Intro Physics Semester I **Name:**

Homework 15: Oscillation: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

1. "Let's play *Jeopardy*! For \$100, the answer is: It is a motion that repeats itself in equal time periods: i.e., a periodic motion. Perfect repetition is an ideal case that is more or less closely approached in reality. The repeated motion is sometimes called a cycle."

   What is a/an ____________, Alex?

   a) inhalation b) exhalation c) rotation d) accleration e) oscillation

2. Say P is the period of an oscillation. Say you observe N oscillations (or cycles) which take time NP, of course, The frequency of the oscillation (cycles per unit time) is given by:

   a) $f = NP$. b) $f = N/P$. c) $f = P/N$. d) $f = 1/P$. e) $f = N$.

3. The MKS unit of frequency is the hertz (Hz). It is derived from the second:

   a) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1}$. b) $1\,\mathrm{Hz} = 1\,\mathrm{s}$. c) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-2}$. d) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{2}$. e) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1/2}$.

4. The linear force law (AKA linear restoring force law, spring force law, Hooke's law force, and simple harmonic oscillator force law) applies approximately to a wide variety of systems. In fact, almost any stable equilibrium system from molecules to bridges and beyond obeys the linear force law for sufficiently small perturbations of any component around the component's stable equilibrium configuration. Components moving under the linear force law alone are in simple harmonic motion. Usually some damping force affects the system. The linear force law in one-dimension with the stable equilibrium point at the origin is given by:

   a) $F = -kx^{1/2}$. b) $F = -kx$. c) $F = -kx^{3/2}$. d) $F = -kx^2$. e) $F = kx$.

5. To be in stable equilibrium, an object must be subject to a restoring force. The restoring force is zero at equilibrium and for any sufficiently small displacement from equilibrium tries to push the object back toward equilibrium. If the kinetic energy built up by the force pushing an object back to equilibrium is too large, then the object can **OVERSHOOT** the equilibrium and an oscillation results. Whether an oscillation results or not depends on the system. In real systems in the absence of any driver force, a damping force will usually dissipate mechanical energy above that of rest at the equilibrium point and cause the oscillation to damp out and the object to come to rest at the equilibrium point again. If the displacement is sufficiently small from a stable equilibrium, then the restoring force will be in most cases linear in the displacement: i.e., for a one-dimensional case

   $$F = -kx \ ,$$

   where k is a constant and x is the displacement from the equilibrium point. The restoring force in the small-displacement limit is often linear because:

   a) of no good reason. b) nature likes discontinuities. c) nature dislikes discontinuities.
   d) nature is indifferent to discontinuities. e) it's lies, all lies.

6. If the linear force alone acts on an object, the object is a simple harmonic oscillator (SHO) and undergoes simple harmonic motion (SHM). Apply Newton's 2nd law to 1-dimensional SHO with linear force $F = -kx$.

   a) $m\dfrac{d^2x}{dt^2} = kx$. b) $m\dfrac{dx}{dt} = -kx$. c) $m\dfrac{d^2x}{dt^2} = -kx$. d) $m\dfrac{d^2x}{dt^2} = -\dfrac{1}{kx}$.
   e) $m\dfrac{d^2x}{dt^2} = -x$.

7. The general solution to the SHO differential equation

   $$m\frac{d^2x}{dt^2} = -kx$$

   is ____________ where amplitude A and phase constant ϕ are set by initial conditions and angular frequency ____________ is determined by the physics of the system itself. The motion of the solution is called simple harmonic motion (SHM).

   a) $x = A\cos(\omega t + \phi)$; $\omega = \sqrt{k/m}$ b) $x = A\cos(\omega t + \phi)$; $\omega = \sqrt{m/k}$
   c) $x = A(\omega t + \phi)$; $\omega = \sqrt{k/m}$ d) $x = A(\omega t + \phi)$; $\omega = \sqrt{m/k}$ e) $x = A(\omega t + \phi)$; $\omega = \sqrt{km}$

8. For a one-dimensional simple harmonic oscillator with mass m, force law $F = -kx$, and solution obeying $a = -\omega^2 x$, the formula for the angular frequency ω is given by:

a) $\omega = m/k$. b) $\omega = \sqrt{k/m}$. c) $\omega = \sqrt{m/k}$. d) $\omega = k/m$. e) $\omega = \sqrt{km}$.

9. A solution to Newton's 2nd law for the simple harmonic oscillator is simple harmonic motion—which is a melodious, uncomplicated motion. The solutions for displacement, velocity, and acceleration for a case when the motion of the oscillating objects starts at time $t = 0$ at its maximum displacement A are, respectively:

   a) $x = A\sin(\omega t)$; $v = \omega A\cos(\omega t)$; $x = -\omega^2 A\sin(\omega t)$.
   b) $x = A$; $v = \omega A$; $a = \omega^2 A$.
   c) $x = A\cos(\omega t)$; $v = -\omega A\cos(\omega t)$; $x = -\omega^2 A\cos(\omega t)$.
   d) $x = A\cos(\omega t)$; $v = -\omega A\sin(\omega t)$; $x = -\omega^2 A\cos(\omega t)$.
   e) $x = A\cos(\omega t)$; $v = \omega A\cos(\omega t)$; $x = \omega^2 A\cos(\omega t)$.

10. In sinusoidal motion, an object's position as a function of time varies like a sine or:

    a) cosine curve. b) tangent curve. c) inverse tangent curve. d) inverse sine curve.
    e) straight line.

11. A sinusoid repeats its behavior every time its argument increases by 2π or $360°$. An oscillation in time described by a sinusoid repeats in a time period called the period of the oscillation. The argument of the sinusoid in this case is $\omega t + phi$, where ω is the angular frequency and ϕ is a phase constant set by initial conditions. For a period P, the angular frequency is given by:

    a) $\omega = 2\pi P$. b) $\omega = P/(2\pi)$. c) $\omega = 2\pi/P$. d) $\omega = 1/(2\pi P)$. e) $\omega = 2\pi P^2$.

12. For simple harmonic motion,

    a) $f = P = \omega$. b) $P = 1/f = 2\pi/\omega$, $f = 1/P^2 = \omega/(2\pi)^2$, $\omega = 2\pi f = 2\pi/P$.
    c) $P = 1/f = 2\pi/\omega$, $f = 1/P = \omega/(2\pi)$, $\omega = 2\pi f = 2\pi/P$.
    d) $P = 1/f = 2\pi/\omega$, $f = 1/P = 2\pi\omega$, $\omega = 2\pi f = 2\pi/P$.
    e) $P = 1/f = 1/\omega$, $f = 1/P = \omega/(2\pi)$, $\omega = 2\pi f = 1/P$.

13. If $x_1(t)$ and $x_2(t)$ are solutions to a differential equation (DE), then the DE is linear if $c_1x_1(t) + c_2x_2(t)$ is also a solution where the c's are arbitrary constants. The differential equation for the simple harmonic oscillator

    $$\frac{d^2x}{dt^2} = -\omega^2 x(t)$$

    is:

    a) linear. b) non-linear. c) neither linear nor non-linear. d) both linear and non-linear.
    e) colinear.

14. The mechanical energy of a 1-dimensional simple harmonic oscillator of mass m and force constant k is given by:

    a) $E = kx_{\text{max}}^2 = mv_{\text{max}}^2$. b) $E = (1/2)kx_{\text{max}}^2 = (1/2)mv_{\text{max}}^2$. c) $E = (1/2)kx^2$.
    d) $E = (1/2)mv^2$. e) $E = mgy$.

15. The projection of uniform circular motion on a line in the plane of rotation is:

    a) uniform circular motion. b) a Tusi-couple motion c) simple harmonic motion.
    d) oblique motion. e) round motion.

16. A phyiscal pendulum is any rigid object held by a free pivot axis: the axis supports the pendulum againnst gravity, but exerts no torque about itself. The equation of motion of the physical pendulum (i.e., Newton's 2nd law applied to the physical pendulum) if only gravity torques it is:

    $$\frac{d^2\theta}{dt^2} = -\frac{rmg}{I}\sin\theta \ ,$$

    where θ is the angle a radius from the pivot axis to the center-of-mass axis makes with a downward vertical, r is the cylindrical radius from the pivot axis to the center-of-mass axis, m is the pendulum mass, and I is the pendulum rotational inertia about the pivot axis. This equation has no simple analytical solution because of:

    a) general considerations. b) the I factor. c) the r factor. d) the $\sin\theta$ factor.
    e) darn good reasons.

17. "Let's play *Jeopardy*! For \$100, the answer is:

$$\sin\theta \approx \theta \ ,$$

where θ is given in radians.

What is a/an ____________, Alex?

a) equality b) approximate equality c) inequality d) trigonometric function
e) small-angle approximation

18. The equation of motion of the physical pendulum (i.e., Newton's 2nd law applied to the physical pendulum) in the small-angle approximation is given by

$$\frac{d^2\theta}{dt^2} = -\frac{rmg}{I}\theta \ .$$

We recognize this equation as the differential equation of:

a) Tusi-couple motion. b) uniform circular motion. c) simple harmonic motion.
d) straight-line motion. e) uniform motion.

19. The period of the physical pendulum (in the small-angle approximation) is

$$P = 2\pi\sqrt{\frac{I}{rmg}} \ .$$

If the pendulum is shrunk to a point-mass bob, we have the simple pendulum. In this case, ____________ and ____________.

a) $I = (1/2)mr^2$; $P = 2\pi\sqrt{r/(2g)}$. b) $I = mr^2$; $P = 2\pi r/g$. c) $I = mr$; $P = 2\pi r/g$.
d) $I = mr$; $P = 2\pi\sqrt{r/g}$. e) $I = mr^2$; $P = 2\pi\sqrt{r/g}$.

20. The period of a simple pendulum is given by

$$P = 2\pi\sqrt{\frac{r}{g}} \ ,$$

where r is the length of the pendulum and g is the gravitational fiedl. A fiducial pendulum period (i.e., period that can be used as a standard for reference or quick estimation) is obtained for a pendulum of length exactly **ONE METER** and assuming that $g = 9.8\,\mathrm{m/s^2}$ exactly. What is this fiducial period to 2 significant figures? To 4 significant figures?

a) 1.0 s and 1.010 s. b) 2.0 s and 2.007 s. c) 2.0 s and 2.01 s. d) 1.0 s and 1.01 s.
e) 1.0 s and 1.1 s.

21. Without using a calculator (you're on your honor here), what is approximately the period of a simple pendulum of length 9 m? **HINT:** Just look at what the simple pendulum period formula and reflect on the answer to the fiducial pendulum period question.

a) 9.8 s. b) 162 s. c) 2.0 s. d) 1.0 s. e) 6.0 s.

22. Time for deep thought. If you are clever (and not like your physics professor) before doing any elaborate dynamical analysis, why should you know that the period of the simple pendulum is independent of mass?

a) Mass is density times volume. Volume never comes into the simple pendulum problem. Ergo mass never comes into the simple pendulum problem.
b) There no mass in the kinematic equations and thus mass can never affect the motion of anything. The simple pendulum is included in the set of anything.
c) Physical intuition.
d) For any particle acted on only by gravity and workless forces of constraint (e.g., normal forces such as those of a frictionless slope or of a frictionless wire for a sliding bead), the only force that can cause acceleration is the component of gravity in the direction allowed by the constraints. This component has mass as one of its factors and is the F in $F = ma$. Thus, the mass in 2nd Newton's

law applied to the system cancels out from both sides of the equation. The acceleration is thus determined by g, the direction of gravity downward, and the directions allowed by the forces of constraint. In principle, the whole kinematics of the particle can be determined from knowing the acceleration for any location and the initial conditions. In practice, it might require elaborate calculations including computer calculations to find the whole kinematics. But mass does not come into the calculation. The simple pendulum fits the case just outlined. The ideal free massless pendulum arm exerts no force in the direction the pendulum bob can move. There is no air drag. The only force that can cause acceleration is a component of gravity. Consequently, the whole kinematics of the pendulum bob is independent of mass. The pendulum period is part of the kinematics. Therefore the pendulum period is independent of mass.

e) Socrates has mass.
All humans have mass.
Ergo all humans are Socrates.

23. You are Galileo Galilei (1564–1642) professor (untenured) of mathematics at the University of Pisa circa 1590. You are red-bearded and feisty (Italian word meaning ...). Tired of people making fun of your redundant name (exactly so in Latin: Galileus Galileus) as if you were Humbert Humbert or some such and bored with dropping balls off the Leaning Tower in the piazza, you seek calm in the adjacent Cathedral of Pisa. There you notice that the Cathedral lanterns oscillate in the wind with a constant period no matter what the amplitude of the oscillation provided the amplitude isn't too large and that period only varies with lantern cord length and not lantern mass as far as you can tell. At once—you are an incomparable genius after all—you realize that pendulums would make great regulators for clocks because:

    a) it has **NEVER BEEN** been thought of before.
    b) it has **BEEN** thought of before.
    c) the hypnotic pendulum swinging motion would induce even deeper slumber in your less-amusing, clockwatching students.
    d) even a bad idea can make money. All that is needed is a great advertizing campaign.
    e) all clocks using pendulums as regulators for the motions of the hands and the energy input to keep the motions going would keep the periods of the hands very constant since small variations in amplitude that arise from somewhat irregular resistive and driver forces would have little effect on the pendulum period. Also the pendulum clocks could be kept synchronized to high accuracy despite varying amplitudes for the pendulum motion and masses for pendulum. Of course, the effective length of the pendulum does affect the period and has to be carefully adjusted for synchronization, but that is easy to do. It's one of those Eureka moments.

24. "Let's play *Jeopardy*! For $100, the answer is: The motion of a system slowing down to rest at a stable equilibrium through a series of decaying oscillations about the equilibrium point."

    What is ____________, Alex?

    a) critically motion b) underdamped motion c) overdamped motion
    d) uniform circular motion e) all wet motion

25. "Let's play *Jeopardy*! For $100, the answer is: The behavior of an oscillatory system driven at its natural or resonance frequency."

    What is ____________, Alex?

    a) underdamped oscillation b) simple harmonic motion c) uniform circular motion
    d) resonance or in resonance e) loco

26. An oscillatory system driven at its resonance frequency will exhibit large oscillations. Every child—before they knew the name of torque—understood this when playing:

    a) on a swing. b) on a ladder. c) soccer. d) hopscotch. e) with matches.

27. There is a simple harmonic oscillator (SHO) that takes a time $\Delta t = 0.75\,\mathrm{s}$ before it begins to repeat. What are its (a) period P, (b) frequency f (in hertz), and (c) angular frequency ω (in radians per second)?

28. A body of mass $m = 0.12\,\mathrm{kg}$ is a simple harmonic oscillator (SHO) with equilibrium position $x = 0$,

amplitude $x_{\text{max}} = 8.5\,\text{cm}$, period $P = 0.20\,\text{s}$, and a linear (spring-like) restoring force.

a) What is the force constant of the linear (or Hooke's law) force?

b) What is the maximum absolute value of the force acting on the body?

c) What is the maximum absolute value of the acceleration of the body?

29. You have a block of mass m sandwiched between a bunch of springs in parallel. The whole system is a 1-dimensional system. The springs are attached to opposing walls. Some springs are from the left and some are from the right. The block sits on a level frictionless floor. The springs are ideal. Each spring has a force constant k_i and equilibrium position x_i for the center of the block: i.e., x_i is where the block center would be in equilibrium if only spring i were attached to the block.

a) What is the expression for the net force on the mass?

b) Derive the appropriate single-spring equivalent k (i.e., force constant) and x_{eq} (i.e., equilibrium position) expressions such that the net force expression changes to

$$F = -k(x - x_{\text{eq}}) \ .$$

Why is the x_{eq} the equilibrium position of the total system?

c) Derive the expression for the total system ω in terms of the individual spring angular frequencies ω_i and the total system period P in terms of the individual spring periods P_i.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x - x_0) f^{(1)}(x_0) + \frac{(x - x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x - x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total } f} = KE_{\text{total } i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \qquad \tau_z = RF_{xy} \sin \gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

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20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1 m_2}{r_{12}^2} \hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2} \hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1 m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \;\approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

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21 **Fluids**

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

$$\begin{aligned}
\text{Pascal's principle} \quad & p = p_{\text{ext}} - \rho g (y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}} \\
\text{Archimedes principle} \quad & F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g \\
\text{equation of continuity for ideal fluid} \quad & R_V = Av = \text{Constant} \\
\text{Bernoulli's equation} \quad & p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}
\end{aligned}$$

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22 **Oscillation**

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$