32161

11 Fluids

— a chapter highly relevant to life-science folks since bodies are mostly fluid

— Fluid include both liquids + gases

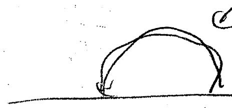
— ~~they~~ both flow — atoms and molecules are NOT rigidly bonded — they slid over each other.

— ideally → a fluid can't resist a shear~~ing~~ force (force tangential to a surface)

→ one ~~that~~ can change shape without changing volume.

No. fluid is quite ideal in this regard, e.g., Water has a surface tension

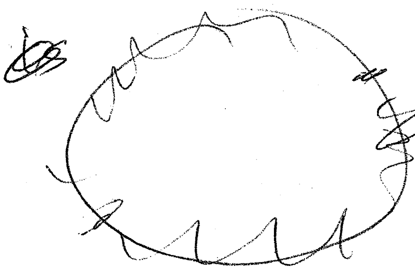
drop



small but can be pulled into a shape

large amounts just spread out.

- in liquids the atoms or molecules "touch" → they have no hard edge, but there is region of strong interaction

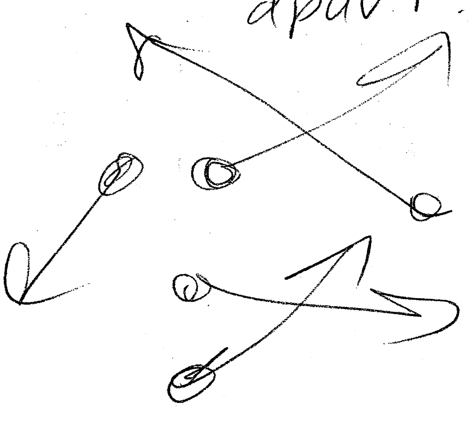


liquid ~~bonds~~ always changing, but strong enough to prevent free expansion

- it takes a lot of force to compress a liquid in everyday world.
- So can be approximated as incompressible for many purposes.

So they don't expand much either. But if the surrounding medium pressure gets too low, they evaporate.

In. Gases → atoms ~~or~~ molecules are free fly & far apart. So gases are compressible → and also freely expand when nothing holds them in



(but still can be approximated as incompressible for very approximate results)

321d

11.1 Density or Mass density

Mass per unit volume

Mass density: understood when you say density unqualified.

[particle density is particles per unit volume for example]

$$\rho = \frac{M}{V}$$

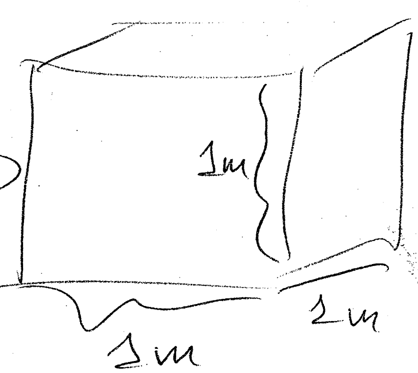
Greek rho $\longrightarrow$ standard physics symbol for density

MKS unit is $\frac{kg}{m^3}$ — No special name or symbol.

liquid water has density

$$1000\ kg/m^3$$

$1000kg \longrightarrow$



Many people find this too small a unit $\longrightarrow$ gives values too big

They prefer the CGS unit g/cm^3

Conversion

factors of unity.

$$1\frac{kg}{m^3} = 1\frac{kg}{m^3} \times \left(\frac{1000\,g}{1\,kg}\right)\frac{1}{\left(\frac{100\,cm}{1\,m}\right)^3}$$

$$= 10^{-3}\frac{g}{cm^3}$$

$$\therefore \quad 1 = 10^{-3}\left(\frac{\frac{g}{cm^3}}{\frac{kg}{m^3}}\right) \quad \text{fraction of unity}$$

$$= \left(\frac{10^{-3}\,g/cm^3}{kg/m^3}\right)$$

So water has density

$1\frac{g}{cm^3}$. ← actually varies a bit with temperature & pressure

Table 11.2 on p. 322 gives some common densities

- they depend on temperature & pressure actually.
- weakly for solids & liquids
- strongly for gases.

321f

Question

What's the density of a human?

a) About that of water

b) About that of steel

c) About that of air

(Wiki: body water)

We marginally float or sink as we'll discuss later that → specific it's water — we are 60% male, 55% female

10.2 Pressure

implies $P_{\text{human}} \approx P_{\text{water}}$

leg mass

- a scalar quantity (a our level) that measures the pushy-outyness of matter. (resistance to compression — but that sounds so inert.)

- gases exert it ~~through~~ collisions — microscopic

↳ and they freely expand to exert pressure.

— but in gases it's sort of explosiveness

322b

Liquids & solids only when compressed

— they do ~~exert~~ it because their atoms which are touching don't like to be squeezed

In vaccuum liquids evaporate — usually — maybe not always — some exception exsit.

— Just in vacuum a solid will just sit pressureless.

— The pushy-outiness (resistance to compression) results (explosiveness) in a **force** on any surface in the material — a normal force

Any surface — a solid walls or just an imagined surface with fluid on both sides

A

Area

If surface has area A,

$F = PA$ is magnitude of the force.

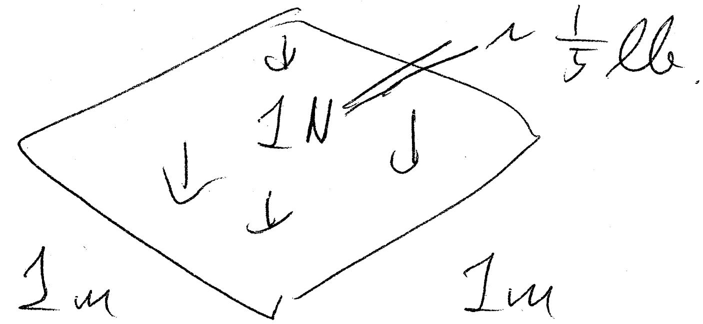
Remember it's pushy-outiness.

P is the pressure

$[P] = 1\,N/m^2$ in MKS units $=$ Pa the Pascal.

which actually is a pretty small unit.

$\boxed{322c}$



Atmospheric pressure at surface of Earth

$$\cong 10^5 \text{ Pa} = 10^2 \text{ KPa}$$

kiloPascals are a reasonable unit. But NIST say STP $P = 101.325$ & $T = 20°$ so two STPs

Other Units

In British system the pressure unit is the psi $\rightarrow$ lb/in^2

P_{air} at surface $\cong 15$ psi

STP = 100 kPa, T = 0°C according to Wiki: Atmosphere (unit) STP

Wiki (air pressure) say mean sealevel is

$$1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa}$$
$$= 101.325 \text{ kPa}$$
$$= 14.696 \text{ psi}$$
$$\cong 15 \text{ psi}$$

$T \cong 20°C$

mean = no single Temperature

Avg Sea level Air Pressure

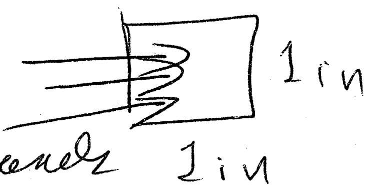
~~1.01325×10^5 Pa = 101.325 kPa = 14.70 psi = 1 atm~~

322d

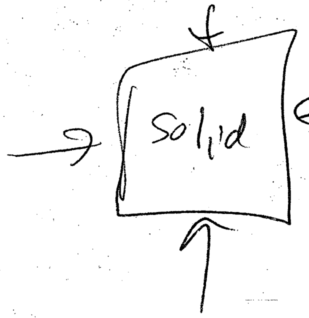
1 atm is atmosphere the unit, not atmosphere air in the spherical shell of gas we live in.

15 psi gives you a visceral feel for pressure.

push on it with 15 pounds of force.



That a lot of pressure force — but usually we don't notice it much because of pressure balance.

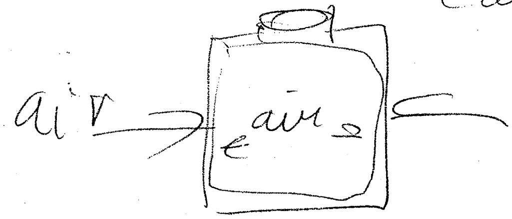


Pressure force on it

— but ~~the~~ the atoms are squished slightly and there's a balance.

322c

can of air



— ~~again~~ a pressure balance

evacuate can and it collapse in — crumples up.

— your body <u>solid</u> and <u>liquid</u> pressure has no trouble resisting air pressure and this is over a wide range from much higher than air pressure to vacuum.

— But the air in your body <u>cavities</u> has a bit more difficulty

— reduced air pressure in airplanes — can cause ear racking — air pocket in ear can expand on takeoff
— contract on landing

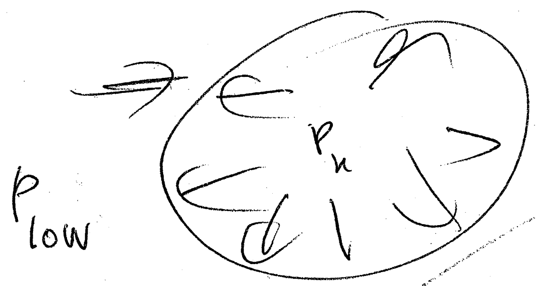
(<u>Cabin pressure</u> is about air pressure at <u>~2700 m</u> (8000 ft)

Wik Cabin Pressure

322A)

— airplanes could keep <u>sea-level</u> pressure if they wanted to, but that's hard on their structure

— shortens lifetime



— too large a pressure difference between inside and out

(Wikipedia 2007 nov 11 cabin pressure)

the plane
— is a bit explosive — so minimize by using cabin pressure reduced from ground level pressure.

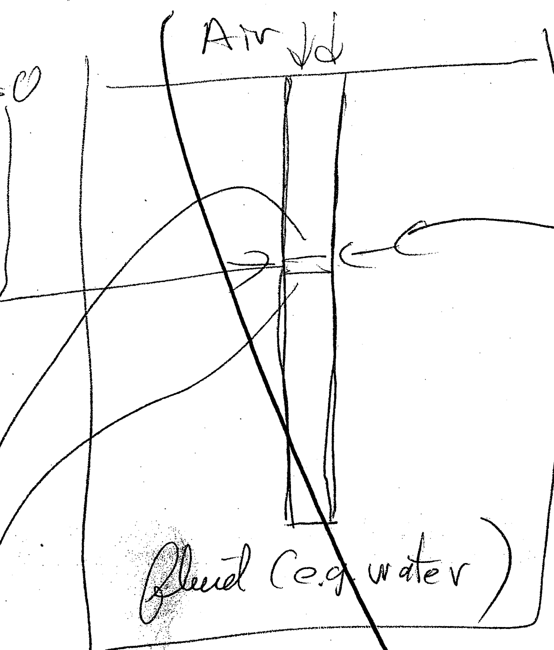
323b

11.3 Pressure at depth in a static medium

Consider an incompressible fluid (at least approximately) first

hydrostatic eq. — stable static equilibrium at all points — any perturbation damps — just take stability as an empirical fact.

Static — nothing is accelerating & nothing moving in our frame of reference



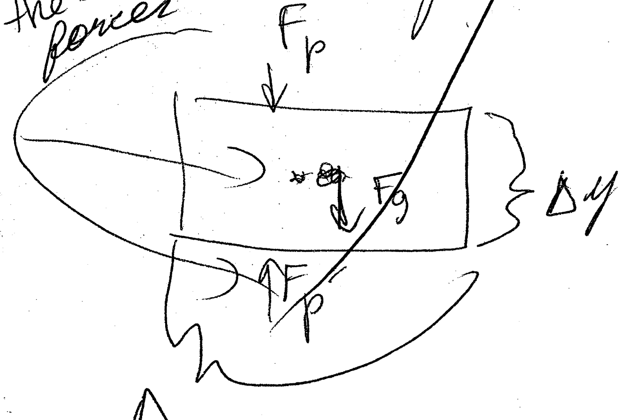
Consider a column of fluid and a slab — our system

In the x-direction $ma_x = \sum_{net} F$ on a slab
only pressure forces

and the pressure forces balance and $a_x = 0$

but vertically we also have gravity

the external forces



$$F_p' - F_p - F_g = ma_y = 0$$

since static

11.3 Pressure at depth in a static ~~Medium~~ Fluid

A key topic relevant to life sciences people

static Fluid

we mean ~~sta~~ every bit of fluid is in static stable equilibrium

stable means small displacement damp out.

almost any persistent static fluid system you see must be stable or you wouldn't see it

But we won't try to mathematically prove stability — Just accept.

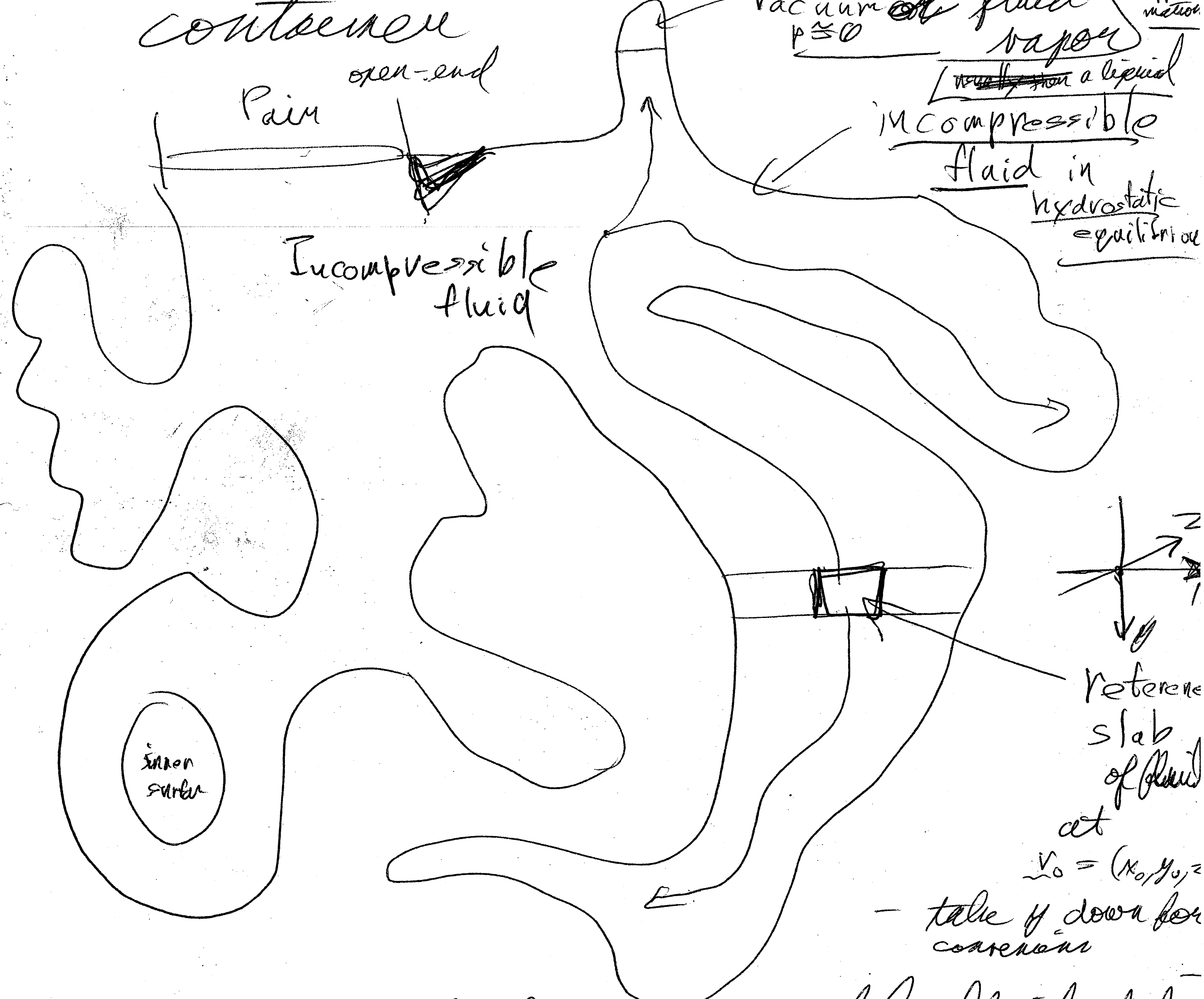
We call this situation (stable) hydrostatic equilibrium

323d) "hydro" means water but the term is generic

We'll now use Newton's 2nd law to find the ~~static~~ hydrostatic pressure formula for incompressible fluids near Earth surface

gas are are never incompressible & liquids only as an approximation

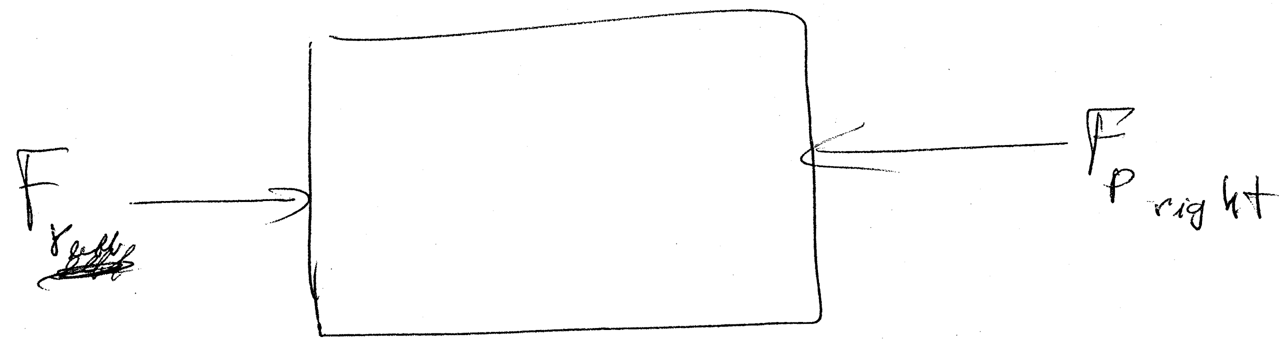
Consider any shaped container



What are the forces on the fluid slab?

external forces

Consider x-direction



In x direction $\sum F_x = m a_x$

but $a_x = 0$ because of equilibrium condition

$$F_{p\,left} - F_{p\,right} = 0$$

$$(P_{left} - P_{right})A = 0$$

$$P_{left} - P_{right} = 0$$

So $P(x) =$ Constant

at least in the branch of the container we are in.

in other branches it must also be constant — in fact, the same constant but that remains to be proven.

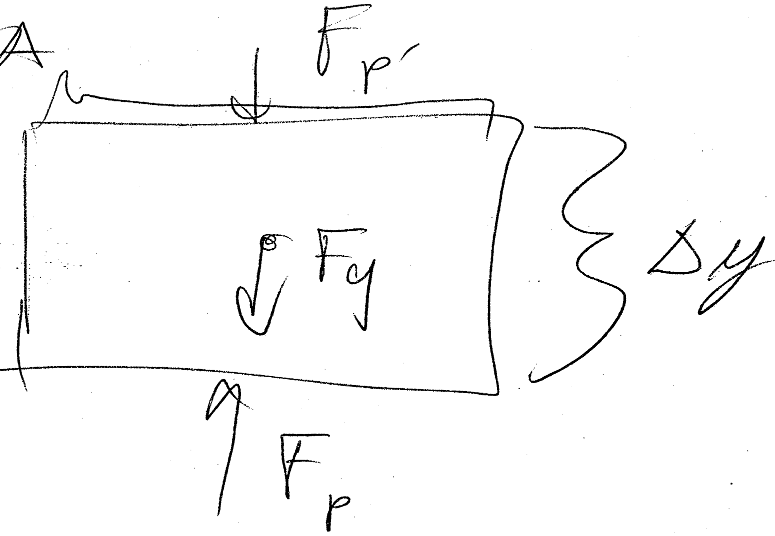
323e

$P(z) =$ Constant

~~too~~ y same argument.

Now what of y-direction

We take y down positive for convenience recall



$$\sum F_y = F_{p'} - F_p + mg = 0$$

$$\therefore \quad F_p - F_{p'} = mg$$

$$(P - P')A = \rho A \Delta y \, g$$

ρ constant since incompressible fluid.

change going down for $\Delta y > 0$

$$\Delta P = \rho g \Delta y$$

So start from our reference slab and go anywhere in short displacement steps $\Delta \underset{\sim}{s}_i$

$$\Delta \underset{\sim}{s}_i = \Delta x_i \hat{x} + \Delta y_i \hat{y} + z_i \hat{z}$$

only a change in y causes a pressure change

$$\Delta P_i = \rho g \Delta y_i$$

So to a general end point y

we have $P(y) = \underbrace{P_0}_{P_0 \text{ of reference slab}} + \underbrace{\sum_i \rho g \Delta y_i}_{\text{all the little step changes}}$

$= P_0 + \rho g \sum_i \Delta y_i$ — independent of x and z coordinates steps.

$$P(y) = P_0 + \rho g (y - y_0)$$

since $\sum_i \Delta y_i = \Delta y_1 + \Delta y_2 + \dots + \Delta y_n$

$= (y_1 - y_0) + (y_2 - y_1) + \dots (y_n - y_{n-1})$

$= y_n - y_0$

323g

So we find
$$P = P_0 + \rho g(y - y_0)$$
in general for an incompressible hydrostatic fluid — no matter what container shape

P depends only on y.

— if gravity turned off ($g = 0$) then $P = P_0$ and pressure is constant

~~If the~~ fluid is compressible, still $P = P(y)$ depends only on y as can be shown no what container shape

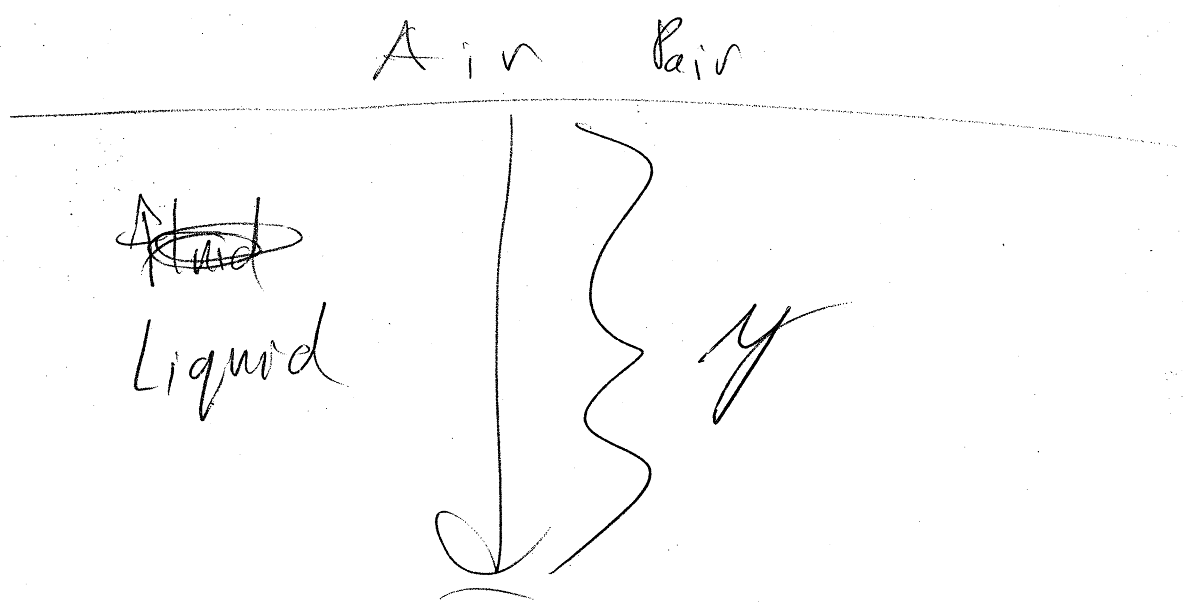
— but our simple formula doesn't apply.

Frequently the reference level is the surface at an air interface

$$\therefore \quad P_0 = P_{air}$$

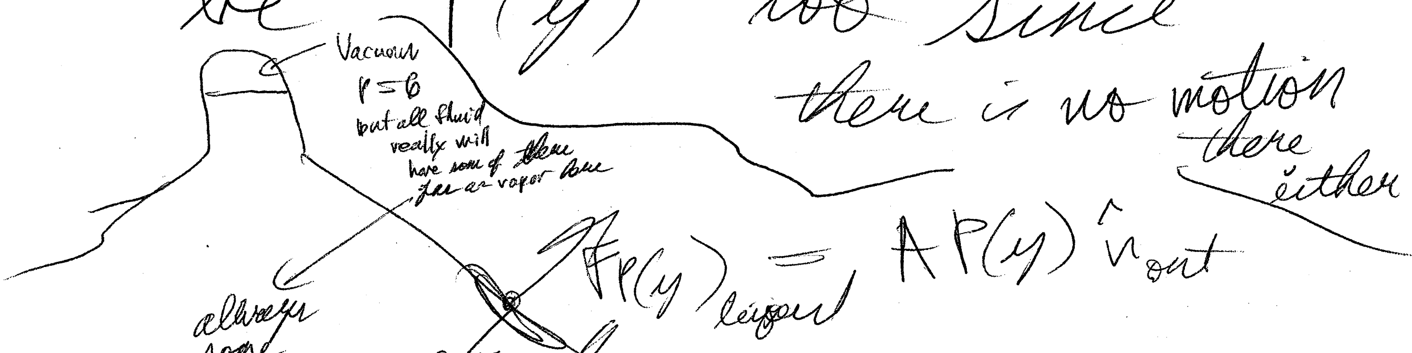
and we set $y_0 = 0$

$$\therefore \quad P = P_{air} + \rho g y$$



323i

Note ~~the~~ pressure at ~~the~~ boundary ~~surfaces~~ must be $P(y)$ ~~too~~ since ~~there is no motion there either~~



Vacuum $P=0$ but all fluid really will have some of ~~these~~ gas or vapor force

always some equilibrium between liquid and gas form of matter

$\vec{F}_{P(y)\,\text{liquid}} = AP(y)\,\hat{n}_{out}$

$\vec{F}_{P(y)\,\text{boundary}}$ ~~AP~~ $= AP(y)\,\hat{n}_{in}$

$F_g = mg$

$F_p' - F_p = mg$ Volume

$\Delta p A = \rho A \Delta y g$

$\Delta p = \rho \Delta y g$

Since incompressible fluid, we find this result for any Δy

$p = \rho g y + p_{air}$

A really simple little formula.

when $y = 0$

$p = p_{air}$

Ex Consider water

$\rho = 10^3 \text{ kg/m}^3$

$g \simeq 10 \text{ m/s}^2, \; p_{air} \simeq 10^5 \text{ Pa}$

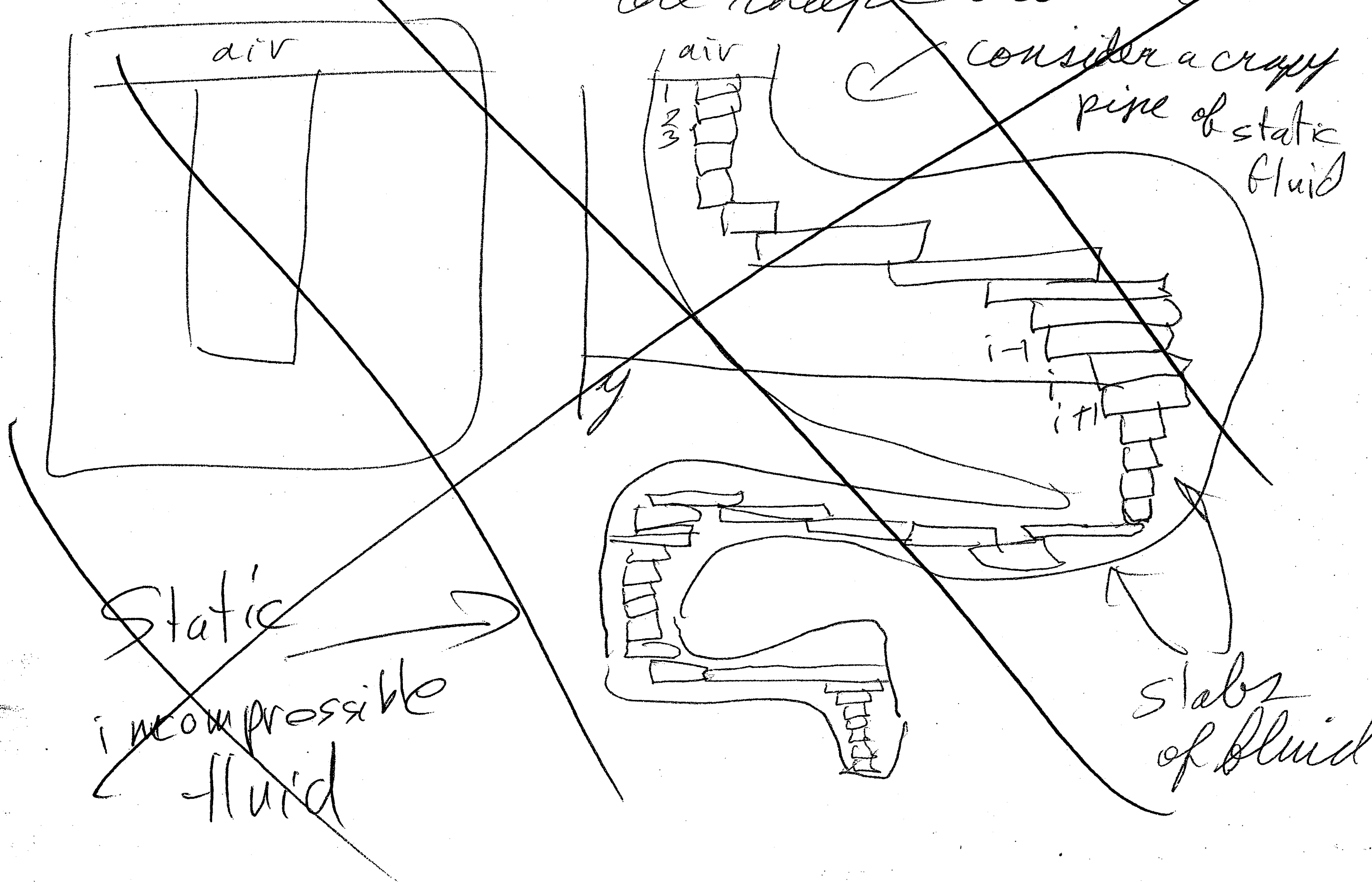
$p = 10^4 y_m + 10^5$ in Pa

323k

$$P \simeq \left(\frac{y_m}{10}\right) + 1 \text{ atm.}$$

— every time you go down 10 m the air pressure increases by 1 atm. — Return to this point in a moment.

~~We just considered a column of fluid, but actually the shape doesn't matter~~



— at every little slabe the horizontal pressure forces balance and cancell. — and the pressure must be ~~constant~~ along every horizontal layer

and for every one

$$\Delta P_i = \rho g \Delta y_i$$

add up from top down

$$\sum_{i=1}^{i} \Delta P_i = \rho g \sum_{i=1}^{i} \Delta y_i$$

$$P - P_{air} = \rho g y$$

$$P = \rho g y + P_{air}$$

It does <u>Not</u> matter that the slabs are displaced.

<u>Compressible fluids</u> — gases, e.g.: air

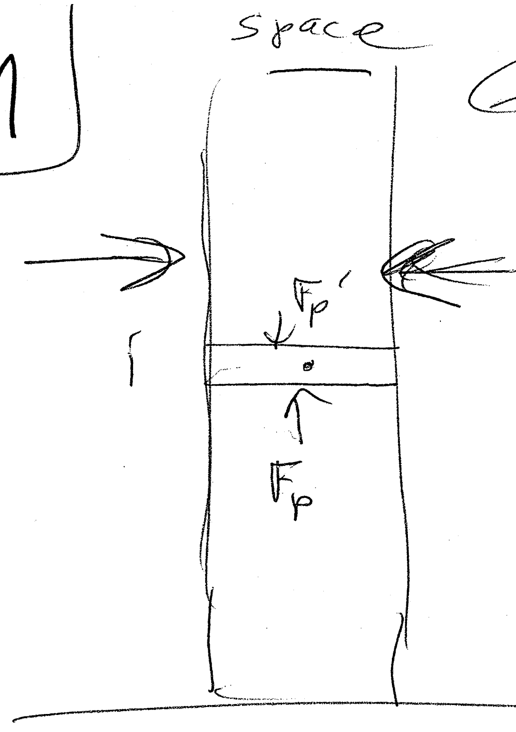
Well ρ is not a constant

322 M

Consider the atmosphere

y

Count y positive up this time



No hard air boundary

in static cases the horizontal forces balance.

ground

$$\Delta P_i = -\rho_i g_i \Delta y_i$$

both density and g are y-dependent.

$$P = -\sum_{l=0}^{i} \rho_l g_l \Delta y_l + P_{ground}$$

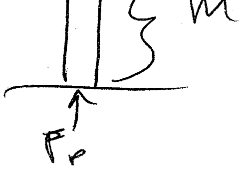
need an integral

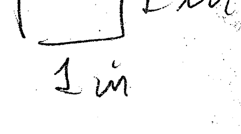
— beyond our scope.

— But one can see as one goes up, pressure decreases

— At ground level $P \cong 15$ psi

— ~~this~~ gives the force needed to support the whole column of air





up to the vacuum of space.

— but for short heights one can approximate air density and g (especially) as constants.

$\rho = 1.21\,\mathrm{kg/m^3}$ (20°C, 1 atm, HRW 323)

$$P \doteq \underbrace{-\rho g y}_{(12\,\frac{Pa}{m})\,y} + \underbrace{P_{ground}}_{10^5\,Pa}$$

$\rho_{air} = 1.2\,\mathrm{kg/m^3}$ at 20°C, 1 atm ⟵ near ground

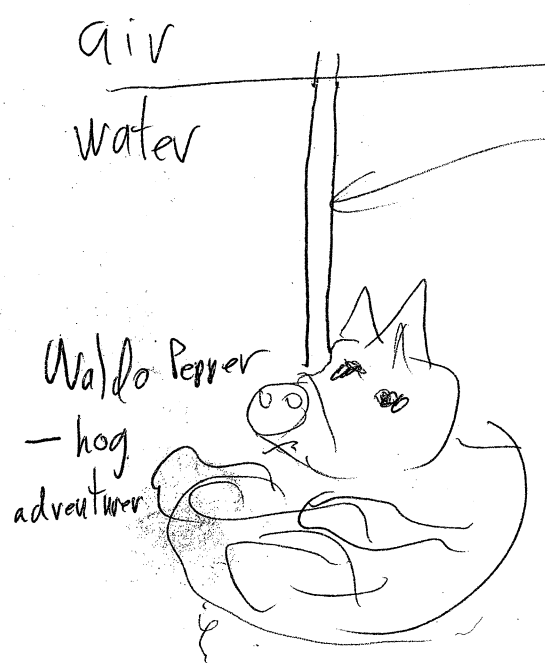
$\rho_{water} = 1000\,\mathrm{kg/m^3}$

Changes of air pressure with Δy are ~ 1000 times smaller than changes in water pressure with the same Δy.

— So actually we often assume air pressure is <u>constant</u> with height for changes of a few meters or even $\lesssim 100$ m, $\Delta y \lessapprox 10^{?}$.

323b

Ex 2 water & snorkeling



air pressure almost ~~constant~~
down the tube
(snorkel) and into
Waldo Pepper

— but

$$P_{water} = \rho g y + P_{air}$$
$$\cong 10^4 y + 10^5 \text{ Pa.}$$

— at 10m down, the outside water pressure is twice the air pressure!!

— you can't expand your lungs to breathe.

— actually 40 cm $\cong$ 15 in is about

the longest tube $\boxed{3\ 2\cancel{3}\text{p}}$
that can be used.
— any deeper and your ~~lungs~~ muscles aren't strong enough to overcome the ~~air~~ water pressure
(Wikipedia 2007 snorkeling)

Question

If the lungs can't expand to overcome water pressure, how is scuba diving possible?

1) It's not (and there is no Jacques Cousteau person).

2) Scuba divers don't breathe

3) They breathe high pressure air.

$\boxed{3\ 23q}$

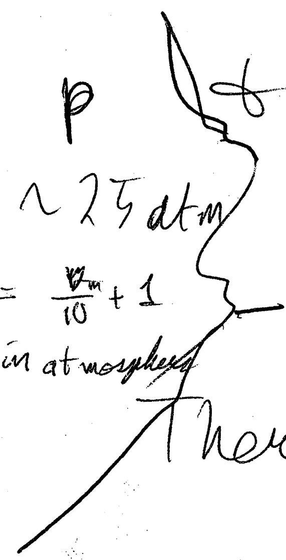
But what about free-diving.

— everyone here has probably gone to the bottom of a pool

p

~25 atm

$p = \frac{d_m}{10} + 1$ in atmospheres



& Dave Mullins of New Zealand holds the free dive record of 244 meters. (Wikipedia 2007 nov 10 freediving)
(on semi-conscious)

There is conscious danger.

— you don't breathe in.

Also mammalian diving reflex

→ includes apnea: suspension of external breathing

— ~~lung~~ muscles aren't used

the body shifts blood ~~a bit~~ to thoracic ~~sustain~~ cavity (whatever that is) to help against lung collapse.

— this all works remarkably well for humans.

— marine mammals are much better.

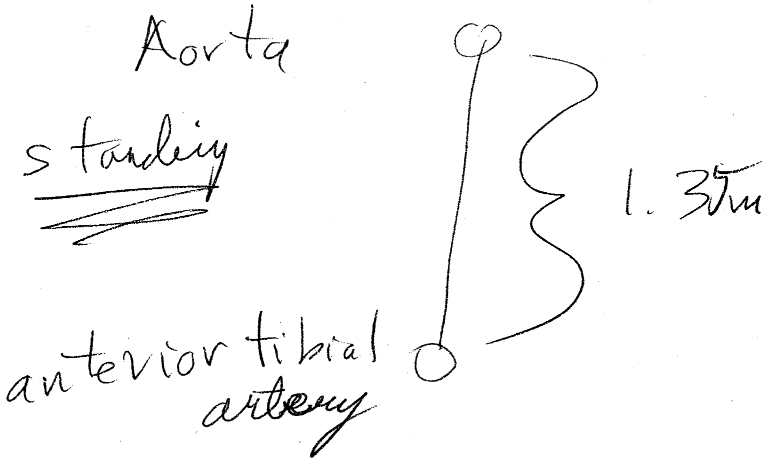
Ex 5 from p. 327

Blood is a fluid much like water in some respects.

— it mostly is water with ~~out~~ some bits of organic stuff.

— Now blood flows always.

— but ~~the~~ lets make the static approximation

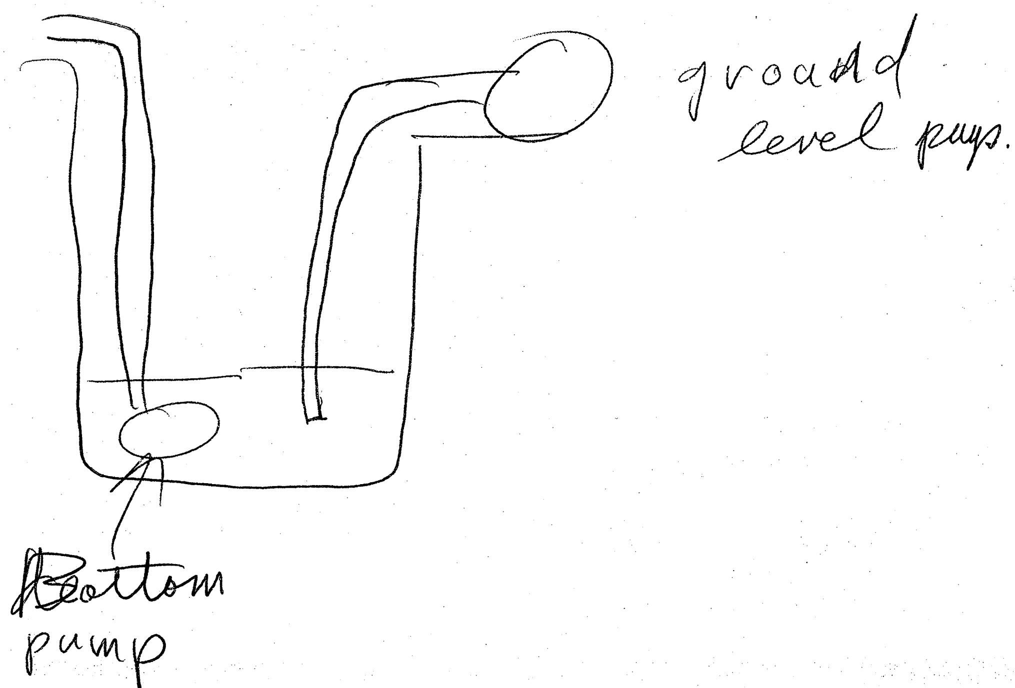


$$\Delta P = \rho g \Delta y$$
$$= 10^3 \cdot 10 \cdot 1.35$$
$$= 1.35 \times 10^4 \text{ Pa}$$

— Pressure of fluids in bodies changes are large, not like P_{air} which varies slowly on human scale.

$\boxed{326b}$

Ex 6 (p. 327) Pumping water

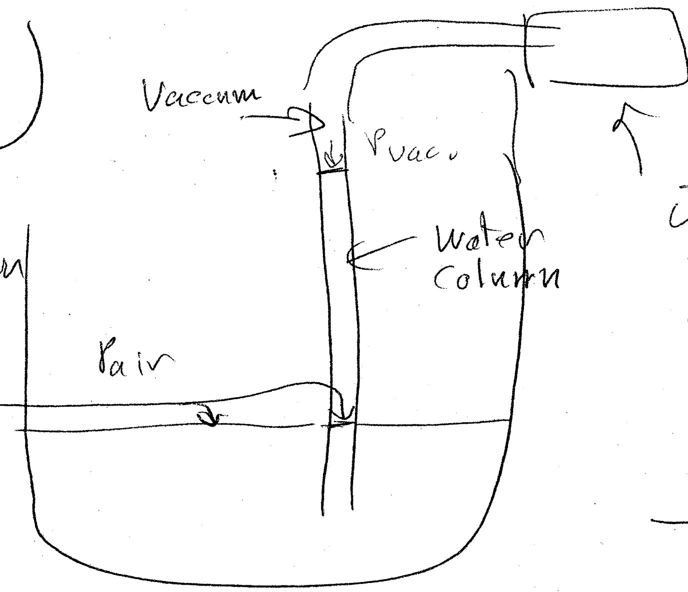


— The Bottom pump can always just push water up if it is strong enough to any heights
↳ Just lifting mass

— But there is a limit to the height a ground level pump can pump water.

3276)

Static situation



P_{air} ~~or~~ there would be motion.

it can only use suction to pull out air.
— but it can only suck out the air that is there. (Actually there must always be some water vapor from the water.)

— If there is no air, there is no more suction.

$$P_{air} \underset{\text{area of tube}}{A} = \underset{\text{of water in tube}}{m} g + \underset{\text{vaccum (vapor)}}{P} A$$

$$P_{air} A = \rho y A g + P_{vac} A$$

$$P_{air} = \rho y g + P_{vapor}$$

$$y = \frac{P_{air}}{\rho g} \simeq \frac{10^5}{10^3 \cdot 10} = 10\ m \simeq 30\ ft$$

You can't suction pump higher than this

Question

Can you drink through a 10 m straw?

Answers

1) Marginally maybe if one really sucks.
2) No way
3) Yes, it's done all the time.
4) I've no idea.
5) Straw crumples

1 & 5 I'd say

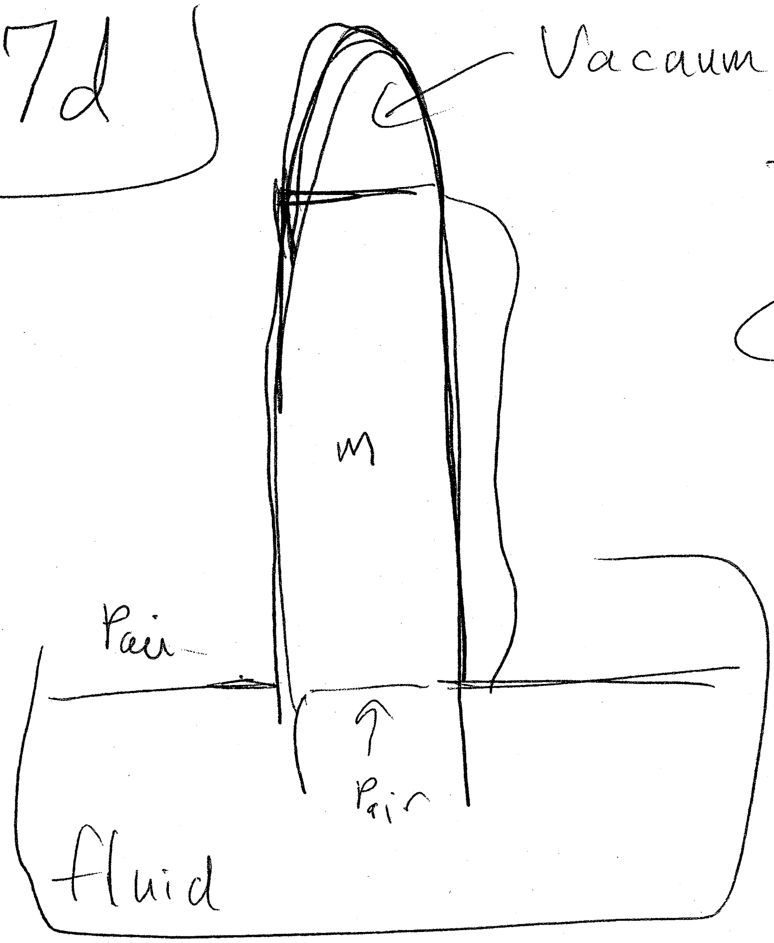
omit?

11.4 Pressure Gauges

The simplest way to measure pressure is a barometer

But Not gauge pressure

327d



Vacuum except for the fluid vapor

↳ always present

— negligible for barometers, but not ultra-high vacuum

$P_{UHV} < 10^{-7}\,\text{Pa}$

$P_{interstellar} \simeq 10^{-11}\,\text{Pa}$

$$P_{air}A = \underbrace{mg}_{\rho A y} + P_{vapor}A$$

$$\therefore\quad P_{air} = \rho g y + \underbrace{P_{vapor}}_{\approx 0}$$

$$P_{air} = \rho g y$$

For any P_{air} fixed: $\rho \uparrow$, $y \downarrow$

So water is not preferred.

— to measure air pressure with water $y \doteq \frac{P_{air}}{\rho g}$

$\doteq \frac{10^5}{10^3 \cdot 10} = 10$ meter

— you'd need a 10 meter barometer. — I think they ~~and pressure changes~~ were played around with

— ~~I believe Early~~

Evangelista Torricelli (1608–1647) (a colleague of Galileo invented the first real barometer using mercury the densest human-environment liquid $\rho_{Me} = 13.6\ g/cm^3$

$\rho_{water} = 1.000\ g/cm^3$

only metal that is a liquid at room temperature — but a bit toxic.

327f

So $P_{air} = \rho_{Me}\, g\, y$

Measure y and calculate P_{air}.

But many people just report y as mm of mercury ~~or~~ (torrs)

$1 \text{ atm} = 760 \text{ torr}$

§11.5

Pascal's Principle

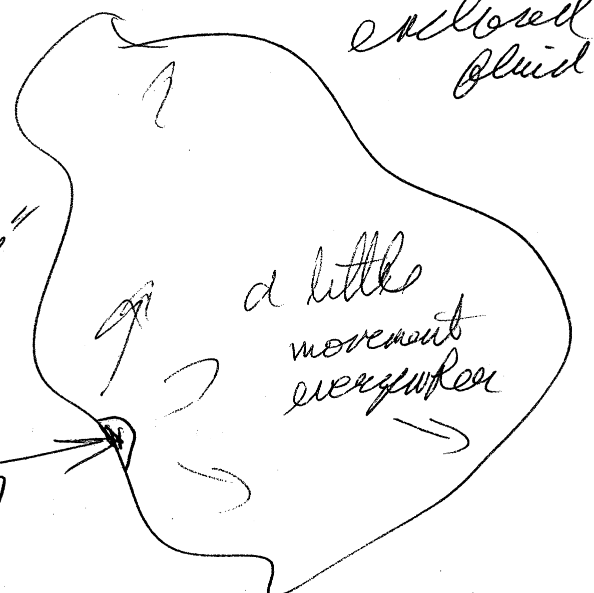
"Any change in the pressure applied to a completely enclosed fluid is transmitted to all parts of the fluid and enclosing walls."

(Not the only or best formulation)

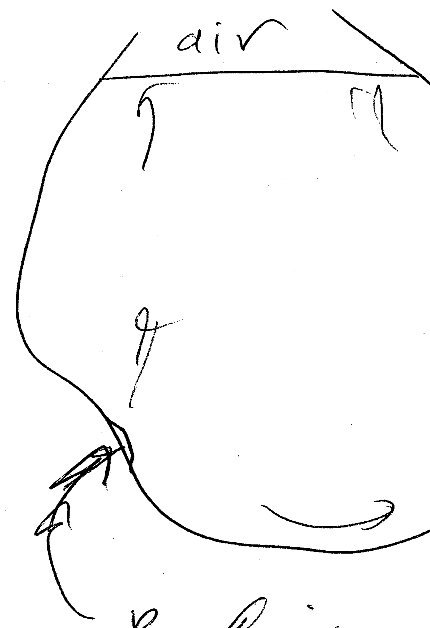
Pascal's Principle

is ~~really~~ not a new law of nature, it follows from the pressure-depth formula really

enclosed fluid



unenclosed fluid



Even "incompressible" fluids compress a little when you push in & decompress when you pull out

$$P(y) = P_0 + \rho g (y - y_0)$$

for incompressible fluid

$$P(y) = P_0 + \sum_i \rho_i g_i \Delta y_i$$

for compressible fluid

— other ways ~~changes~~ P_0 by ΔP and $P(y)$ changes by ΔP

Push in and no change because P_{air} is the boundary condition

$$P = P_{air} + \rho g y$$

— the fluid level just increases a bit.

Question How fast does the pressure change get transmitted?

328d)

1) Instantly — really, really instantly

2) months, years

3) at of order the speed of sound in the fluid.

Here it's 3.

— There is a hydrodynamic ~~of~~ event to change the pressure

— and a little bulk flow ~~throughout~~ — but not fluid parcels moving far.

a compression/rarefaction wave propagates ~~throughout~~ & probably echoes and reflects in a complex damped oscillation pattern — viscosity — fluid friction does this.

But ~~the~~ seed of sound is very high!

3~~28e~~ 328e

$v_{air} = 343\ m/s$
$v_{water} = 1483\ m/s$
1 Atm
20°C
HRW400

~~As these oscillations propagate~~
the ~~change~~ signal propagates quickly
&
and for small enough systems
seems instantaneous.

Really the way it is in most hydraulic systems

Use fluids to transmit forces over distances and around complex twists and turns
+ create mechanical advantage
factor by which an input force is multiplied.

Often better than using gears + chains.

Just 2024dec02 Tuesday a hydraulic machine
||
excavator (Wik ~~tells me~~ — I've never known before.

328 #

Hydraulic systems are really moving flow systems, but pressure change signaling is so fast & bulk flow is so slow that one can often regard them as static systems that change instantaneously if that makes sense.

in a Pascal Principle sense

Bernoulli's ~~principle~~ equation sort of justifies using the static pressure-depth relationship with slow flows

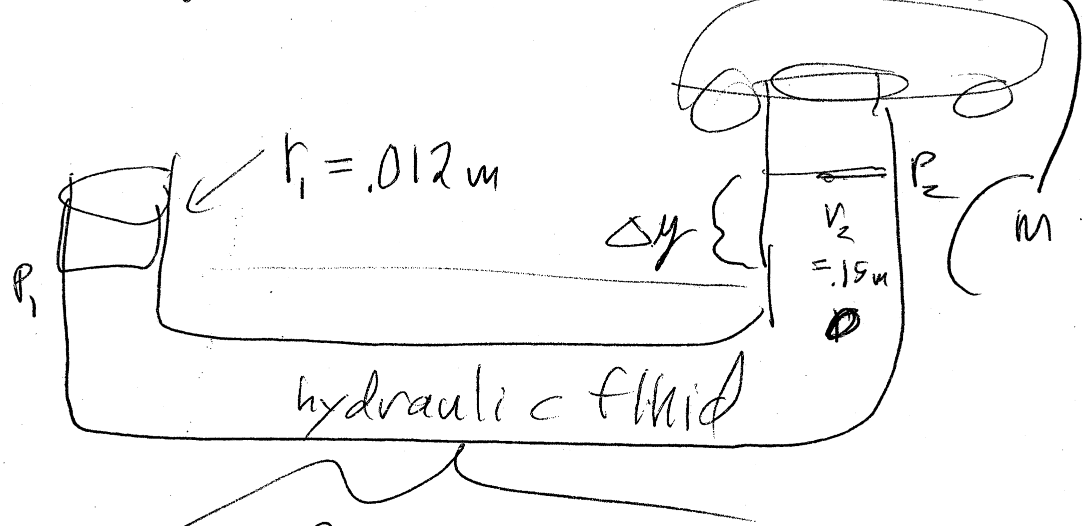
Ex 8 Car lift

→ a hydraulic device to lift cars — one sees them in garages and, in fact, when I was a tike my dad used to let me operate his

Having Father who was a car mechanic, you'd think I'd know something about cars but I don't !!

— up & down doesnt 3286
the cars it was great fun.

Holding up a car



$W = 20500\,N$

$m \simeq \frac{W}{g}$

$= 2000\,kg$

$= 2$ tonne

$= 2\,Mg$

Mega gram as I like to call them.

$\rho = 800\ kg/m^3$

→ some kind of oil.

— why not water?

— probably less corrosion, greater range of fluidity

↳ higher boiling / lower freezing point.

— less dense.

From pressure-depth

$$\underbrace{P_2 = P_1 - \rho g \Delta y}$$

Count Δy as positive up.

We ~~can~~ implicitly subtract off air pressure from both sides

328 h)

— air pressure is a background pressure everywhere

↳ and over a few meters air we've argued virtually a constant.

We solve for F_1

— how much force to lift the car?

$$\frac{F_2}{A_2} = \frac{F_1}{A_1} - \rho g \Delta y$$

$$F_1 = A_1 \left(\frac{F_2}{A_2} + \rho g \Delta y \right)$$

$$= \frac{A_1}{A_2} F_2 + \rho g \Delta y A_1$$

$.8^2 \cong .6$

$$= \left(\frac{.012}{.15}\right)^2 \cdot 20000 + 800 \cdot 10 \cdot 1.1 \cdot a_1$$

$$\cong 120 + 3 \times 10^4 \times 10^{-4} \quad \leftarrow 1.2 \times 10$$

$$\cong 120 + 3 \cong 200\text{ N}$$

Ans
127N

328, i

— to lift the car you must accelerate the whole fluid and give it momentum. — So more force is needed for that and to counter resistive viscosity.

in hydraulic systems small changes in applied force to accelerate/decelerate / overcome viscous resistance must go on all the time.

But even if you use a small force to produce a big one by mechanical advantage — you get nothing for free — energy is conserved.

328i

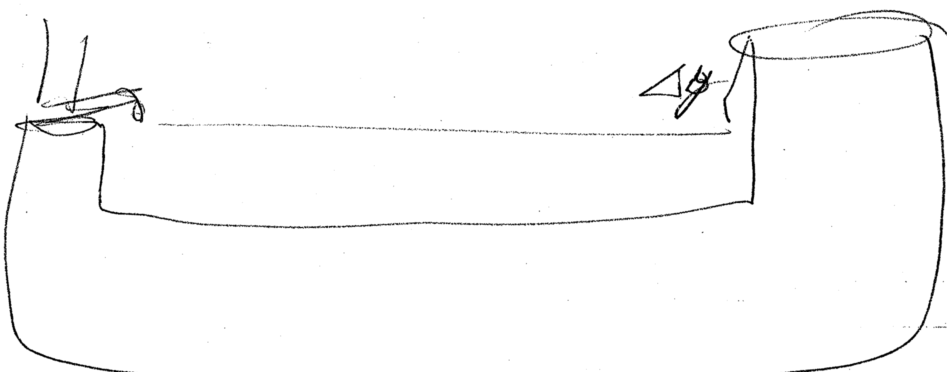
So with 127N ≤ 30lb

You can hold the car with a hand.

you can lift a 20500N car.

Mechanical Advantage is ~ 200 $= \frac{20500}{127}$

How is it?



Well the pressure in the fluid is high everywhere & the walls have the same pressure as has the fluid at each height.

Not just you, but all the walls hold the car up.

— You are just part of the wall.

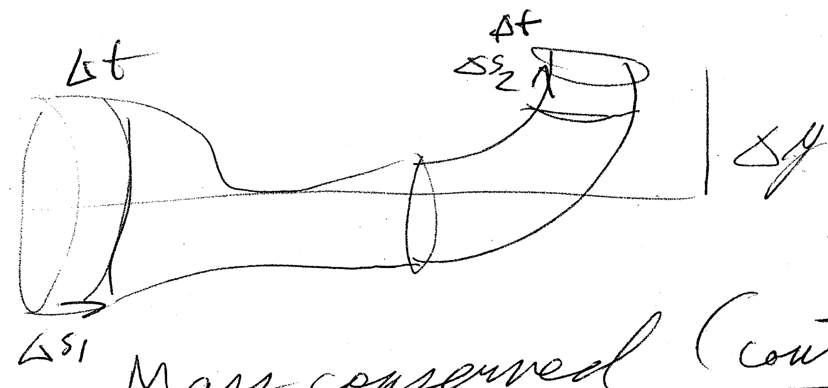
Recall $\quad$ 328K

$$P_2 = P_1 \cancel{+} \rho g \Delta y$$ for incompressible fluid

$$\frac{F_2}{A_2} = \frac{F_1}{A_1} - \rho g \Delta y$$

$$\frac{F_2}{A_2}\Delta s_1 = \frac{F_1 \Delta s_1}{A_1} - \rho g \Delta y \Delta s_1$$

$$F_2 \frac{\Delta s_1 A_1}{A_2} = F_1 \Delta s_1 - \rho g \Delta y \Delta s_1 A_1$$



Mass conserved (continuity equation)

mass in = mass out in Δt $\Big\}$ $$\rho_1 \Delta s_1 A_1 = \rho_2 \Delta s_2 A_2 = \rho \Delta V$$

$\rho_1 = \rho_2$ for an incompressible fluid.

$$\frac{\Delta s_1 A_1}{A_2} = \Delta s_2$$

$$W_2 = W_1 - \underbrace{\rho \Delta V}_{m \text{ of fluid lifted}} g \Delta y$$

3282

So ~~W~~ W_1 goes into W_2
and some loss/gain
do to changing gravitational
PE.

— This is a static approximation
where KE terms are
considered negligible

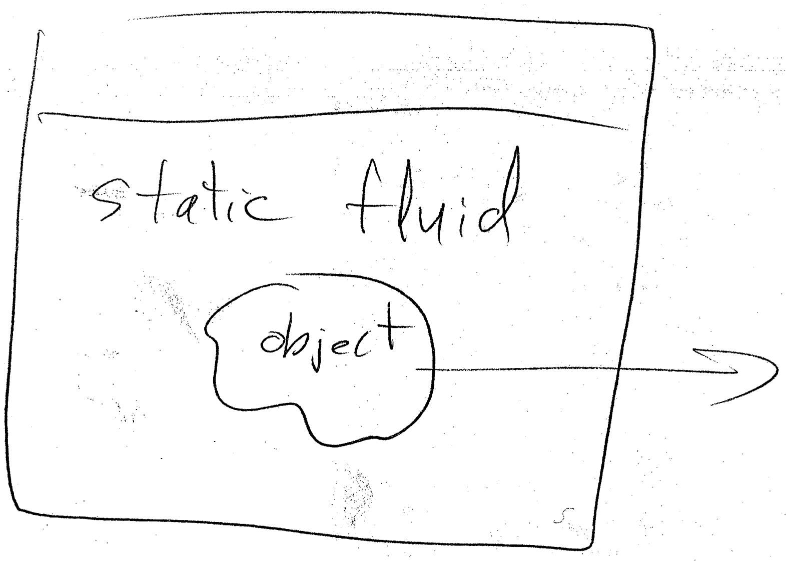
331b | 11.6

Archimedes's Principle

If Pascal has his principle — well Archimedes has a principle (287–212 BCE) too. — & earlier.

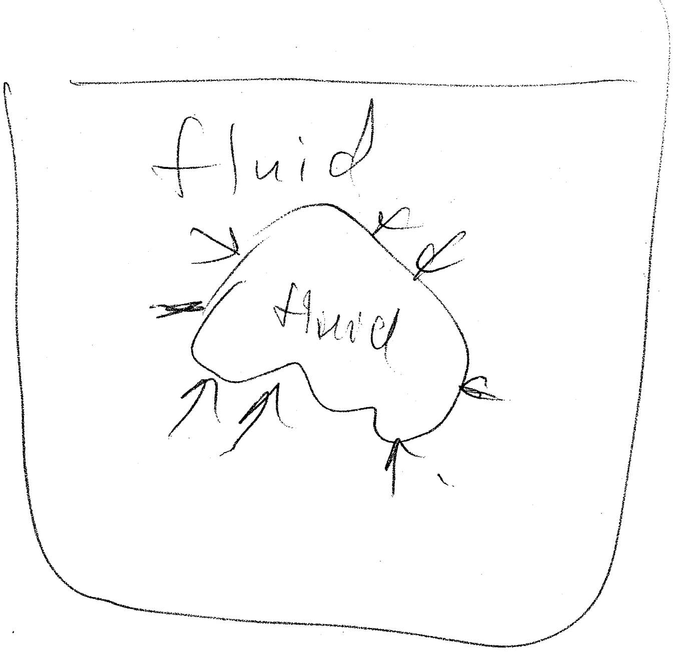
— it's pretty easy to understand — and explain why things float.

Prove actually using — what else — Newton's 2nd law.



pull out the object and replace with the fluid.

Static fluid



Well the "fluid" object replacement just sits there.

Remember $F = ma$ is always true and its true component by component.

in y direction

$$ma_y = 0 = \sum F$$

$$= F_b - m_f g$$

The net force of pressure in the y-direction. — buoyant force

Complicated to calculate in a vector addition sense.

331d)

General Formula - all other special cases refer back to this only near Earth's surface where g constant of course.

fully immersed object,

but $F_B = m_f g$

This is Archimedes's principle as a formula

In words:

"The buoyant force magnitude equals ~~is~~ magnitude of the weight of the fluid displaced!"

Now if we put our original object back in the fluid, the gravity force would be different, but the buoyant force just set by the surrounding pressure forces would not.

$$\Delta P \equiv P_I - P_O$$

$$- P_I)V = \Delta(\tfrac{1}{2}mv^2 + mgy)$$

$$\Delta P = \Delta(\tfrac{1}{2}\rho v^2 + \rho g y)$$

$$\Delta(P + \tfrac{1}{2}\rho v^2 + \rho g y)$$

So sort of like a conservation of energy eqn.

$$+ \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

An energy per unit volume along a streamline is constant

Bernoulli's equation.

— it really applies along a streamline

for inviscid, incompressible ~~fluid~~ fluid.

— but we can use it approximately in other contexts.

~~$v = 0$ or~~

~~$P = -\rho g y + \text{Constant}$~~

~~Our old result pressure decreases with altitude.~~

of density ρ

$\rho V = m$

ime

~~and~~ narrow one so Pressure is constant over each

sure

it cross-sectional area.

rgy

mgy)

sure

rough

eory

ricto

ow

line

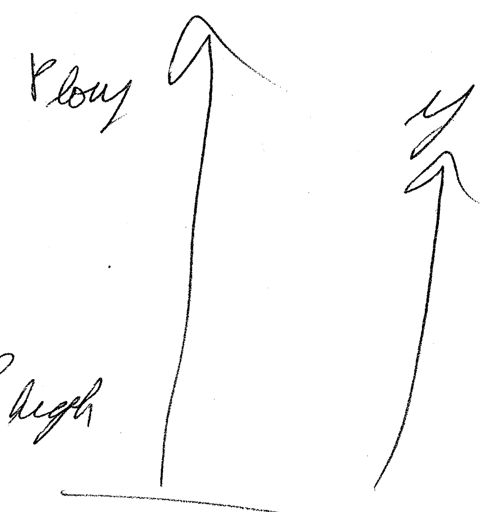
Note if $v = 0$

or a constant.

$$P = -\rho g y + \text{Constant}.$$

This is our old static result recovered. ($P - P_0 = \pm \rho g (y - y_0)$
+ for y increasing down, − negative for y increasing up.

Pressure decreases with altitude



Sort of odd ~~that~~ we've recovered it from a moving fluid derivation

— Still ponder the deep significance.

~~[illegible]~~ If no velocity no speed up and pressure work just goes into ~~[illegible]~~ grav PE. But this also means there is equilibrium everywhere if not static

— I guess it means if there is no motion, there ~~is no~~ must be hydrostatic equilibrium or there would be motion

340b

11.10

Applications of Bernoulli's equation

— ~~both~~ quantitative and qualitative.

Let's specialize for a moment to a $\Delta y = 0$ case.

Then $P + \frac{1}{2}\rho v^2 = \text{Constant.}$

This makes clear an interesting effect

if $v \uparrow$ $\quad P \downarrow$

A way to i~~nterp~~ interpret this ~~for~~ incompressible is ~~that random~~

341a

energy changes. According to the work–energy theorem, the work equals the change in the total mechanical energy:

$$W_{\text{nc}} = E_1 - E_2 = \underbrace{\left(\tfrac{1}{2}mv_1^2 + mgy_1\right)}_{\substack{\text{Total mechanical} \\ \text{energy in region 1}}} - \underbrace{\left(\tfrac{1}{2}mv_2^2 + mgy_2\right)}_{\substack{\text{Total mechanical} \\ \text{energy in region 2}}} \tag{6.8}$$

Figure 11.31*b* helps us understand how the work W_{nc} arises. On the top surface of the fluid element, the surrounding fluid exerts a pressure P. This pressure gives rise to a force of magnitude $F = PA$, where A is the cross-sectional area. On the bottom surface, the surrounding fluid exerts a slightly greater pressure, $P + \Delta P$, where ΔP is the pressure difference between the ends of the element. As a result, the force on the bottom surface has a magnitude of $F + \Delta F = (P + \Delta P)A$. The magnitude of the *net* force pushing the fluid element up the pipe is $\Delta F = (\Delta P)A$. When the fluid element moves through its own length s, the work done is the product of the magnitude of the net force and the distance: Work $= (\Delta F)s = (\Delta P)As$. The quantity As is the volume V of the element, so the work is $(\Delta P)V$. The total work done on the fluid element in moving it from region 2 to region 1 is the sum of the small increments of work $(\Delta P)V$ done as the element moves along the pipe. This sum amounts to $W_{\text{nc}} = (P_2 - P_1)V$, where $P_2 - P_1$ is the pressure difference between the two regions. With this expression for W_{nc}, the work–energy theorem becomes

$$W_{\text{nc}} = (P_2 - P_1)V = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$

By dividing both sides of this result by the volume V, recognizing that m/V is the density ρ of the fluid, and rearranging terms, we obtain Bernoulli's equation.

BERNOULLI'S EQUATION

In the steady flow of a nonviscous, incompressible fluid of density ρ, the pressure P, the fluid speed v, and the elevation y at any two points (1 and 2) are related by

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \tag{11.11}$$

Since the points 1 and 2 were selected arbitrarily, the term $P + \frac{1}{2}\rho v^2 + \rho g y$ has a constant value at all positions in the flow. For this reason, Bernoulli's equation is sometimes expressed as $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$.

Equation 11.11 can be regarded as an extension of the earlier result that specifies how the pressure varies with depth in a static fluid ($P_2 = P_1 + \rho g h$), the terms $\frac{1}{2}\rho v_1^2$ and $\frac{1}{2}\rho v_2^2$ accounting for the effects of fluid speed. Bernoulli's equation reduces to the result for static fluids when the speed of the fluid is the same everywhere ($v_1 = v_2$), as it is when the cross-sectional area remains constant. Under such conditions, Bernoulli's equation is $P_1 + \rho g y_1 = P_2 + \rho g y_2$. After rearrangement, this result becomes

$$P_2 = P_1 + \rho g(y_1 - y_2) = P_1 + \rho g h$$

which is the result (Equation 11.4) for static fluids.

11.10 APPLICATIONS OF BERNOULLI'S EQUATION

When a moving fluid is contained in a horizontal pipe, all parts of it have the same elevation ($y_1 = y_2$), and Bernoulli's equation simplifies to

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \tag{11.12}$$

Thus, the quantity $P + \frac{1}{2}\rho v^2$ remains constant throughout a horizontal pipe; if v increases, P decreases and vice versa. This is exactly the result that we deduced qualitatively from Newton's second law at the beginning of Section 11.9, and Conceptual Example 14 illustrates it.

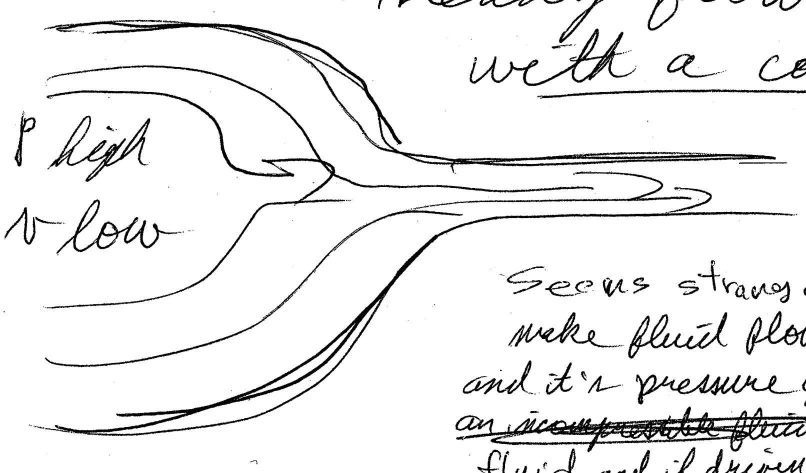
~~parts~~

fluids, is that the some of the energy stored in the compression of the atoms gets converted to bulk macroscopic KE

Our Bernoulli equation applies approximately.

— for compressible gases, ~~the~~ some of the random KE of the particles gets converted to bulk macroscopic KE

Consider steady flow in a pipe with a constriction.



Seems strange that you ~~squeeze~~ make fluid flow into a constriction and it's pressure goes down. But ~~that it is~~ ~~an incompressible fluid~~ it is an incompressible fluid and if driven thru a constriction in steady flow it "decompresses" by converting pressure energy to bulk kinetic energy.

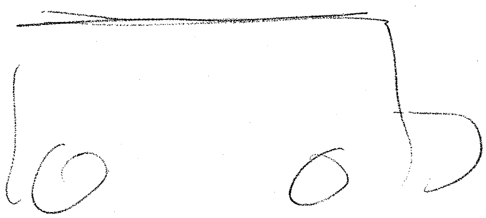
This interesting result can be used to explain a lot effects qualitatively

$p + \frac{1}{2}\rho v^2 + \rho g y =$ Constant

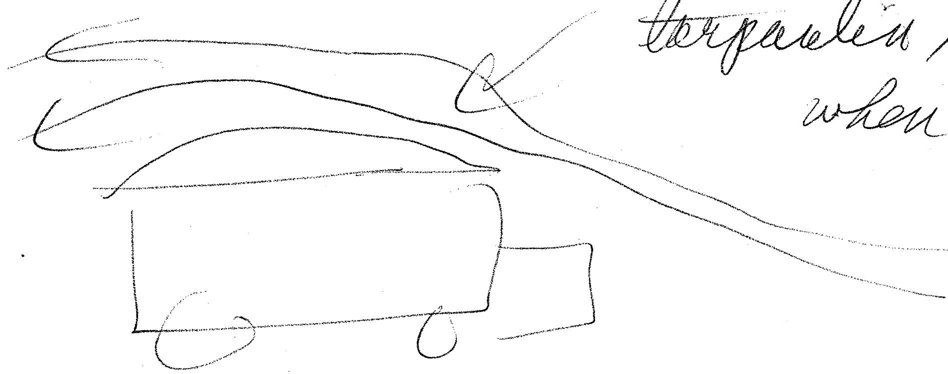
for an incompressible inviscid fluid, but it applies approximately to fluids with viscosity & compressible ones like air

Ex 14

flat tarpaulin in truck at rest



tarpaulin bellows up when truck moving — which can also be viewed as truck at rest and air flowing around it.



The truck is a constriction on the flow.

Air must flow faster around it and the flowing air pressure drops.

The air below tarpaulin is a rest (with respect to truck) and has normal pressure.

342b

This effect is actually the Bernoulli lift.

Domo with shift of paper

— blow over it

— the fast moving air from your pursed lips has low pressure.

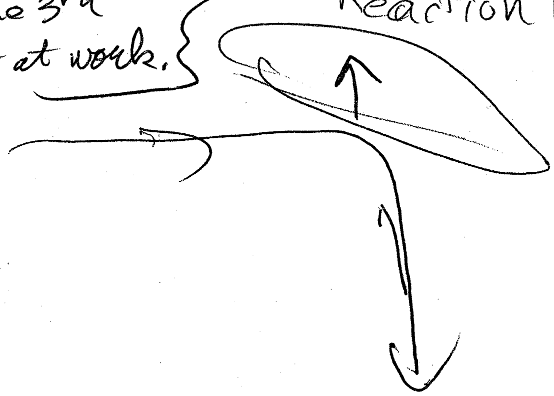
Put it very close below your lips.

Omit if done earlier

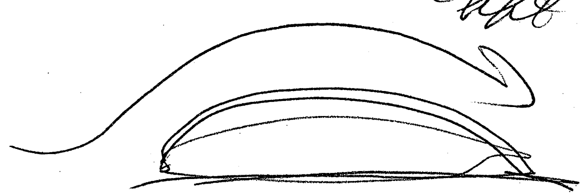
Aerodynamic lift consists of Reaction lift + Bernoulli lift

The 3rd law at work.

Reaction lift



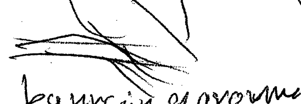
Bernoulli lift



— weaker but stabilizing

Actually stronger usually

— but unstable → think of a speed boat



bouncing around

by reaction lift

Omit if short (out) of time

Then there are baseball pitches

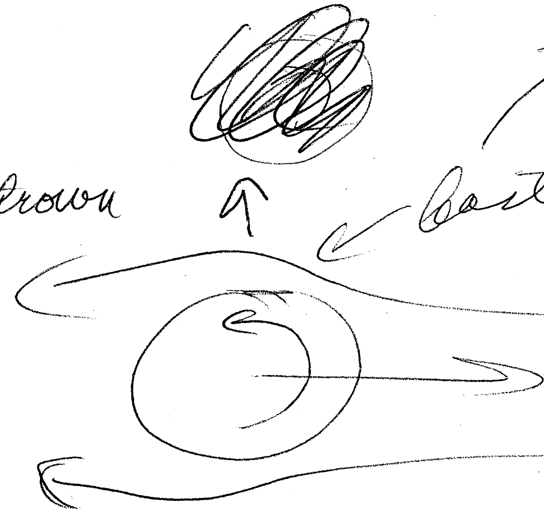
— fastball

— natural spin of thrown ball

lowered pressure

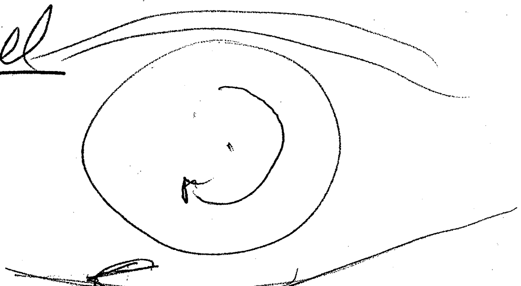
faster air relative to the ball.

Side view



— tends to level the flight and make it less parabolic

— curve ball ~~ball~~



faster air

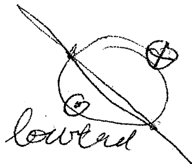
lowered pressure

— the pitch ~~tends~~ to drop

— also some horizontal motion

— away from a right-handed batter by a right handed pitcher

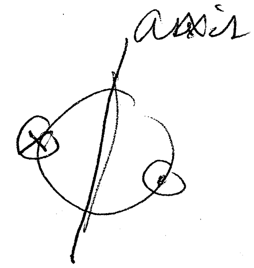
axes of spin tilted toward the plate



34 2d

— so it moves away from a right-handed batter thrown by a righted pitcher.

— tilted the other way



it curves ~~into~~ in toward the batter.

→ a screwball which few pitchers use because it's rather hard on the arm — unless they are really smart about it.

Knuckle ball — thrown with the fingertips

— has low spin and ~~is uns~~ lacks the stabilizing effect of rotation

You don't want to control it too much — unpredictabil is it's value

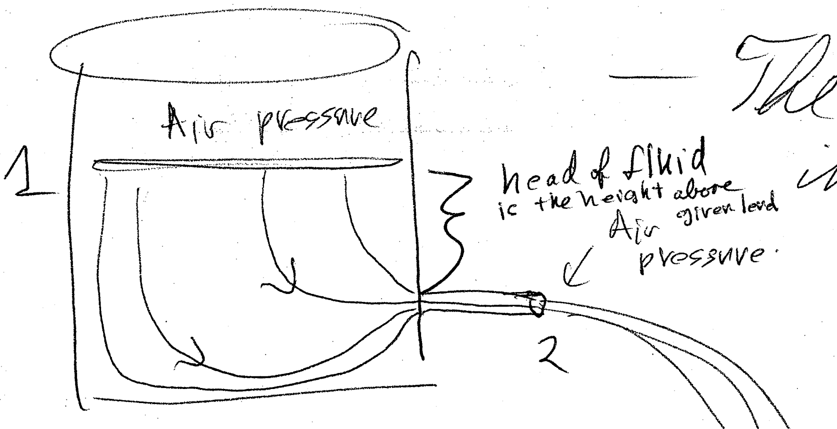
— hard to hit, hard to control and catch.

— thrown slow, so it's easy on the arm.

343b

Ex 16

Water flowing out of a ~~tank~~ (an open tank)



Do hmk problem as an example ?

— The streamlines in general might be quite complex, but they must begin a the surface

Question

Why can't the streamlines begin at a wall.

1) The fluid pressure force keeps any gaps the opening between fluid and wall.) Irrelevant

2) fluid can't flow out of the wall
3) Matter (in this context) can't be created — it is conserved. } Yes both

Well it doesn't matter which streamline we follow for Bernoulli's equation.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

P_{air} in both cases.

Solve for v_2 the efflux speed

$$\frac{1}{2} v_2^2 \rho = \frac{1}{2}\rho v_1^2 + \rho g(y_1 - y_2)$$

$$v_2^2 = v_1^2 + 2g(y_1 - y_2)$$

$$v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)}$$

- a pretty familiar looking formula
- We keep encountering it in different contexts.

But it's ~~interp~~ meaning is often a bit different.
g is NOT the fluid acceleration

If the tank is big and the spout small

$$v_1 << 2g(y_1 - y_2)$$

and $$v_2 = \sqrt{2g(y_1 - y_2)}$$

But we don't have to make this approximation

Recall the equation of continuity for an incompressible fluid.

$$v_1 A_1 = v_2 A_2$$

$$\therefore \quad v_1 = v_2 \frac{A_2}{A_1}$$

$$\therefore \quad v_2^2 = \left(\frac{A_2}{A_1}\right)^2 v_2^2 + 2g(y_1 - y_2)$$

$$v_2 = \sqrt{\frac{2g(y_1 - y_2)}{1 - (A_2/A_1)^2}}$$

a correction factor

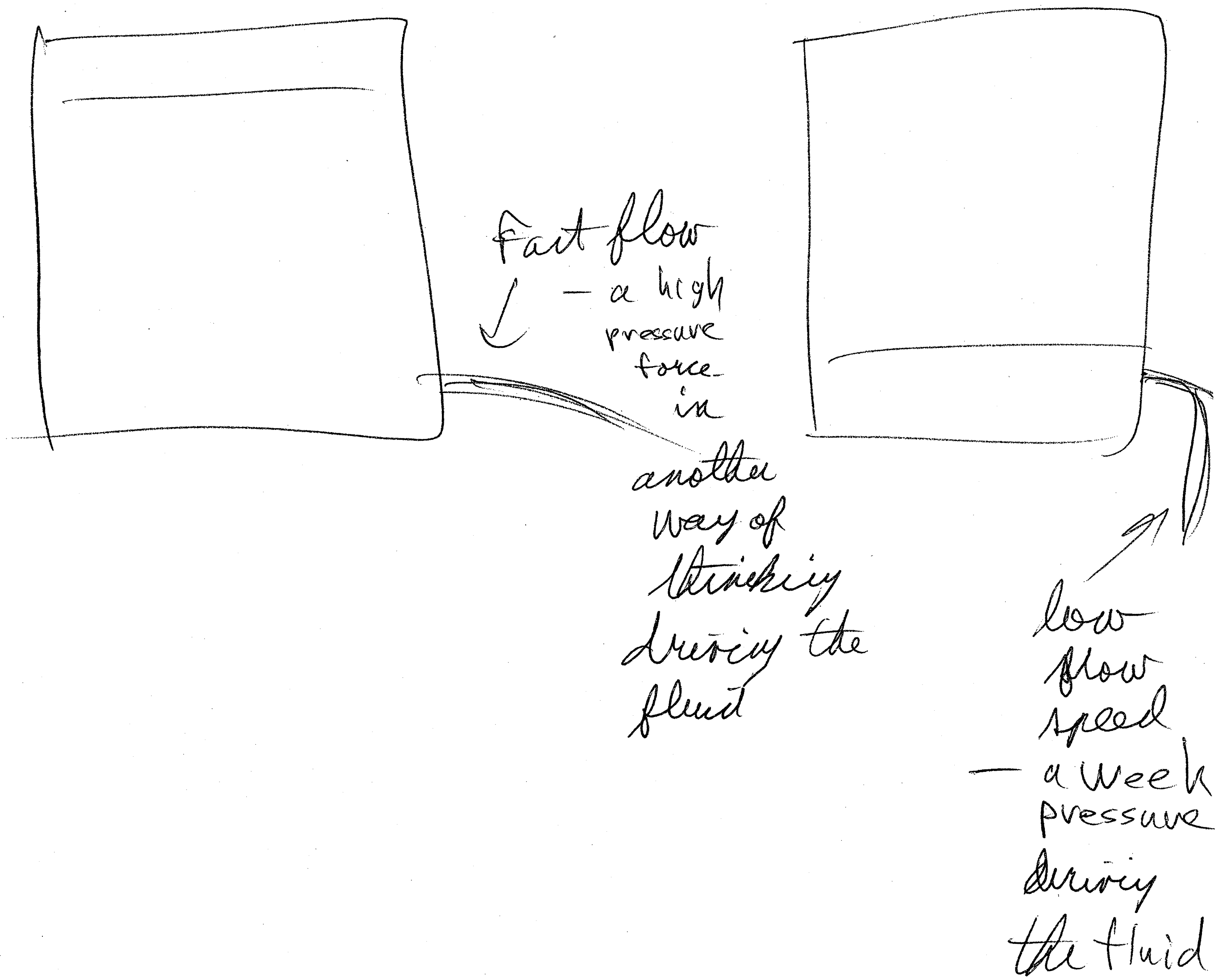
That is small if $A_2 << A_1$

344d

$$v_2 \cong \sqrt{2g(y_1 - y_2)}$$

is recovered

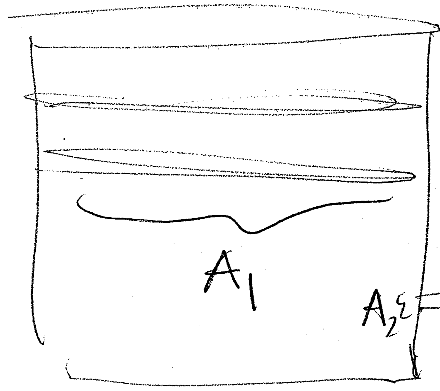
Note v_2 goes to zero as $y_1 \to y_2$



3456

Omit !!

Actually the whole evolution can be solved.



We are assuming quasi-static evolution so that we can assume the Bernoulli equation applies at any time.

$$y_1 = y_{10} - \frac{V(t)}{A_1} \quad \text{← volume flowed out}$$

Bernoulli's equation See p. 346

$$V(t) = A_2 \int_0^t v_2(t)\,dt$$

$$v_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = 2g(y_{10} - y_2) - 2g\frac{A_2}{A_1}\int_0^t v_2(t)\,dt$$

~~Move to other side and~~ Differentiate with respect to time.

$$2\cancel{v_2}\,\dot{v}_2\left(1 - \left(\frac{A_2}{A_1}\right)^2\right) = 0 - 2g\frac{A_2}{A_1}v_2$$

$$\dot{v}_2\left(1 - \left(\frac{A_2}{A_1}\right)^2\right) = -g\frac{A_2}{A_1}$$

$$\dot{v}_2 = -g\,\frac{A_2/A_1}{1-(A_2/A_1)^2} = -g\,\frac{R}{1-R}$$

$$v_2 = -g\frac{R}{1-R^2}t + \text{Constant}$$

$$v_2 = -g\frac{R}{1-R^2}t + \sqrt{\frac{2g(y_{10}-y_2)}{1-(A_2/A_1)^2}}$$

See p. 34

~~We~~ start with flow assumed. No acceleration phase.

But note $v_2 = 0$ when $y_1 = y_2$

$$t_{end} = \frac{1}{g}\frac{1}{R}\sqrt{2g(y_{10}-y_2)(1-R^2)}$$
$$= \sqrt{\frac{2}{g}(y_{10}-y_0)\left(\frac{1}{R^2}-1\right)}$$

3 45c

$$V = A_2\int_0^t v_2\,dt \quad , \quad y_{01}-y_1 = \frac{V}{A_1}$$

$$= A_2\left[-\frac{1}{2}g\frac{R}{1-R^2}t^2 + \sqrt{\frac{2g(y_{10}-y_2)}{1-R^2}}\,t\right]$$

$$V(t_{end}) = A_2\left[\underbrace{-\frac{1}{2}g\frac{R}{1-R^2}\sqrt{\frac{2}{g}(y_{10}-y_2)(1/R^2-1)}}_{-\sqrt{\frac{g}{2}(y_{10}-y_2)\frac{1-R^2}{(1-R^2)^2}}\;=\;-\frac{1}{2}\sqrt{\frac{2g(y_{10}-y_2)}{1-R^2}}} + \sqrt{\frac{2g(y_{10}-y_2)}{1-R^2}}\right]t_{end}$$

$$= A_2\frac{1}{2}\sqrt{\frac{2g(y_{10}-y_2)}{1-R^2}}\sqrt{\frac{2}{g}(y_{01}-y_2)\left(\frac{1}{R^2}-1\right)}$$

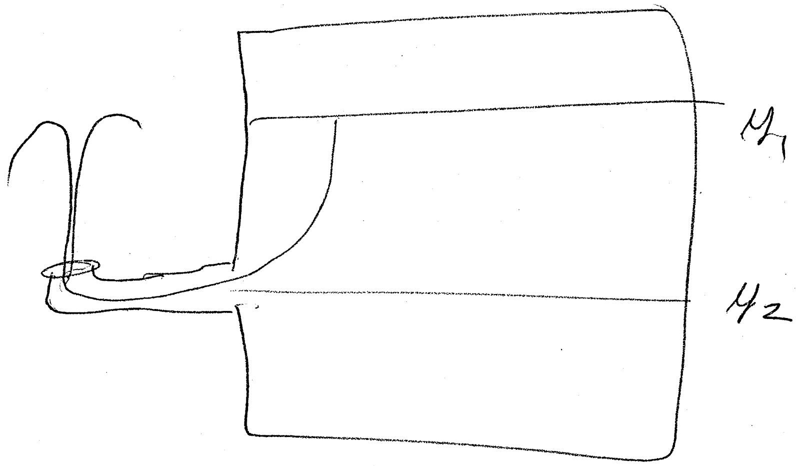
$$= A_2(y_{01}-y_2)\underbrace{\sqrt{\frac{1/R^2-1}{1-R^2}}}_{\frac{1}{R}=\frac{A_1}{A_2}}$$

$$= A_1(y_{01}-y_2)$$

and so all is consistent.

345d

Spout turned up.



$$v_2 \approx \sqrt{2g(y_1 - y_2)}$$

How high does a water particle rise?

— Use ~~conserv~~ work-energy theorem $W_{nc} = \Delta E$ on a bit of fluid mass.

$$W_{nc} = 0, \quad \cancel{m}g(y_f - y_2) = \frac{1}{2}\cancel{m}v_1^2$$

$$y_f - y_2 = \frac{1}{2}\cdot 2g(y_1 - y_2)$$

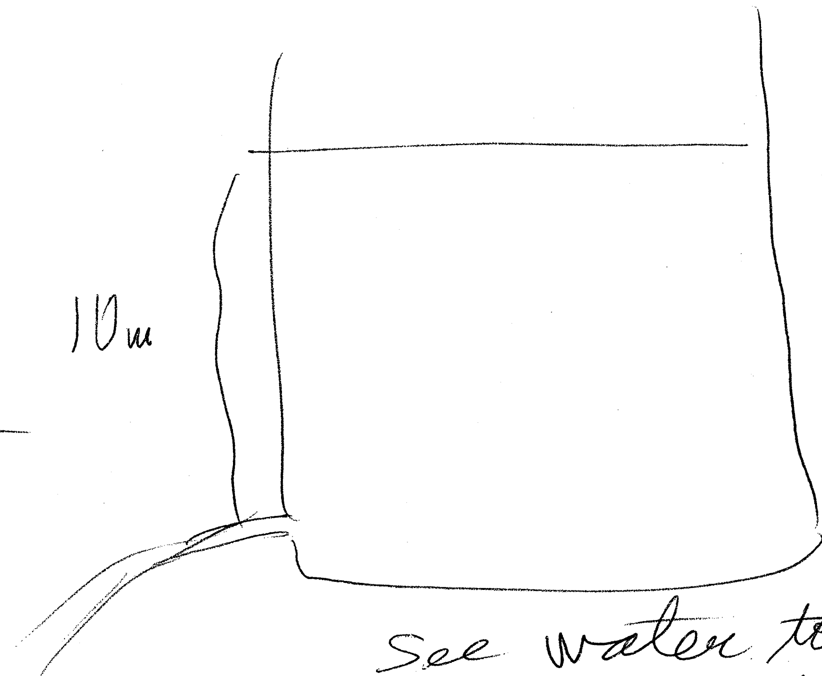
$$y_f = y_1$$

But we've assume **no loss** of energy to **viscosity**.

— and there will be turbulence and breakup into spray.

Ex. Calculation.

$$v_2 = \sqrt{2g(y_1 - y_2)}$$



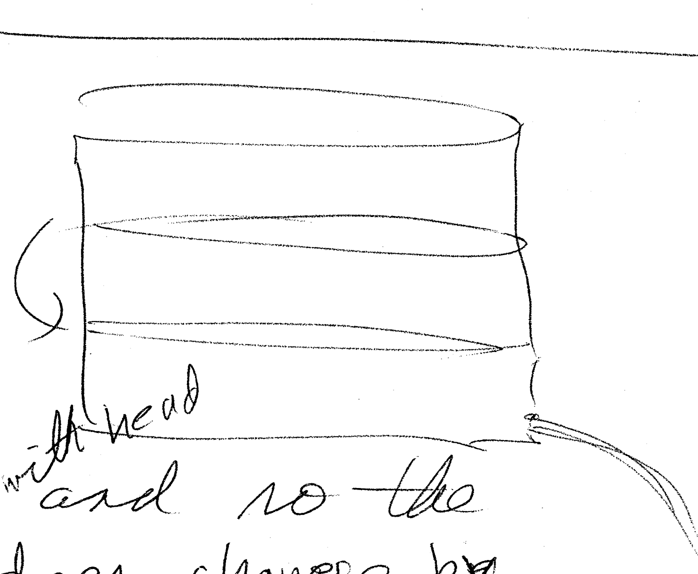
$$v_2 = \sqrt{2 \cdot 10 \cdot 10}$$
$$= \sqrt{200}$$
$$\cong 14\,m/s$$

See water tower on p. 346b

Ex Water Clocks

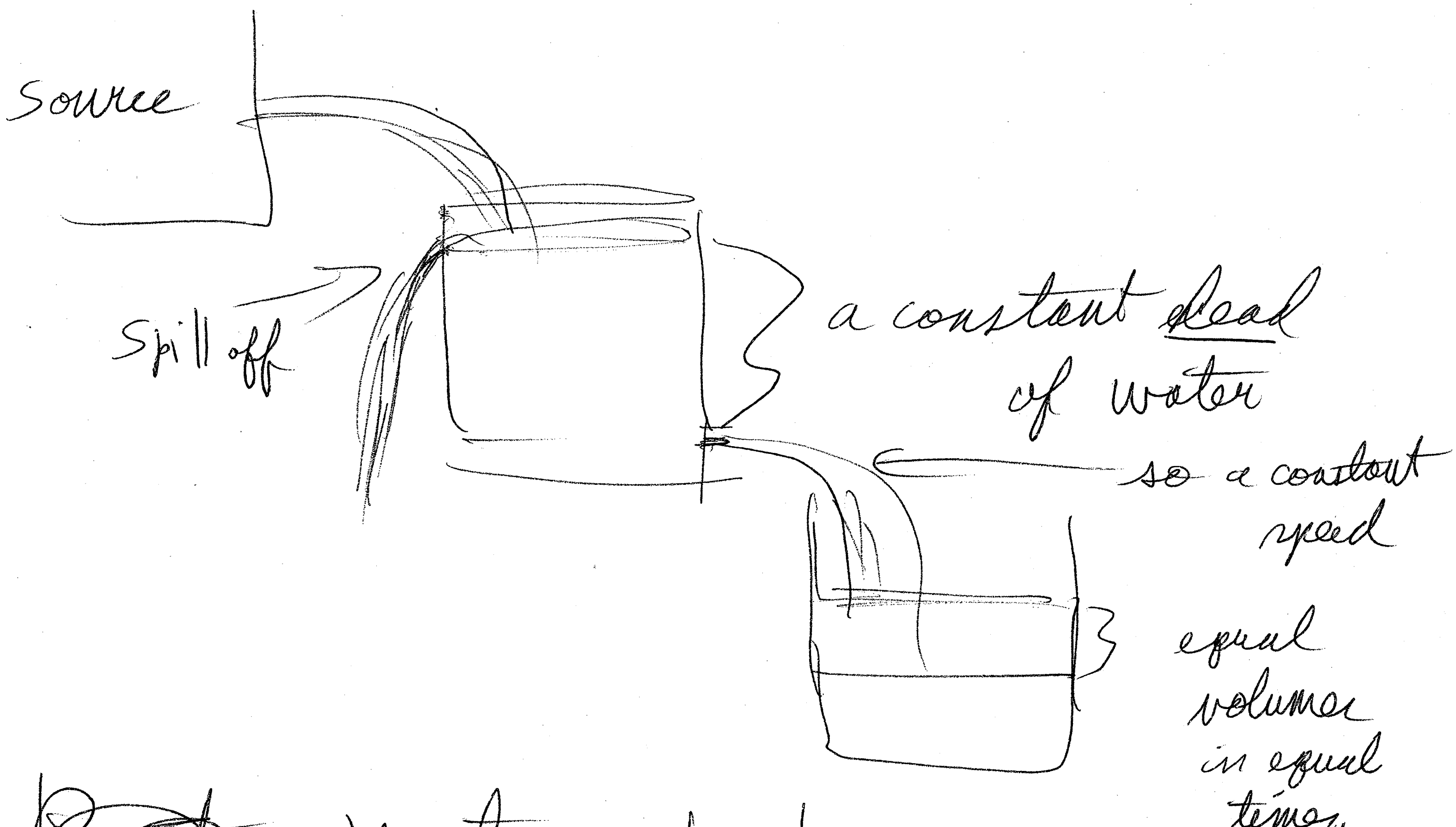
Simplest idea

- measure time by volume fall.



But v_2 varies with head and so ~~the~~ volume change does change b~~y~~ equal amounts in equal time, 34-5f)

Ktesibios (~285 - 222 BC)
invented an improved water clock



~~But~~ Water clocks could be made quite elaborate
— Made to turn dials, mechanisms
— Some ~~elaborate~~ ones were made in Greco-Roman antiquity and in China in Middle Ages.
— Could be made very accurate — but only

up to some point.

- evaporation was a major problem. —

so is viscosity but that shouldn't be a problem for the steady Ktesibian water clock. Is a problem for my Bernoulli equ theory

- So they lost out to mechanical & then electric & electronic clocks

where the flow is faster than otherwise. But water expands a bit when heated above 4°C

- There are still some for show purposes.

11.11 Viscous Flow

- Just a brief word
- the friction of fluids
- turns mechanical energy into heat.

345g)

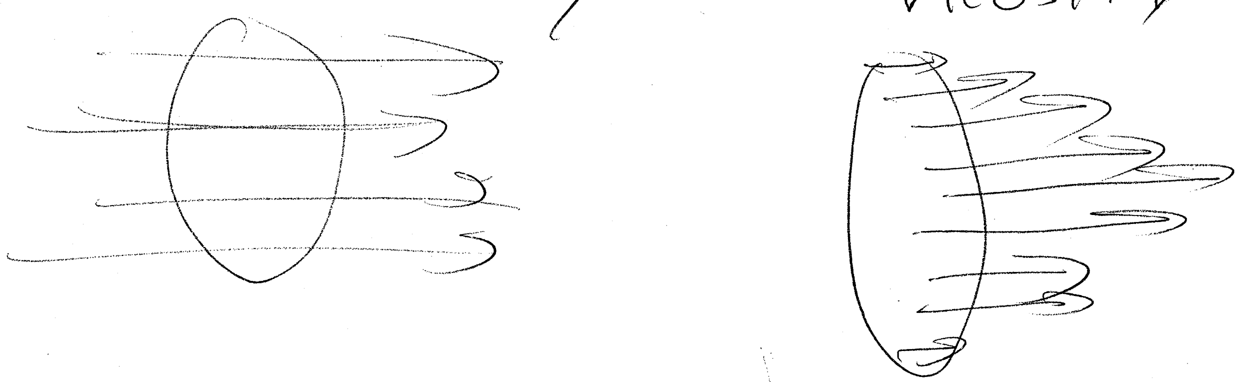
— slows fluid motion

— tends to cause a layering effect.

— slowest near solid walls.

no Viscosity

Vicosity



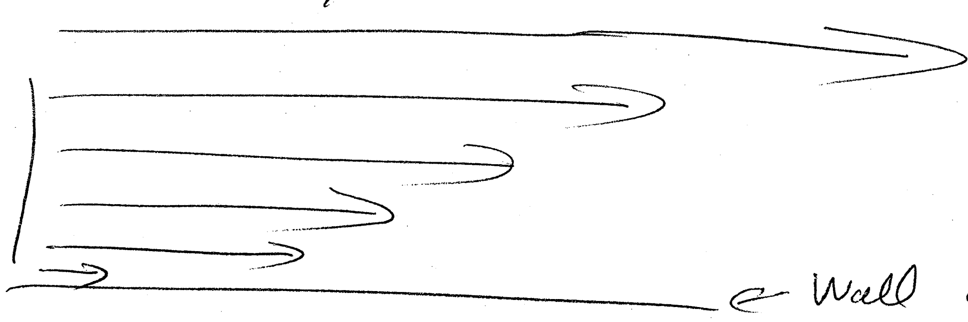
— in rivers water slow near banks

— fastest near the center of the channels.

— ~~Viscosity with~~

2-d

each layer slows the next.



← wall at rest

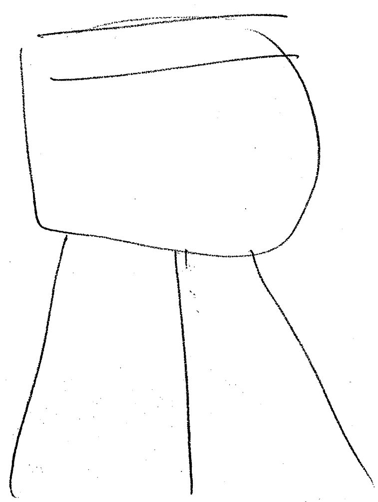
346b

— Of ~~no~~ course, no discreet layer

— a continuous variation in speed.

Omit — stick in earlier, on p. 346e

Water Towers



Up high to keep pressure in our ~~taps~~ taps

— actually need valves ~~to~~ and the like to keep the pressure from being too high & making a taps into jets.