Chapter 1

Introduction & Mathematical Concepts

— I've already given the philosophical intro on the syllabus, so we just jump into the deepend now.

Units

— physics is an exact science
— to understand ~~the~~ the physics of nature we need accurate + precise measurements.

answer not wrong

answer known to many digit places

— so we need accurate standards of measure for the quantities of interest — e.g., length, time, mass

→ we need precisely defined units that are the same/valid for all cases

— For discrete quantities, this is no problem.

- if there are 10 cows in the field, there are 10 cows in the field precisely.

cattle have their own natural unit — the cow.

But for continuous quantities (length, time, etc) nature has not been so kind.

There are no macroscopic objects that are precisely identical.

[In physics jargon everything much bigger than a molecule is macroscopic even microbes are macroscopic → an abuse of language but that's the convention]

- all stones are different
- all snowflakes

- animate things are somewhat better constrained

[also it is used rather loosely - sometimes macro is just bigger than microbes]

and so ~~many~~ some ~~measurements~~ units started out as defined by a human object

- foot
- inch from a thumb width maybe.
- hand as in the height of a horse (which doesn't seem a very convenient unit actually).

— after a long history of units & measures people arrived at having a single ~~well me~~ ultra-carefully maintained object to be the defining object of a unit at least in some key cases.

all other measurement scales are calibrated by it — usually at several removes

— the meter was once ~~the~~ defined as distance between two marks on a platinum iridium bar

& and the kilogram was (& still is though perhaps not for much longer) as a platinum-iridium cylinder

These objects are both kept Sèvres outside of Paris

Why Paris you ask?
Well why not.
— La cité de lumière et l'amour.
— if you were a keeper of weights and measures would you choose Peoria?

actually I believe it was a political deal

- England got Greenwich mean time & the prime meridian
- France got the meter & the kilogram
- America got nothing: our trade negotiators were off the ball in the 1880s or so when these things were decided

But using single objects isn't so great.

- metal expands when heated
- mass changes by accumulation of dust.

the single objects have to be ever so carefully maintained and even then it's hard to be sure they are exactly stable to all the precision we need.

- and what if they are destroyed somehow in war — e.g., WWII?

So what was desired
was to set basic units
by physical "objects"
or things
that we believe to be

exact in ~~the~~ principle.

— So the meter is now
defined as the distance
light in a vacuum travels

in $\dfrac{1}{299\,792\,458}$ seconds.

— In the theory of special relativity
~~the~~ vacuum light speed is invariant.
all observers ~~meas~~ measure the
same speed — which does weird
things to time — but that's a
story ~~for~~ another day.

and we believe that theory is accurate
beyond anything we can measure.

— I should say a great deal of well-established modern physical theory is inextricably linked.

but not all

→ if one part is somehow wrong, then there is something wrong with a lot of it.

→ all of modern experiments and a good deal of modern technology tells us that well-established results ~~are~~ right to any degree and ~~~~ we can presently test it.

and SR is part of that inextricably linked theory.

— So you don't need to go to that bar in Sèvres — (I. bar of metal)

→ anyone with the right equipment can do a fundamental calibration.

→ usually only national bureaus of standards do that kind of thing.

— What of the second.

— it's defined as the time for exactly 9 192 631 770 wave cycles

of emission of particular
kind of a Ce-133 atom.
element isotope

— why? One for some reason
it's easy to deal with in
the lab.

~~H~~istorically the second was

$$\frac{1}{24}\cdot\frac{1}{60}\cdot\frac{1}{60}=\frac{1}{86400}$$

of a solar day.

but length of the day varies
and is slowly increasing with
time.

— It's a macroscopic object and so
its periods ~~cannot~~ (i.e., days)
can't be exactly identical
even in principle.

But Ce-133 is microscopic
& quantum mechanics (the story
of another day) tells us
all unperturbed Ce-133 atoms
are absolutely in principle identical

— this is true of all simple microscopic objects.

→ for example every electron is exactly in principle like every other one.

They are sort of like <u>Platonic forms</u> (ideas)

Omit?
One of the founders of QM Heisenberg was ~~the~~ son of a professor of <u>Greek</u> studies & this may given him some openness to the idea of Platonic forms

Nowadays most basic units are based on natural things we think are absolutely invariant in principle.

<u>Macroscopic mass</u> though is still linked to that kilogram cylinder.

<u>microscopic mass</u> is defined by the mass C-12 atom (ordinary), but ~~hitherto~~ it hasn't been possible to use that for macro measurements, ? but maybe some day ~~soon~~ it will be.

These natural ~~of~~ standards

— the speed of light

— the Ce-133 frequency

— mass of C-12 atom

we believe are unchanging and in principle absolute quantities according to modern ~~the~~ physics theory.

— because of the inextricably connection of modern physics supported by a host of experiments and ~~your cells~~ the fact that your cell phone works.

This Doesn't mean we can measure them perfectly, but people who try can always approach them more & more closely to perfect measurements and anyone ~~at~~ anywhere and at any time in the future.

those people are often — but not only — researchers at National Bureau of Standards.

Basic & Derived Units

There are 7 base units,

those of

length meter

time second

mass kilogram

Mass by the way is not weight although the two are closely connected on the Earth's surface.

— as we'll discuss more fully later mass is resistance to acceleration.

— objects in free-fall are weightless, but a massive object is still harder to get moving than a small one

and 4 others we won't annoy ourselves with now.

~~amt~~ omit

current ampere (Wikipedia)

temperature kelvin

quantity of (almost) elementary objects mole

luminous intensity candela ← why do we need this?

maybe because it's based on model of the human eye.

exactly it.

— the subjective effect of light on human needs a base unit for historical and practical reasons

Système International or metric

These are the SI units ~~that~~ are universally adopted in science as standard → of course, ~~we~~ science uses all other kinds of units when convenient.

[e.g., in astronomy, the mass of the sun = the solar mass is a convenient unit for measuring the mass of stars that range from ~ $\frac{1}{80}$ to ~100 or more solar masses.

This is a physically meaningful unit in this context

— the kilogram is a convenient size for human scale objects.]

Of course, conventional units outside of science & engineering can be other~~wise~~ things.

- e.g., miles, feet, slugs (don't ask) ~~in~~ British units in US.

— like many scientists, I think we should junk British units, but the changeover would be a bit painful

we should at least get rid of Fahrenheit for temperature — everyone knows or can learn Celsius anyway.

from the base units derived units can be obtained.

— speed is the ratio of $\frac{\text{distance}}{\text{time}}$

and so has units of $\frac{\text{length}}{\text{time}}$ or in SI m/s.

— Volume is the product of 3 lengths so has SI units of m^3

— Some derived units have special names

e.g., the joule $= kg \frac{m^2}{s^2}$

the unit of energy which we'll get to

the unit of power (in physical sense) is J/s = watt

Power is energy per unit time.

— a 100 W light light bulb use 100 J every second
↳ mostly ~~going into heat~~ overall ~~rather~~ not going into visible light — only ~5%. (Wikipedia)

We also have multiples of base SI ~~[illegible]~~ units denoted by prefixes
~~[illegible] units~~ ~~(When one just limits to the base units and non practical [illegible] that is MKS.)~~

kilo k (small) 10^3

capitals for increase

not for kg or km

e.g., a kilogram = 10^3 gram
~~oddly~~ for practical or historical reasons we consider the kilogram ~~the base unit rather than~~ the gram.

kilometer = 10^3 m

centi c 10^{-2} ~~cm~~ e.g., centimeter = 10^{-2} m.

small letters for decrease

~~Smil 12-16~~
Smil 2-16

mega — M 10^6 MJ
gasoline has ~ 45 MJ/kg of chemical energy

micro — μ 10^{-6}

to release on combustion | 1-12b

— only a fraction of which goes into kinetic energy

20 to 30% (Wikipedia)

facts at the fingertips, — only a keyword & a click away

Kilogram is ~~not the~~ base unit not the gram due to some foul up in the old days

The set of ~~of one just used~~

~~the~~ base units meter, kilogram, second, and ~~an~~ unprefixed units derived from them, then ~~that~~ is called

MKS for meters, kilograms, seconds.

base set. No prefixes except kilogram.

When one is just ~~doing~~ elaborate ~~calcs~~ calculations when an intuitive feel for sizes is not needed then it is best to use MKS units.

— since then you don't need to worry about conversions,

~~[illegible]~~ and writing the units explicitly

— the units will work to be MKS units if you do the math correctly.

consistent MKS

CGS = centimeters, grams, seconds are a rival consistent set. It is just as good as MKS but is only used by astronomers. It is what I use in my ~~professional~~ [illegible] work

But people don't just stick to MKS.

- they use units that are physically or otherwise meaningful for the quantities they are treated.

- of course, sometimes they use units out of pure obstinancy — there is no other reason for Fahrenheit.

So since we won't stick to MKS for good (and bad) reasons, we need to deal with unit conversions (which are often a great bore).

to release on combustion

11-13

only a fraction of which goes into the kinetic energy of a car

20 or 30%. (wikipedia)

wikipedia is so good to have these facts at the finger-tips literally

Unit Conversions

- sort of a bore, but we have to put up with it.
  - convenient units are not always <u>base units</u> or even <u>SI units</u>

But it's really just algebra with the <u>units</u> being like "unknown" x's that have to be cancelled.

"Soap" Jessica Tate "I never really found out what x was".

e.g. $(10x) \times (13\frac{y}{x})$

$= 130y.$

Ex. 1

Convert 30 kg
to g (grams)

This is so cinchy that one hardly needs to follow a procedure, but we will

$$1\,kg = 1000\,g$$

$$\therefore\ 1 = \frac{1000\,g}{1\,kg}$$

Cutnell doesn't use the term, but it's common — a factor of unity.

$$30\,kg$$

$$\therefore = 30\,kg \times 1$$

Multiplying by 1 is always allowed

$$= 30\,\cancel{kg} \times \frac{1000\,g}{1\,\cancel{kg}}$$

cancel ~~the~~ "unknown" kg

$$= 30 \times 1000\,g$$

$$= 30000\,g.$$

Ex. 2 Angel Falls

in Venezuala

— world's highest falls.

— 979.0 m

Niagara Falls 51 m.

Shoshone Falls 64.7 m

— as person's whose hometown is near Niagara Falls ~~falls~~ I find this hard to accept. — The mighty Niagara is second to none.

More water goes ove Niagara I'm sure.

find the height in feet. (a perfectly useless number I'd say).

$$1\,m = 3.281\,ft$$

$$\therefore 1 = \frac{3.281\,ft}{1\,m}$$ factor of unity.

$$\therefore h = 979.0\,m$$
$$= 979.0\,m \times 1$$
$$= 979.0\,\cancel{m} * \frac{3.281\,ft}{1\,\cancel{m}}$$
$$\simeq 3000\,ft.$$

Note on the board | 1-16

I will very often use only 1 digit ~~accuracy~~ of precision and ie 1 digit accurate

— this is what one does in any fast me~~ntal~~ntal calculation.

And I'll have to do it when people ask me to work out problems, I've not prepared.

In fact, on homework & test problems it often pays to do the calculation to one digit accuracy in your head before you use your calculator to calculate a precise number

Ex 3. Convert 10 m/s into ~~miles~~ mi/hr = mph

Question for class

- what is special about 10 m/s?
- wait, look at watch.
- what is special in sports?
- wait till they give.

Often calculators go astray — but so does the head sometimes.

1-17

— about as fast as a human can run

if the human in question is an Olympic sprinter.

But Usain Bolt can do a bit better 9.65 m/s in the 200m.

$$v = 10\,\tfrac{m}{s}$$

$$= 10\,\tfrac{m}{s} \times 1 \times 1$$

$$1 = \frac{1\,km}{10^3\,m} \cdot \frac{1\,mi}{1.609\,km}$$

$$1 = \frac{3600\,s}{1\,h}$$

$$= 10\,\cancel{\tfrac{m}{s}} \cdot \frac{1\,\cancel{km}}{10^3\,\cancel{m}} \cdot \frac{1\,mi}{1.609\,\cancel{km}} \cdot \frac{3600\,\cancel{s}}{1\,h}$$

$$\cong 10 \cdot \frac{3.6}{1.6}\,\frac{mi}{h}$$

$$= 22.5\,\frac{mi}{h}$$

Question for Class

What is $1\,m/s$ in $\frac{mi}{h}$ Approximately?

$\sim 2.25\,\frac{mi}{h}$

What is special about $1\,m/s$ ~~in human~~ for the human condition?

~~about~~
— of order walking speed.

Omit? Yes? It is a strategy for checking your ~~work~~ equations, but frankly at the Intro physics level, I've ~~never~~ seen ~~any~~ much use for it. There will be no explicit questions on dimensional analysis.

Dimensional Analysis

— we usually think of dimensions as being the 3 spatial dimension

— maybe plus time or the 4th dimension.

— but one can think any quantity as ~~being a di existing~~ ~~in an abstract~~ extending in an abstract dimension

e.g., the dimension of mass where the unit of "length" is the kilogram.

You can analyze the ~~units~~ dimensions or units of an equation to see if it is dimensionally correct, which is not the same as correct.

Ex. $x = v t$ or $x = v^2 t$

$L = \frac{L}{T} T = L$ $\qquad$ $L = \frac{L^2}{T^2} \cancel{T} = \frac{L^2}{T}$

You are you ... Your are not; you — wrong

1 - 19

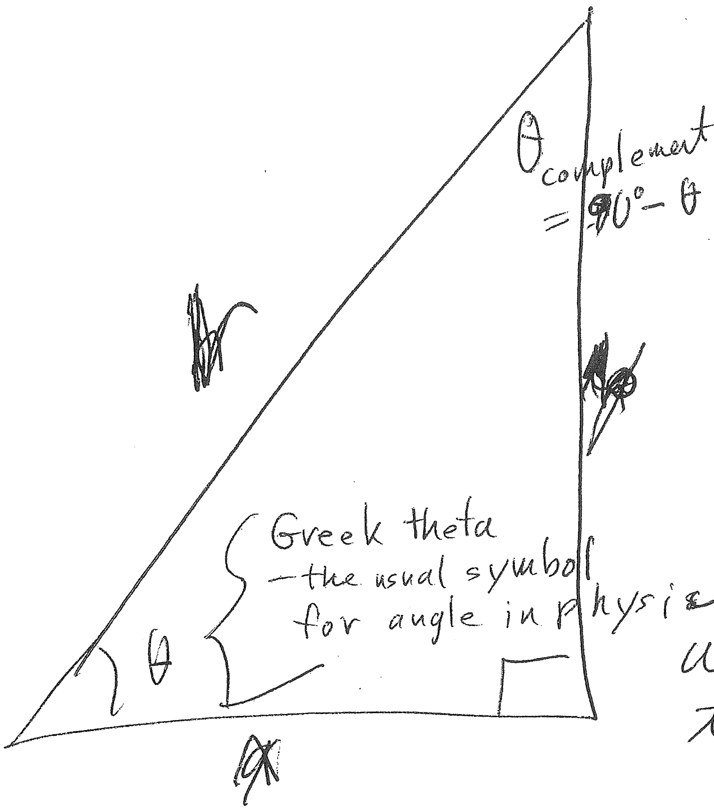
Trigonometry — I'm going to go a bit beyond the ~~text~~ here since we will need a bit beyond later. Some folks may know it already.

↳ triangle measurement roughly speaking

Gr. trigon + metron
triangle + measure (Wikipedia)

Most people I expect are somewhat familiar with the subject which just deals with the relationships of angles and ~~sides (or re~~ the ratios of sides of a right angle triangle

The Absolute ~~values~~ sizes of h hypotenuse a adjacent o opposite are unspecified since only their ratios enter trigonometry.



Greek theta — the usual symbol for angle in physics and I will use it all the time — get used to it.

Recall the Pythagorean Theorem

$$h = \sqrt{x^2 + y^2}$$

Thus if you've specified

x & y

or x & h

or y & h

you've specified the third side and the whole ~~sh~~ shape of the triangle just by saying the lengths of any two sides.

this includes the angles of course.

On the other hand, ~~the~~ a single angle (other than the right angle which understood as already given) specifies the ratios of all the sides via the trigonometric function.

The 3 basic trigonometric functions (trig functions) are

sine ⟹ sin

cosine ⟹ cos

tangent ⟹ tan

These function ~~along~~ with θ as their argument give the ratios of the sides

$$\frac{y}{r} = \sin\theta$$

$$\frac{x}{h} = \cos\theta$$

$$\frac{y}{x} = \tan\theta$$

Note

~~$r = \sqrt{\ }$~~

$$r^2 = x^2 + y^2$$

$$1 = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2$$

$$1 = \cos^2\theta + \sin^2\theta$$

a trig. identity.

~~Of course, from~~ sin, cos, tan and other trig functions are **transcendental functions**

But they aren't the only transcendental functions

which means they can't be evaluated by a finite series of algebraic operations

↳ they are not exactly expressible as polynomials.

You have to add up an infinite series of powers of θ to evaluate them exactly

Ex. $\sin\theta = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + \ldots = \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \ldots$

It's a weird thing that are exact numbers like sine of most angles or π that we know are exact, but we can't in principle know them exactly in terms of rational numbers – they're irrational

Which can't be done in practice.

But you can always evaluate enough terms ~~that you have them~~ as accurately as you need.

– this is what calculators and computers do

– and how ~~tables of~~ trig function are created.

Of course, you can ~~inverse~~ inverse ~~the functions~~

means inverse in this context – not "one over"

$\theta = \sin^{-1}\left(\frac{y}{h}\right) = \arcsin\left(\frac{y}{h}\right)$

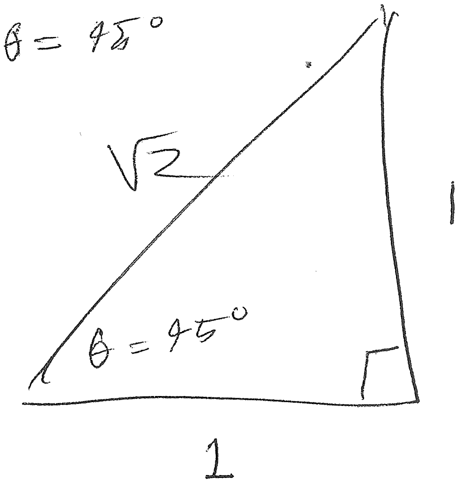
$\theta = \cos^{-1}\left(\frac{x}{h}\right) = \arccos\left(\frac{x}{h}\right)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \arctan\left(\frac{y}{x}\right)$

and calculators do this too.

Standard angle triangles

1 $\theta = 45°$



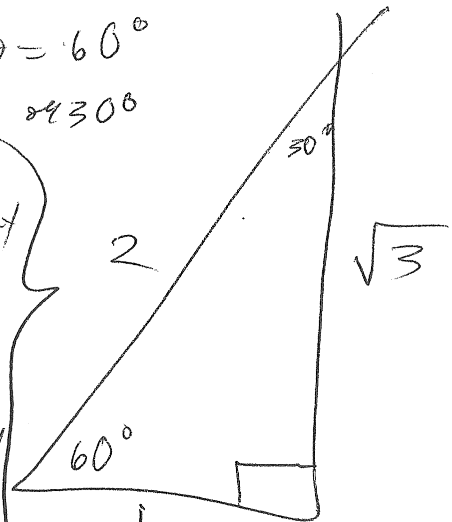
$\sin\theta = \frac{1}{\sqrt{2}} = .7071\ldots$

$\cos\theta = \frac{1}{\sqrt{2}} = \ldots$

$\tan\theta = 1$ One of the few exact rational number ~~values~~ for a trig function.

$\sqrt{2} = \sqrt{1^2 + 1^2}$

2 $\theta = 60°$ or $30°$



These are tricky to prove but one can do it using a hexagon

construction.

$\theta = 60$ $\sin\theta = \frac{\sqrt{3}}{2} = .8660\ldots$

$\cos\theta = \frac{1}{2}$

$\tan\theta = \sqrt{3} = 1.7321\ldots$

irrational numbers that trail on forever without becoming periodic

$\theta = 30$ $\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\tan = \frac{1}{\sqrt{3}}$

~~So~~ $2 = \sqrt{1^2 + \sqrt{3}^2}$

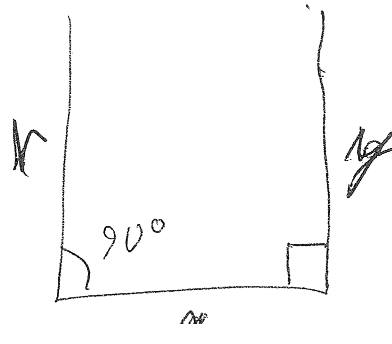
Note $h = \sqrt{a^2 + b^2}$

so $1 = \sqrt{\left(\frac{a}{h}\right)^2 + \left(\frac{b}{h}\right)^2} = \sqrt{\cos^2\theta + \sin^2\theta}$

$1 = \cos^2\theta + \sin^2\theta$ a trig identity for all θ.

3. $\theta = 90°$

Limiting case



~~$\theta = 90°$~~

$\sin\theta = \frac{y}{r} = 1$

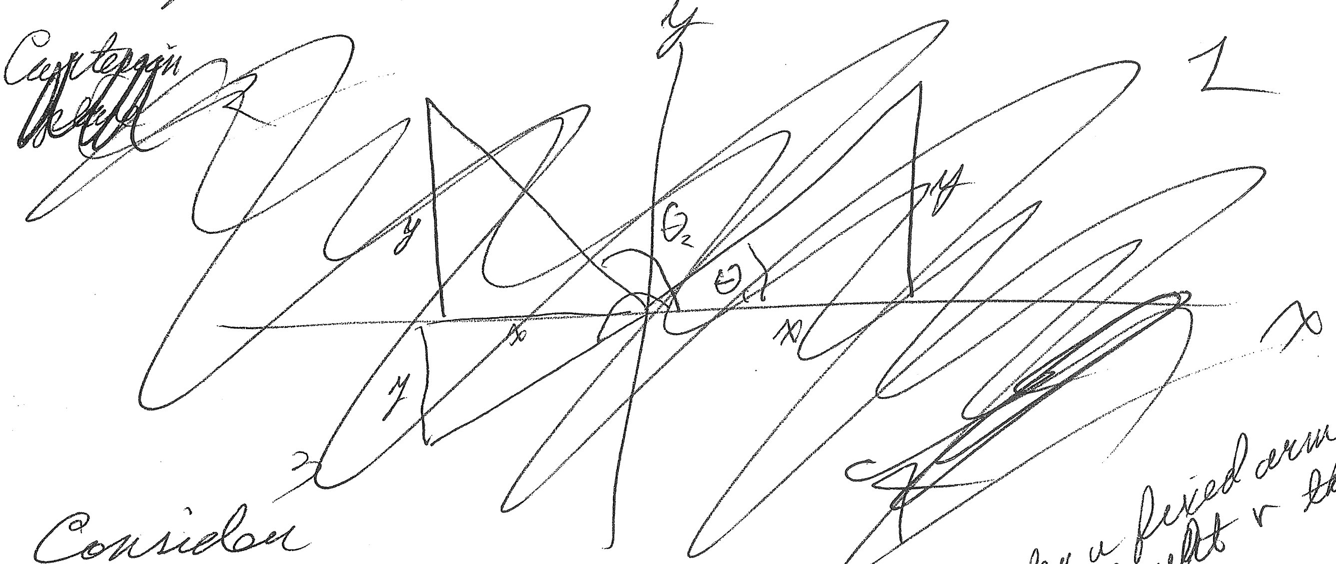
$\cos\theta = \frac{0}{0} = 0$

$\tan\theta = \frac{\infty}{0} = \infty$

These are limiting values what you get as $\theta \to 90°$

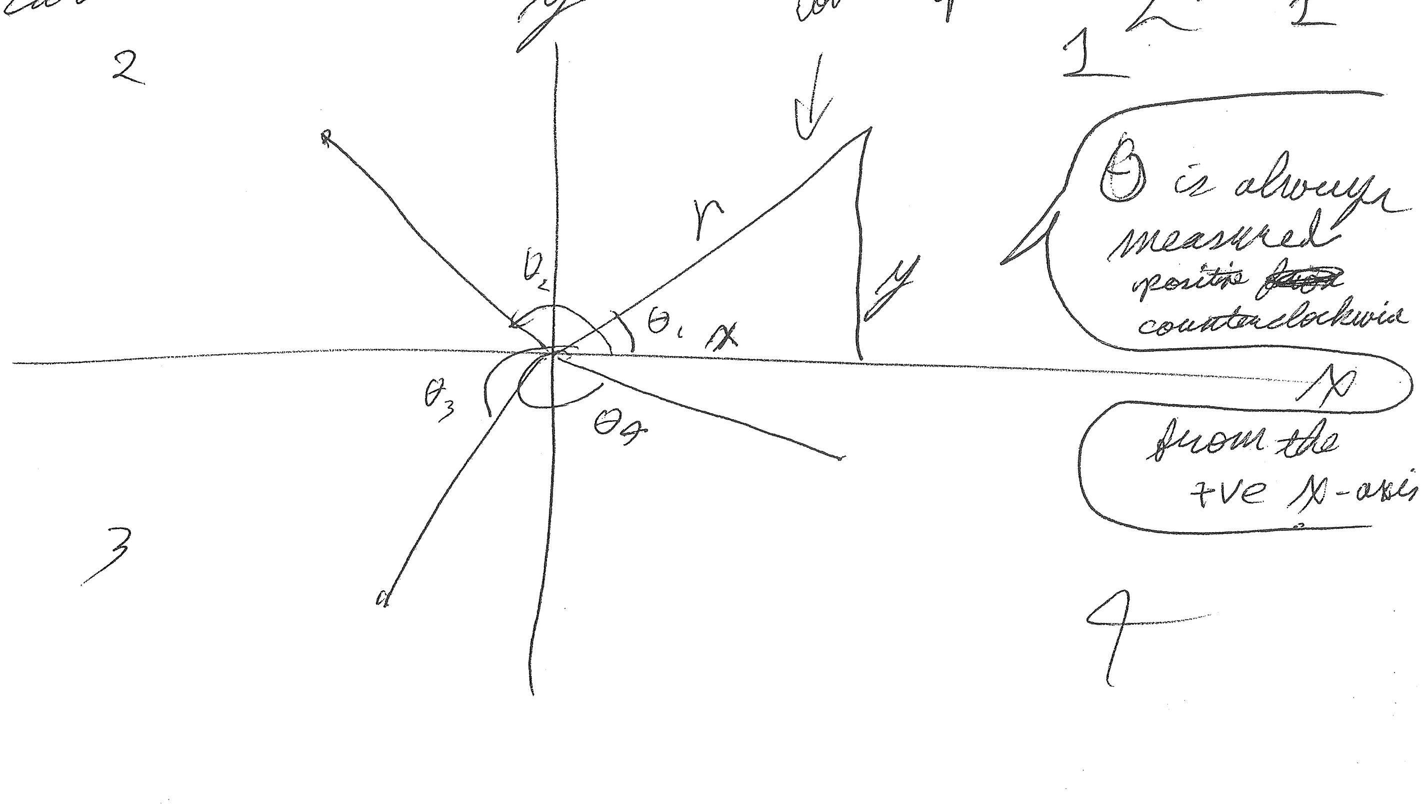
1-24a

The trig functions can be extended to angles $> 90°$ and $< 0°$ – your calculator knows how to do this.



Consider the Cartesian plane

consider a fixed arm of length r that you can rotate



Define the functions as before, but now x & y can become negative.

1-24b

$$\sin\theta = \frac{y}{r}$$

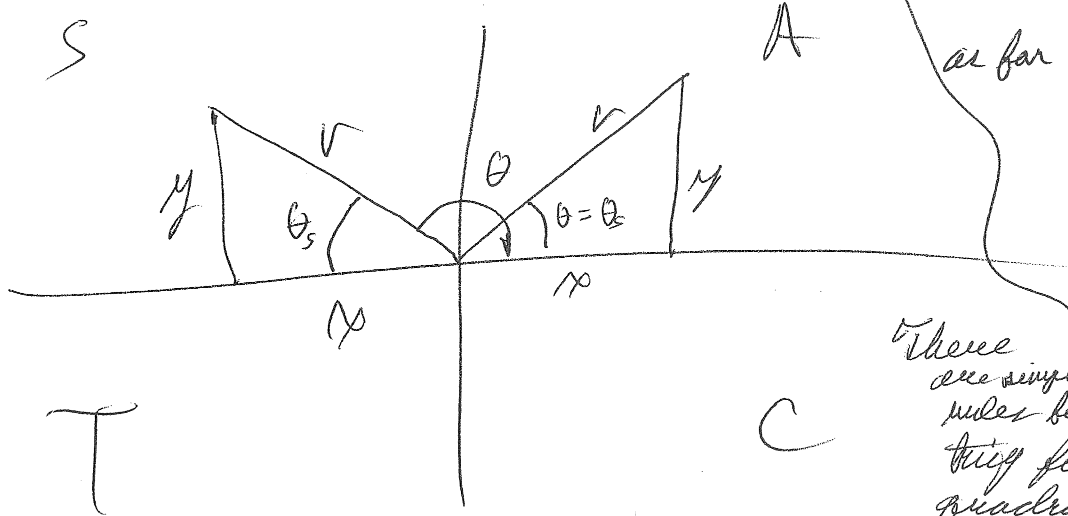
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

for all x and y as θ is increase or decreased.

as for ~~[illegible]~~ cos one likes.

CAST rule



There are simple rules for finding the trig functions in other quadrants based on values in quadrant 1, but omit them.

2nd quadrant or S quadrant

Omit?

$$\sin\theta = \frac{y}{r} = \cancel{\sin\theta_s}$$

$\cancel{\sin(180^\circ - \theta)}$

$= \sin(180^\circ - \theta)$

$= \sin\theta_{sup}$ where $\theta_{sup} \in (0, 90)$ and so we know it's 1st quadrant value is.

so if $\theta = 135^\circ$

$$\sin 135^\circ = \sin 45^\circ$$

$$\text{but } \cos\theta = \frac{x}{r} = -\frac{|x|}{r} = \cancel{\cos\theta_{sup}}$$

$$= -\cos(180 - \theta) = -\cos\theta_{sup}$$

e.g. if $\theta = 135^\circ$

... 45°

1 - 25a

$$\tan\theta = \frac{y}{x}$$

$$= -\frac{y}{|x|} \quad \cancel{= -\tan\theta_s}$$

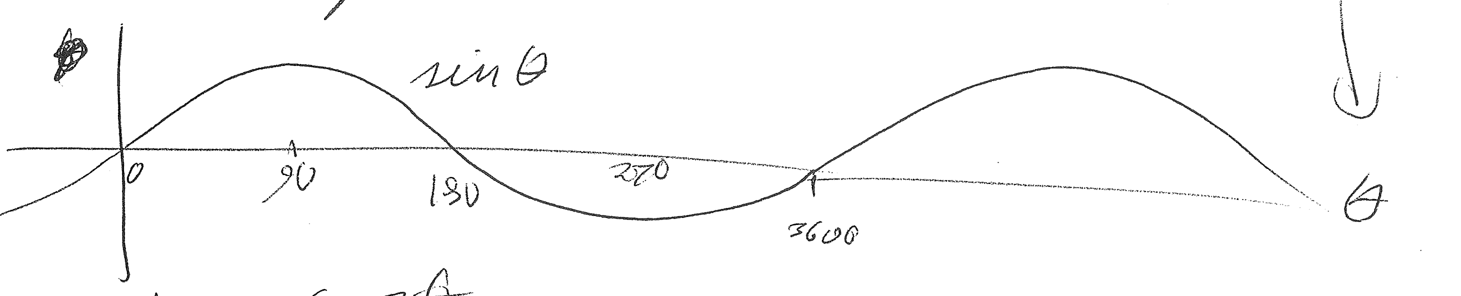
$$= -\tan(180-\theta)$$

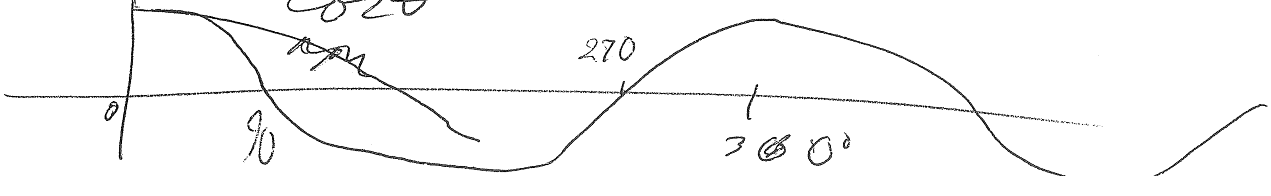
$$= \cancel{-\tan(180-\theta)} = -\tan\theta_{sup}$$

e.g., $\tan 135^\circ = -\tan 45^\circ$

- and one could go on to find expressions for the function in all quadrants from the function in the 1st quadrant.
- the calculator does that for you.
- But ~~the~~ ~~trig~~ are <u>periodic functions</u> they repeat every 360° and you can plot them

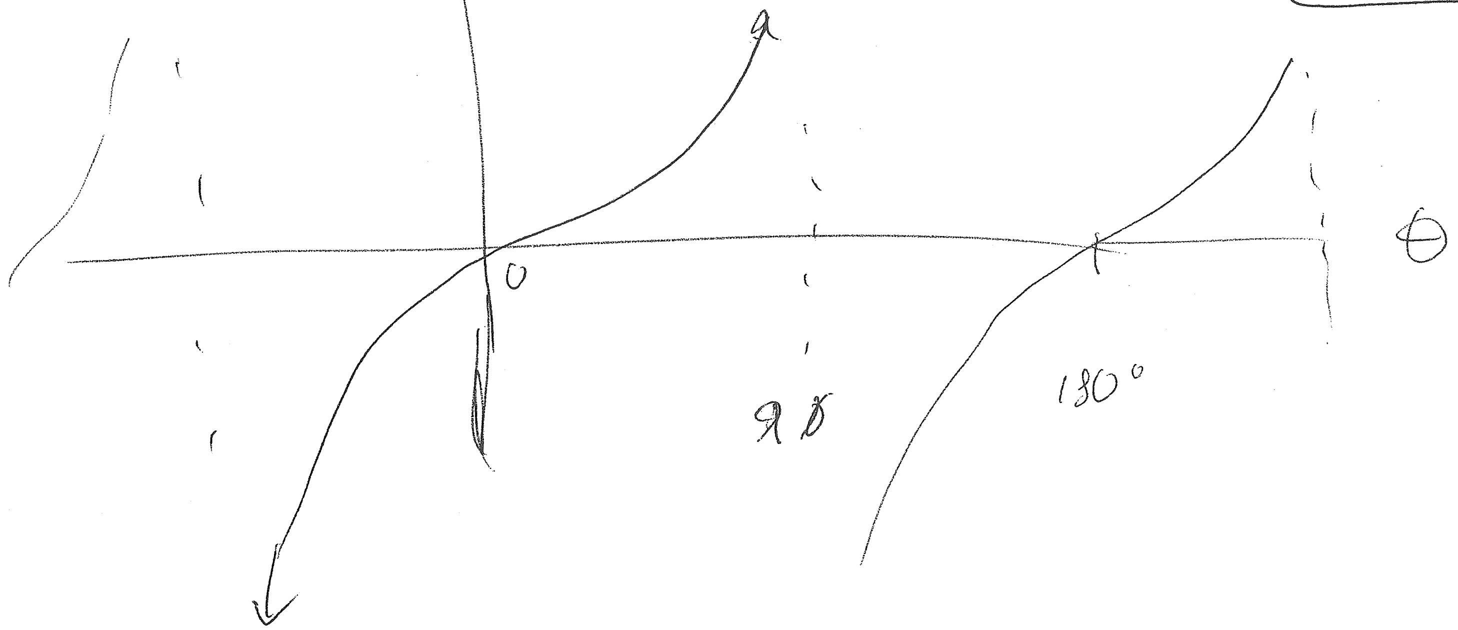
The theta axis





1-2

$\tan\theta$ actual repeats every $180°$

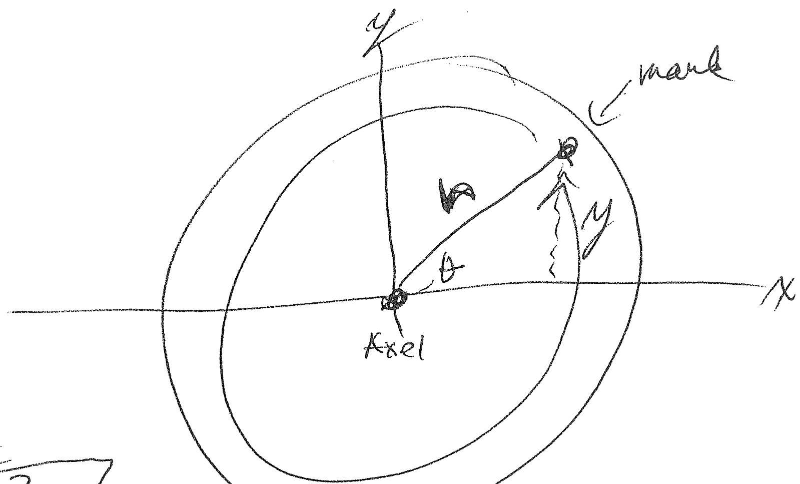


infinities
or singular points.

$\sin\theta$ and $\cos\theta$ are called
sinusoidal functions
because they are sine-like

The sinusoidal curves are common — they turn up for any rotational motion.

Consider a wheel turning at a constant angular rate (cycles per second)



as the mark goes around and around.

But what is it's y-height on Cartesian plot?

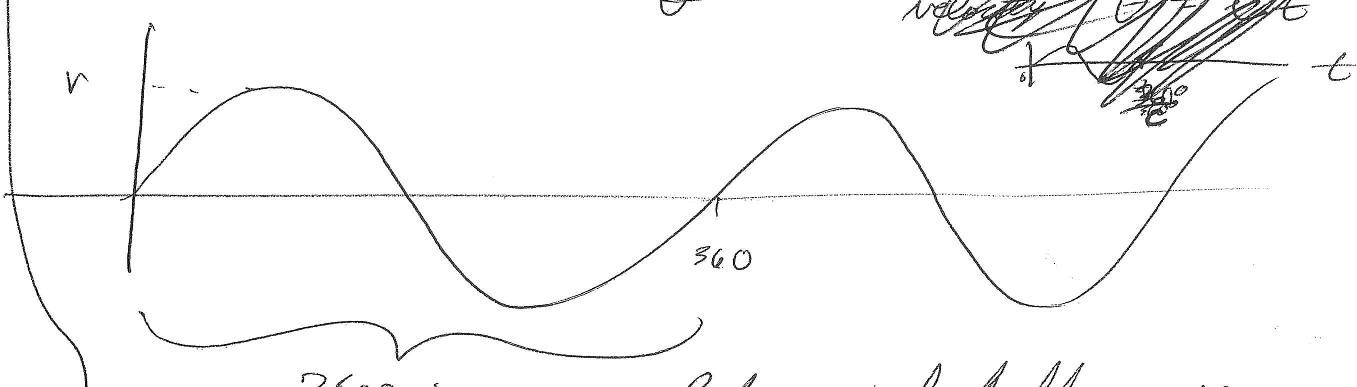
Omit?
Not just a mathematical curiosity since mechanical devices have to convert linear to rotational motion and vice versa.
?

Well $\frac{y}{r} = \sin\theta$

$y = r\sin\theta$

as θ increase from zero to ∞

For a constant angular velocity



360° is a complete period of the wave.

Say

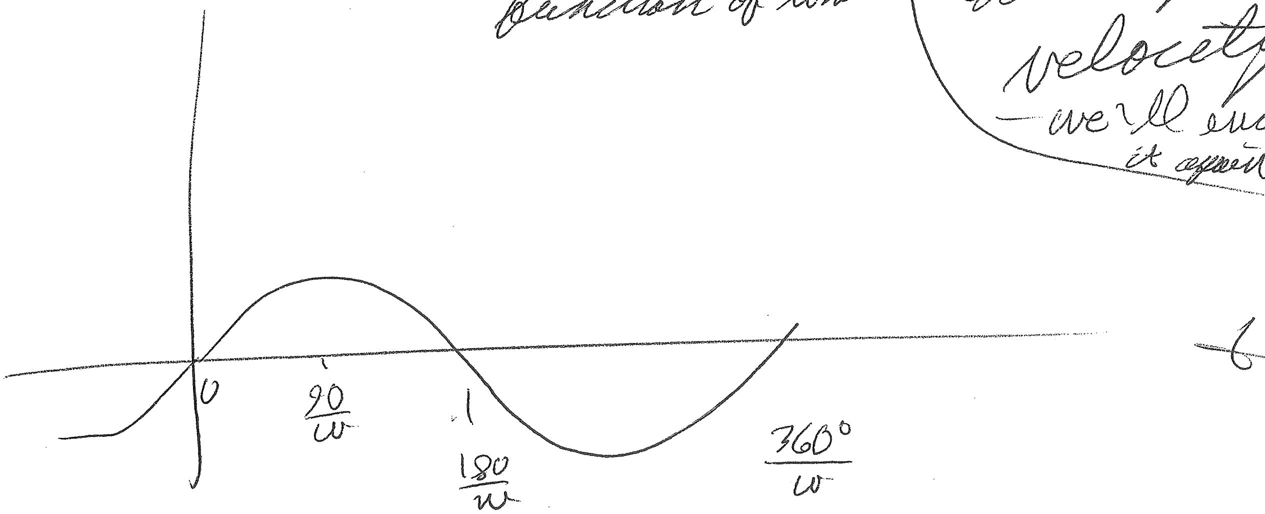
$\theta = \omega t$ where

ω = Greek omega

called the angular velocity.

— we'll encounter it again.

Then you can plots sinusoids as function of time



We will return to sinusoids at a later time.

— They turn up in a lot of places besides ~~wheel~~ rotational motion e.g., Alternating current (AC) and alternating voltage are sinusoidal functions of time.

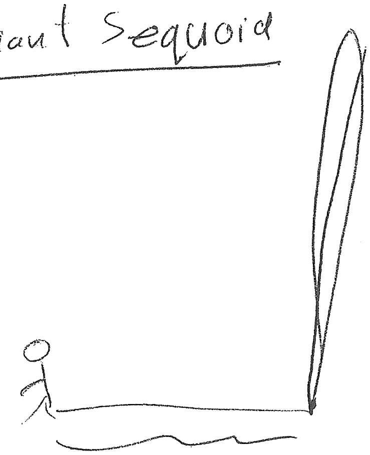
Question
What is the time period of ~~a~~ standard North American AC?
Wait 15 seconds How many cycles per second?
60 cycles = 60 Hertz.

So $\frac{1}{60}$ s for 1 cycle.

Ex. From astronomy

Ex. Height of a Redwood

a Giant Sequoid

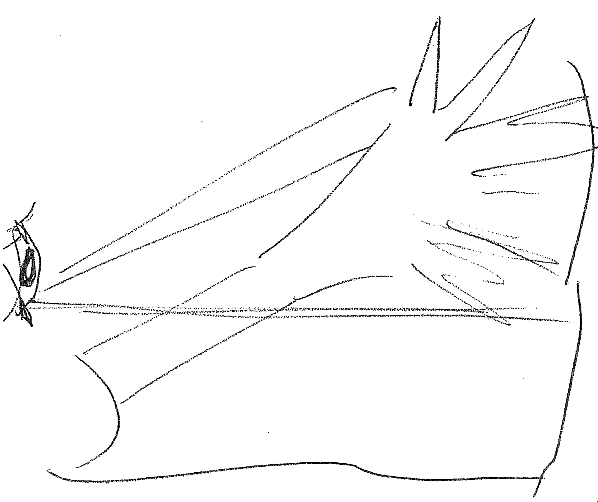


You stand at the base and the tree seems to go up to the sky.

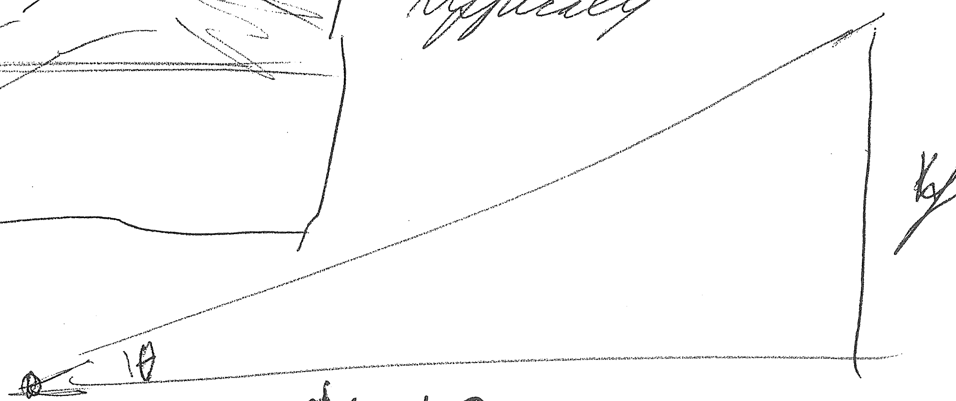
100 steps ≅ 100 m

you walk ~~1000 m~~ away and measure its **angular height** with 2 spread hands at arm's length

$\approx 36°$



~18° typically



$x = 100\,\text{m}$

the baseline

your height is negligible compared to a redwood

(this is not a super precise ...)

$$\tan\theta = \frac{y}{x}$$

$$y = x\tan\theta = 100\,\text{m} \times .7265$$

$$\cong 73\,\text{m}$$

Scalars + Vectors

— a scalar is quantity that has no associated direction in real space → it just has a magnitude

e.g.,
— temperature, density

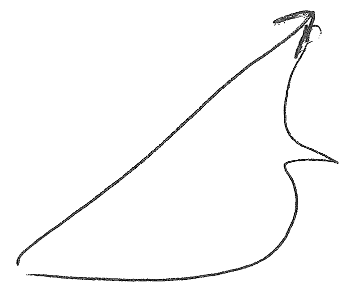
— a vector is a quantity with both a magnitude and a direction in real space (in more advanced physics can be in an abstract space; and math this definition needs revision, but it's fine for our needs)

The (~~representive~~ prototype) vector is displacement to use the physics jargon word.

↓

which is distance and direction specified

Graphically we can represent a vector as a line with an arrow head



← arrow head gives the direction

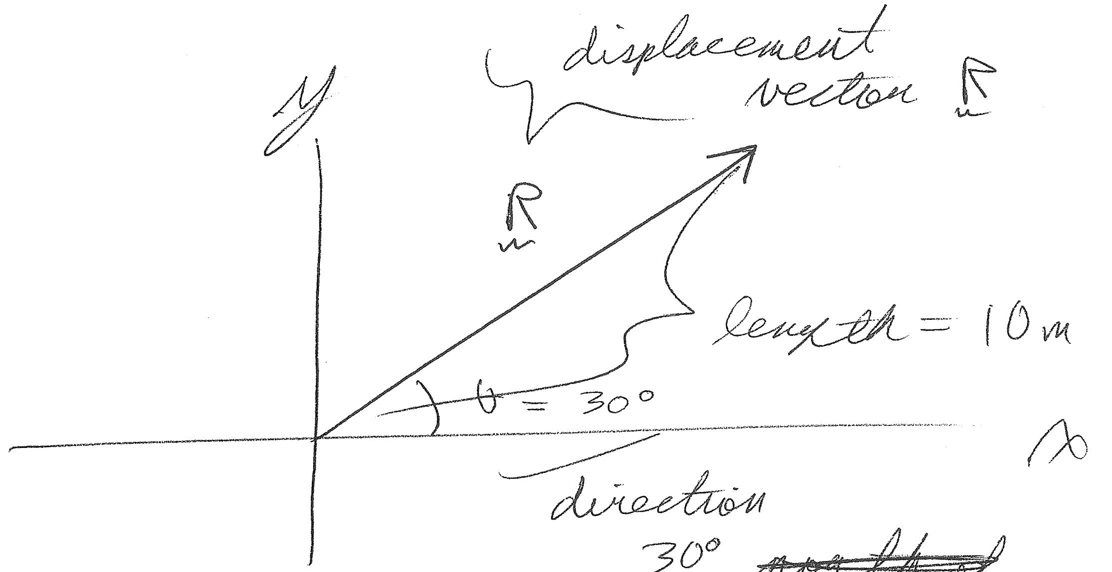
length represents the magnitude

To represent a vector in symbols there are several forms

- boldface **A** but this is hard to distinguish on a whiteboard
- overhead arrow as in the text $\vec{A}$
- or under squiggle $\underset{\sim}{A}$

all meaning vector A

This is the one I use on the board since I ~~then~~ find squiggles easy to write → you all may think that is all I do write



direction
30° ~~north of~~ counterclockwise from the x-axis.

Both length & direction need to be specified to specify a vector completely.

— but the direction can be specified in several ways. — with respect to different axes or using components.

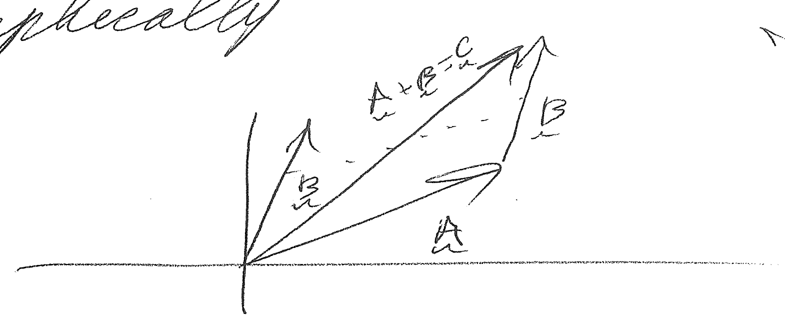
Vector Addition

↳ this is a defined process. One needs a new rule because they are not scalars

$\underset{\sim}{A} + \underset{\sim}{B} = \underset{\sim}{C}$ ← resultant vector

makes sense.

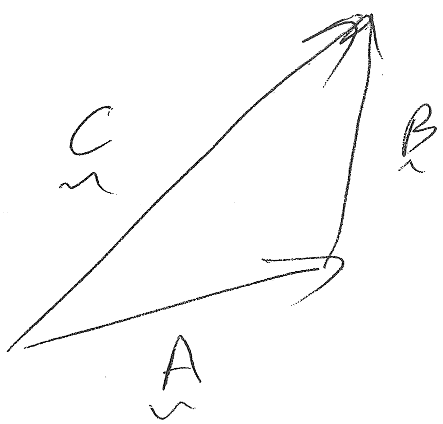
Graphically



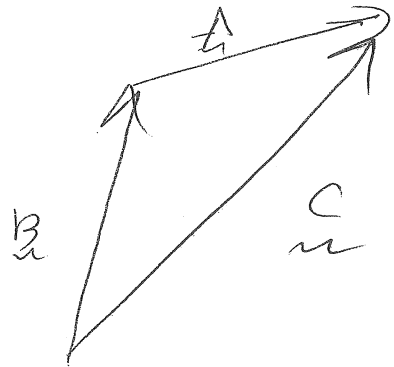
"transport" B to a head-to-tail ~~tail~~ position

$$\underset{\sim}{A} + \underset{\sim}{B} = \underset{\sim}{B} + \underset{\sim}{A}$$

vector addition is commutative.

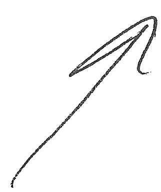


and

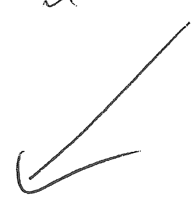


Negation of a vector

if $\underset{\sim}{A}$



then $-\underset{\sim}{A}$



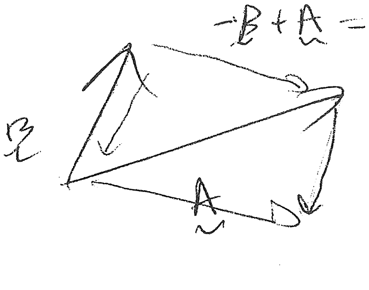
— same magnitude, but opposite direction (opposite "sense" in math jargon)

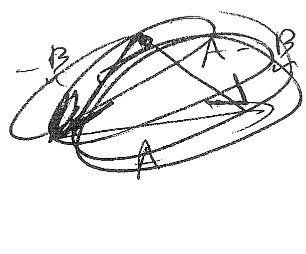
sense – one of two opposing directions in which a vector can point. (Ba-1102)

Vector subtraction

$$\underset{\sim}{A} - \underset{\sim}{B} = \underset{\sim}{A} + (-\underset{\sim}{B})$$

$-\underset{\sim}{B} + \underset{\sim}{A} = \underset{\sim}{A} - \underset{\sim}{B}$

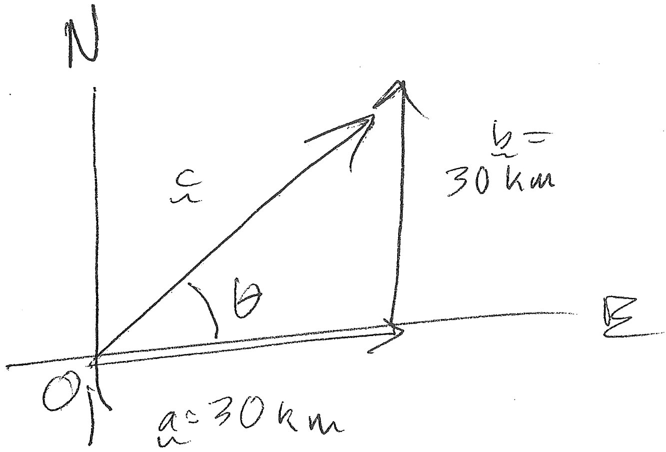




Ex You displace

30 km east,

then 30 km north



~~What~~ distance have you traveled?

wait 15 ~~sec~~

60 km.

But what is your displacement from the origin?

$$\underset{\sim}{c} = \underset{\sim}{a} + \underset{\sim}{b}$$

without the squiggle I mean magnitude

$$c = \sqrt{a^2 + b^2} \quad \text{by Pythagorean theorem}$$

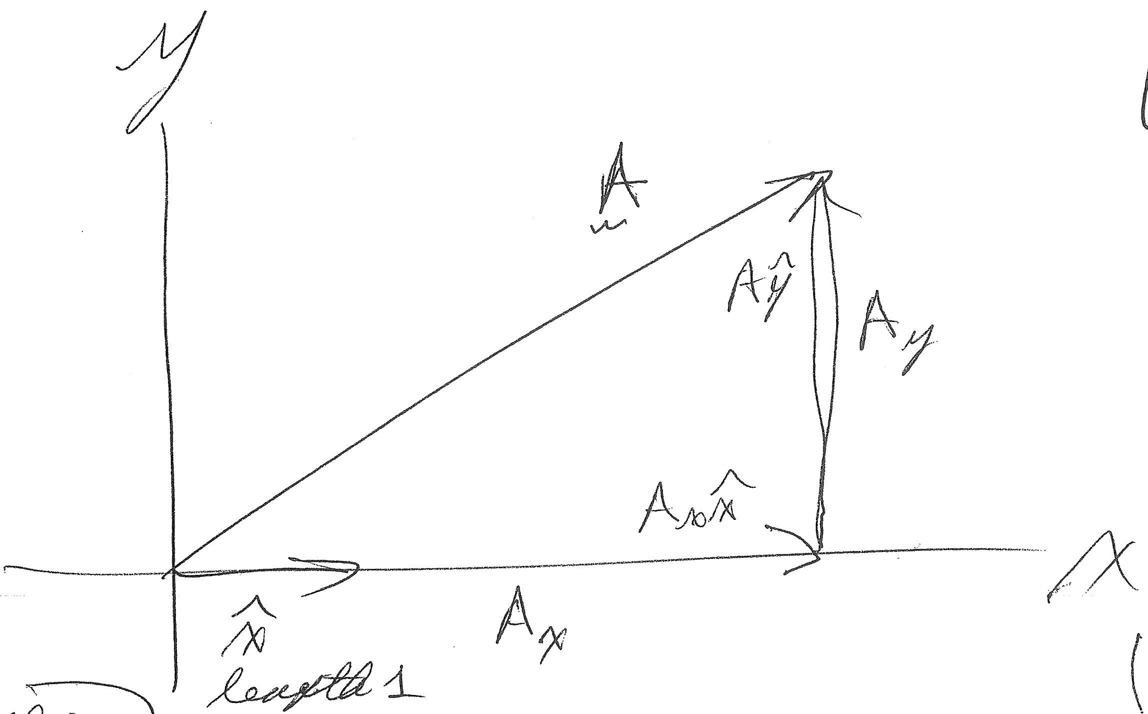
$$\simeq \sqrt{2000}$$

$$\simeq 45 \text{ km}$$

Question Direction from east? 45°, one of those special triangles - the isosceles right triangle

Vector Components

- now I know what you are thinking,
  it's tedious to deal with ~~with~~ two things that have to be dealt with in different ways
  - i.e., length & direction
- Is there some more ~~power~~ flexible way to deal with vectors?
- Yes you can break them into <u>components</u>
  relative to some <u>coordinate system</u>.



ordered pair notation.

$$\underset{\sim}{A} = A_x \hat{x} + A_y \hat{y} = (A_x, A_y)$$

So this should be valid just by vector addition.

As should $\underset{\sim}{A} + \underset{\sim}{B} = (A_x + B_x, A_y + B_y)$

scalar x-component

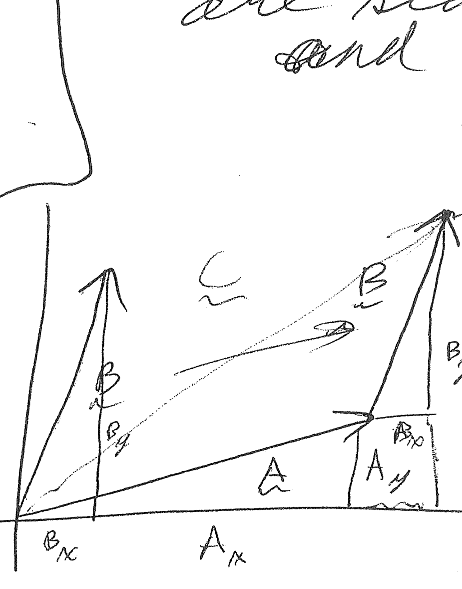
a unit vector in the x-direction

scalar y component

a unit vector in the y-direction.

pointed hat ^ means unit vector

The components are scalars and add and subtract that way. This works correctly as we can show graphically



x-coordinate is $A_x + B_x = C_x$

y-coordinate is $A_y + B_y = C_y$

So these do add to give the components of vector $\underset{\sim}{C}$

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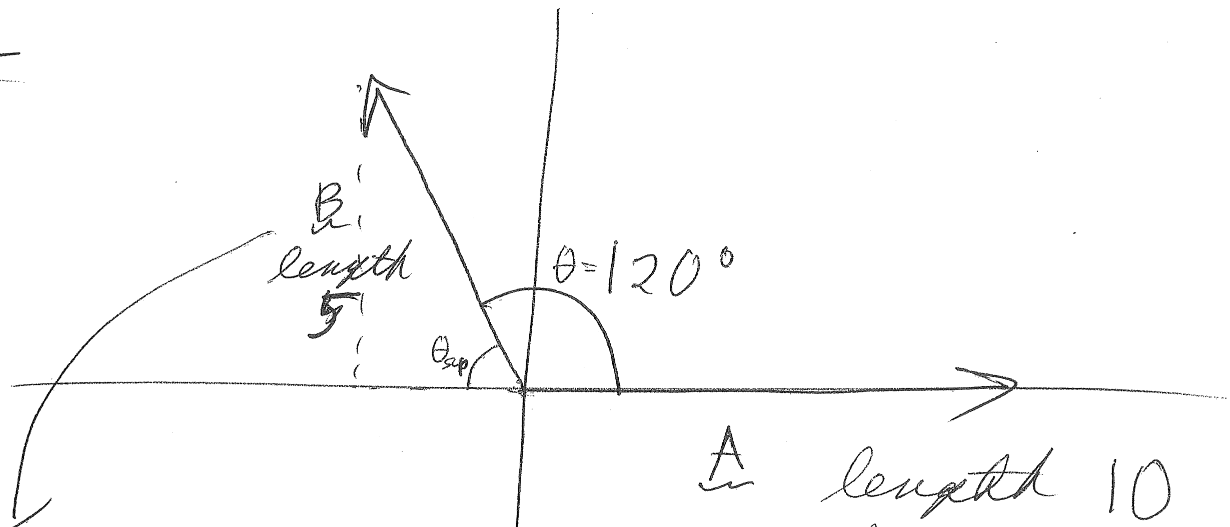
Thus graphical vector addition is indeed equivalent to adding components as scalars.

$$\underset{\sim}{A} + \underset{\sim}{B} = (A_x + B_x, A_y + B_y)$$

Ex.



$\underset{\sim}{A}$ length 10
direction all
in the x-direction

$\underset{\sim}{A} = (10, 0)$

we have to find the components of $\underset{\sim}{B}$ with trig.

omit ?
$B = 5$ the magnitude

$$B_x = -B \cos\theta_{sup} = B\cos\theta$$
$$B_y = B\sin\theta_{sup} = B\sin\theta$$

You can always use the angles measured clockwise from the x-axis which is the standard view

$B_x = -5 \cdot \frac{1}{2} = -2.5$

$B_y = 5 \cdot .866 \cong 4.33$

$$\underset{\sim}{A} + \underset{\sim}{B} = \cancel{(A_x}$$

$$= (10 - 2.5,\ 0 + 4.33)$$

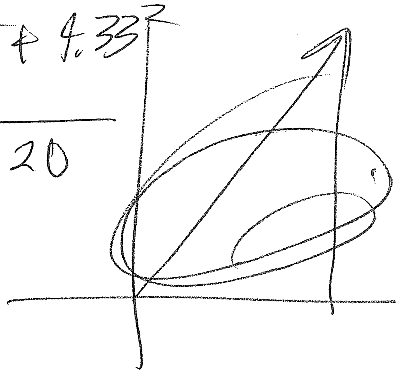
$$= (7.5,\ 4.33)$$

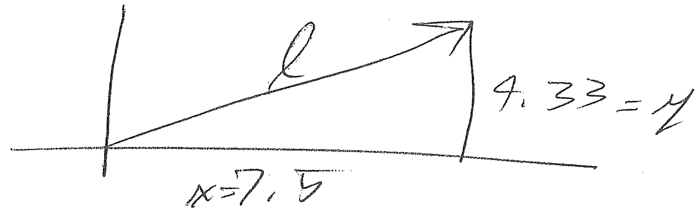
$$l = \sqrt{7.5^2 + 4.33^2}$$

$$= \sqrt{55 + 20}$$

$$= \sqrt{75}$$

$$\simeq 8.5$$





Not bad.

$$\theta = \tan^{-1}\left(\frac{4.33}{7.5}\right) \simeq 30 \quad {\sim .60}$$

$$x = l \cos\theta$$

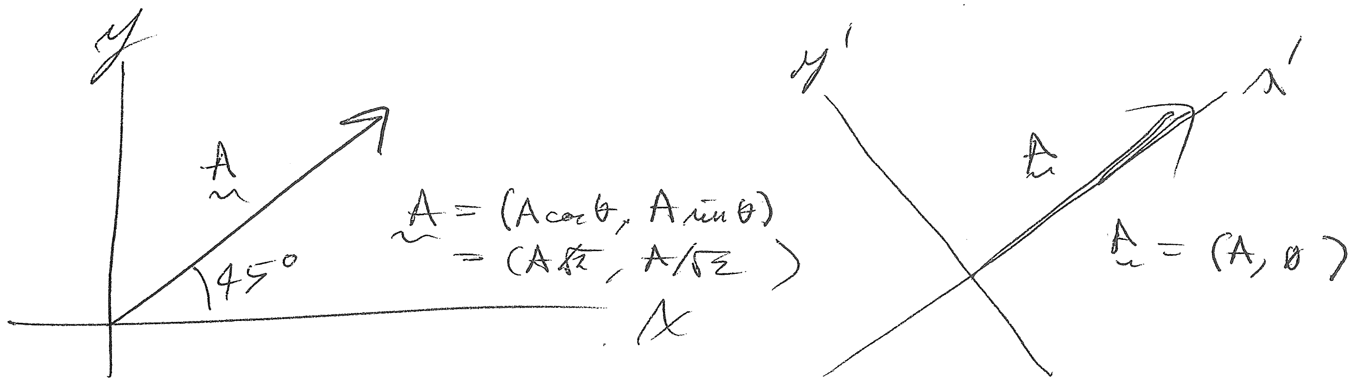
$$l = \frac{x}{\cos\theta} \simeq 8.66$$

Did in math!

Vector Components are Not Unique

— They are determined relative to a coordinate system.

Ex.

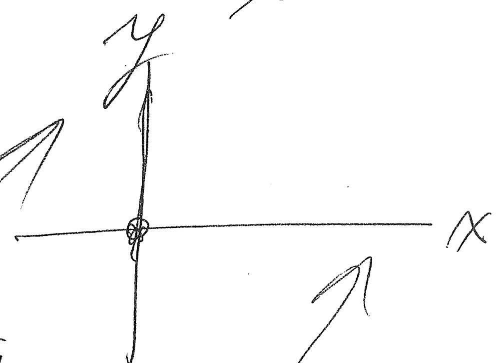


Same vector, but the components are different for different sets of axes.

— But in either coordinate system, the ~~axes~~ components are correct and you will deduce the correct behavior for the vector.

<u>But</u> usually you can choose the coordinate system for convenience of calculation.

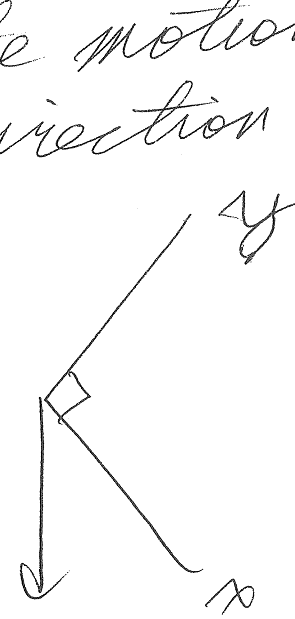
Ex In this course we'll consider falling objects — lots & lots of times. And projectile motion maybe a little less often.



the y-axis perpendicular to the ground

so align the x-axis with the ground

thus for dropped objects the motion is only in ~~one~~ the y-direction and is scalar-like

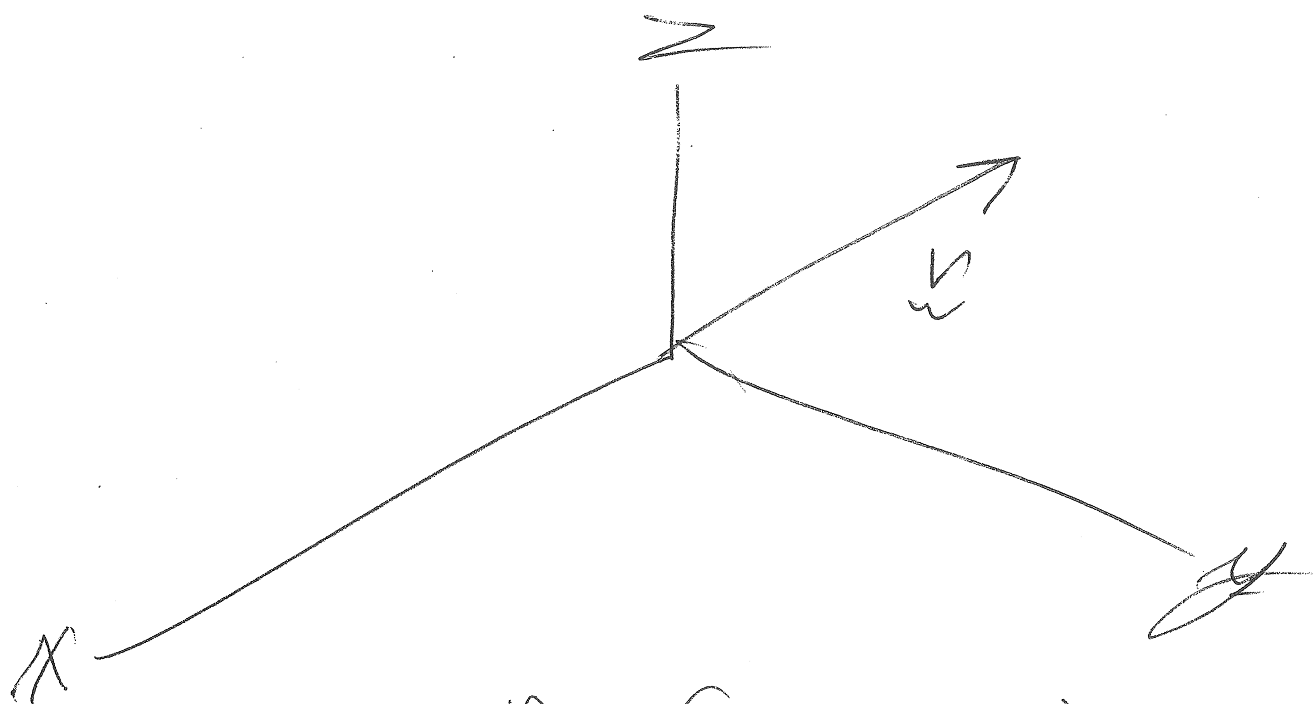


← if you choose the axes this way then you'd need two components to describe the motion and everything would be a mess.

You'd get the same answer if you did it [illegible]

Three Dimensional Vectors

They are easy enough in principle, but ~~they~~ they are beyond the scope of this class.



$$\underset{\sim}{r} = (x, y, z)$$

They just have 3-components.