\sim}{A}_{sides} = 0$$

$$\therefore \oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \int \underset{\sim}{E}_{top} \cdot d\underset{\sim}{A}_{top} = \frac{q_{encl}}{\varepsilon_0}$$

which must also be normal to the surface

$d\underset{\sim}{A}_{top}$ is so small $\underset{\sim}{E}_{top}$ is a constant.

$$\therefore \underset{\sim}{E}_{top} = \frac{q_{encl}}{\varepsilon_0 A_{top}}$$

or $$\underset{\sim}{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

The Gaussian surface can be made

24-60)

~~the~~ vanishingly small (in a macroscopic sense)

and so this

result is exact (in a macroscopic sense)

Recall our result for an infinite plane of charge

$$\underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where $\hat{n}$ points outward from the surface, on both sides

Up close to a finite plane — just at its surface, we should have

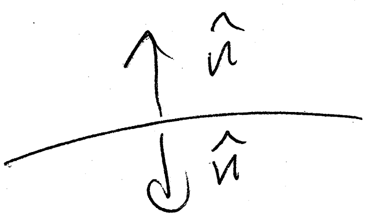
$$\underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \text{ also.}$$

Now up close our conducting

surface ~~one~~ the surface charge itself should give

$$\underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

for $\hat{n}$ in and out



Conductors in electrostatic equilibrium self-regulate to do this. Self-regulation is an important aspect of steady current conduction too.

But all the other charge and applied fields in the world cancels the inside pointing E-field and double the outside pointing E-field (Tipler-751)

Ex 24.7

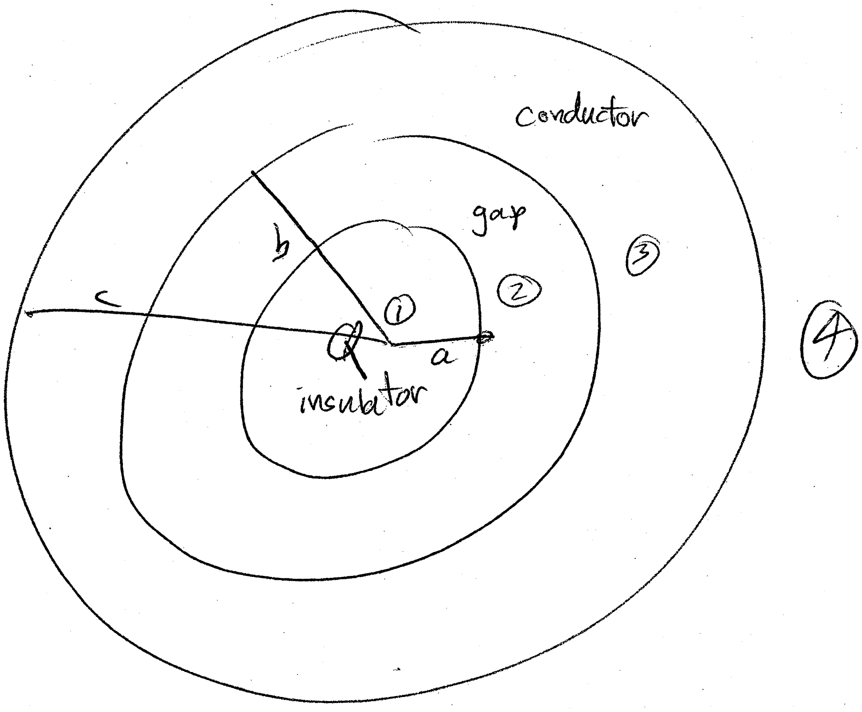
We have an insulating sphere with net charge Q uniformly spread throughout

24-62

There is a concentric conducting spherical shell with a vacuum gap between.

the shell is overall neutral

Find $\underset{\sim}{E}$ everywhere and where separated charge is.



Gauss's law

① $\underset{\sim}{E} = \dfrac{q_{encl}}{4\pi\varepsilon_0 r^2}\hat{r} = \dfrac{Q\left(\frac{r}{a}\right)^3}{4\pi\varepsilon_0 r^2}\hat{r}$

$= \dfrac{Q}{4\pi\varepsilon_0 a^2}\left(\dfrac{r}{a}\right)\hat{r}$

② $\underset{\sim}{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

(3) $\underset{\sim}{E} = 0$ by the fact ~~that the~~ shell is a conductor

$\therefore$ $q_{encl} = 0$

$\therefore$ ~~+~~ $-Q$ must be spread on the inner shell surface.

$\sigma_{\substack{\text{inner}\\\text{shell}\\\text{surface}}} = \dfrac{-Q}{4\pi b^2}$ it must be uniform by symmetry.

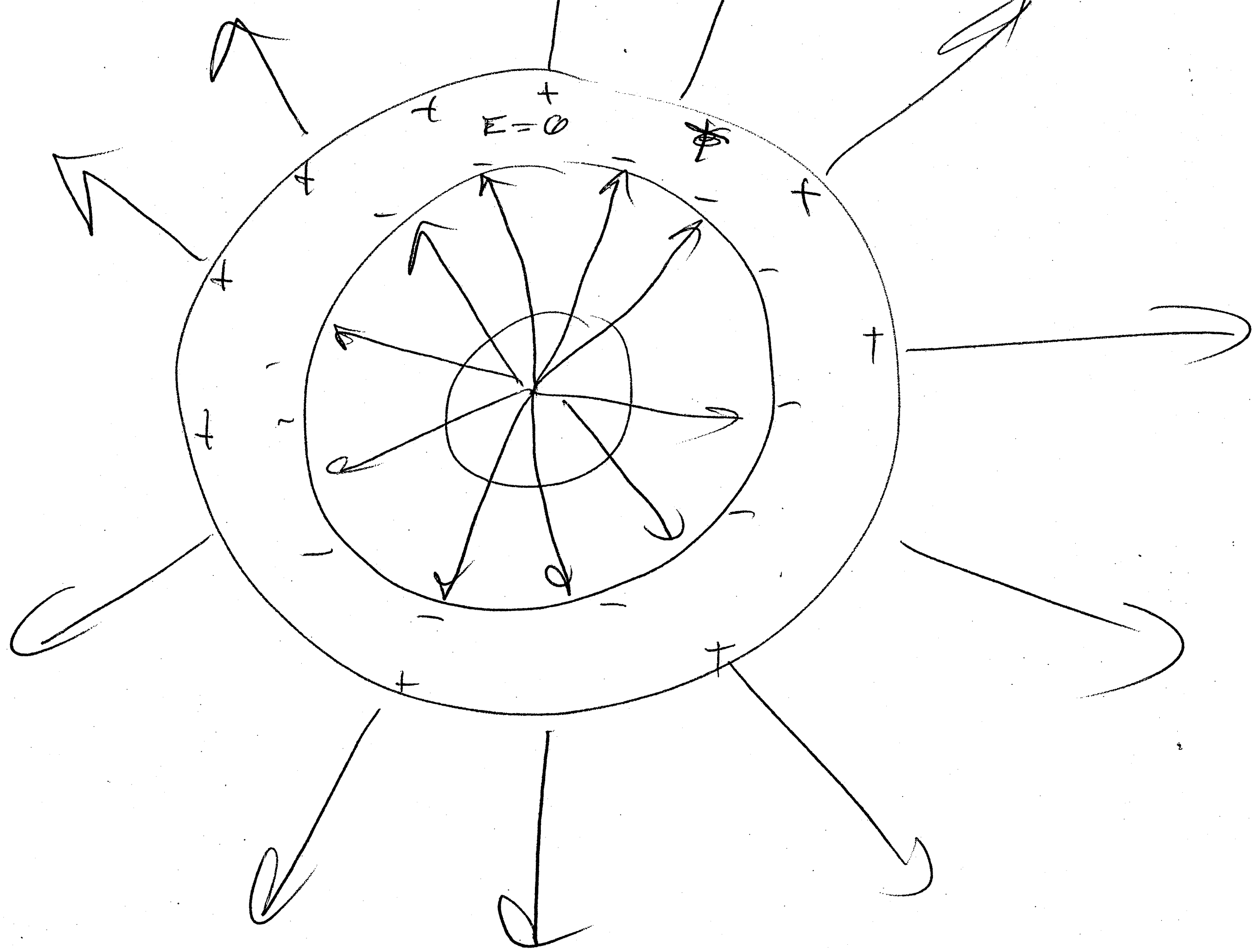
(4) $E = \cancel{\dfrac{Q}{4\pi r^2}} \; \dfrac{Q}{4\pi \varepsilon_0 r^2} \hat{r}$

~~also~~ since Q is the net charge enclosed (the shell is neutral overall).

$\sigma_{\substack{\text{outer}\\\text{shell}\\\text{surface}}} = \dfrac{Q}{4\pi c^2}$

and it must be uniform by symmetry.

24-64



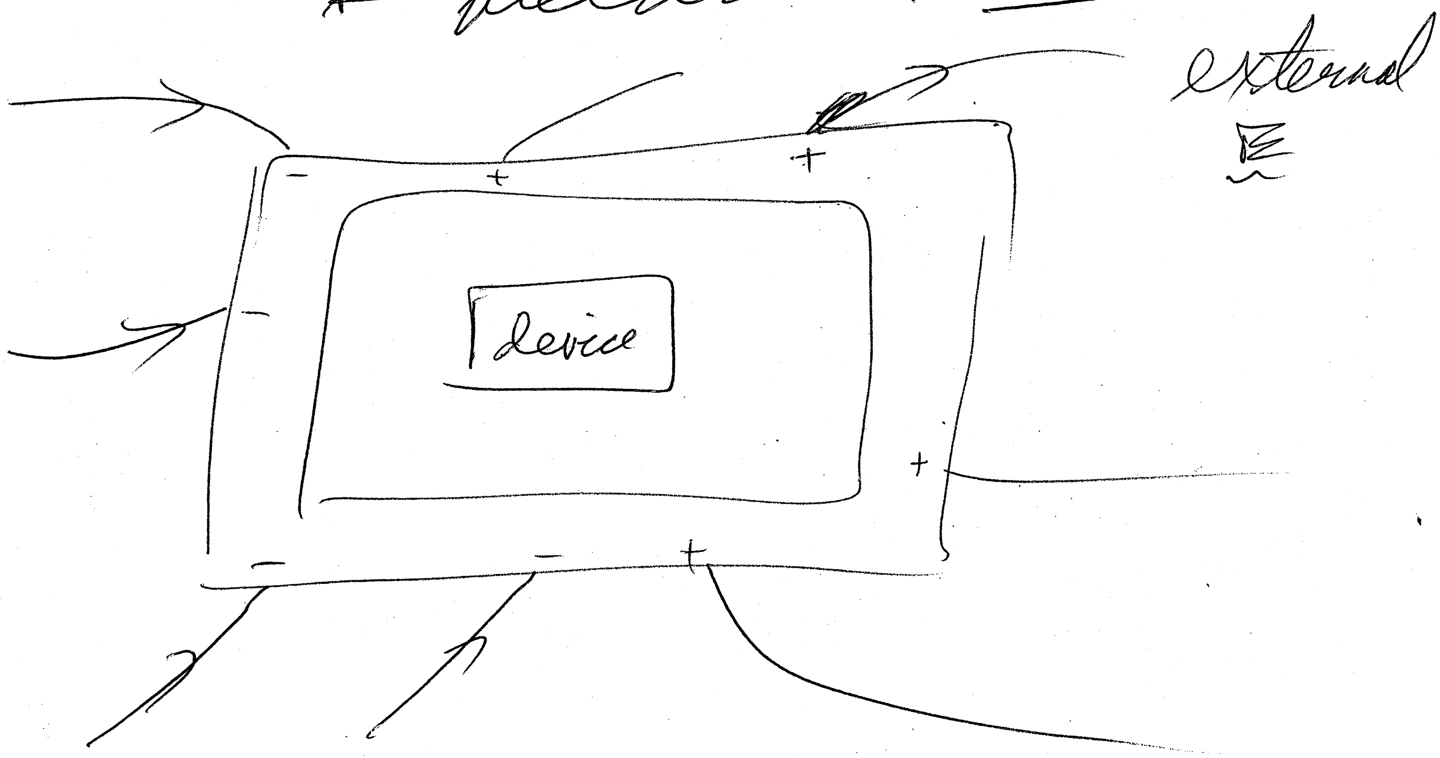
Once you get ~~the~~ hang of it all such highly symmetrical Gauss's law plus conductors are pretty similar and easy — it's when you start breaking symmetries that

the going gets tough
(and yours truly bugs out)

Shielding

These are called Faraday cages. (Wiki)

If you put something in a conducting container it is shielded from external ~~m~~ $\underset{\sim}{E}$-fields — static ones



24-66

— to be perfect
the external E-field
must be absolutely
static

— and the container must
completely enclose.

<u>But</u> in practical cases one
doesn't need these conditions
absolutely.

e.g., protect amplifiers ~~that~~
from stray electrical fields
that would introduce noise
into radio or other transmissions
(WP-569)

Serway suggests that good conductors
reach equilibrium in 10^{-16} s !!
— I'm not sure if this number is right

24-68

Obviously how quickly the conductor charge adjusts to cancel the external field depends on many things

— size & shape of ~~on~~ the Faraday cage (It's thickness too.

— strength and rate of change of external fields.

Still the canceling response is pretty fast.

— e.g., a car hit by ~~lightn~~ lightning — the E-field doesn't ~~ever~~ penetrate and current flows thru the outer body to ground. (Wik)

— e.g., depending on how metallic it is, your cellphone may have no reception in an elevator. (Wik)