Intro Physics Semester I **Name:**

Homework 13: Gravity: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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014 qmult 00100 1 4 2 easy deducto-memory: Newton gravitation law

1. "Let's play *Jeopardy*! For \$100, the answer is: He/she discovered the gravitation law (AKA law of universal gravitation) of classical physics. This law shows that the same gravity that holds on Earth also holds throughout the space—insofar as classical physics applies."

   Who is ____________, Alex?

   a) Galileo (1564–1642) b) Isaac Newton (1643–1727) c) James Clark Maxwell (1831–1879) d) Albert Einstein (1879–1955) e) Emmy Noether (1882–1935)

   SUGGESTED ANSWER: (b)

   Wrong answers:
   d) He discovered general relativity which supercedes classical gravity physics.

   Redaction: Jeffery, 2008jan01

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014 qmult 00110 1 1 3 easy memory: Newton's Apple

2. William Stukeley (1687–1765) recorded a conversation with Newton at Kensington, 1726 April 15 (less than year before Newton's death at 84):

   "when formerly, the notion of gravitation came into his mind. It was occasioned by the fall of a/an __________, as he sat in contemplative mood. Why should that ____________ always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre."

   a) peach b) pear c) apple d) orange e) sparrow

   SUGGESTED ANSWER: (c)

   Wrong answers:
   d) Now how many orange trees are there in England? OK, OK, in greenhouses. But there were very few greenhouses at Woolsthorpe Manor in Woolsthorpe-by-Colsterworth, Lincolnshire in 1666.
   e) The fall of the sparrow.

   Redaction: Jeffery, 2008jan01

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014 qmult 00120 1 1 1 easy memory: gravity attracts always
Extra keywords: physci

3. Gravity is the force between systems with mass and it is:

   a) always attractive. b) always **REPULSIVE**, except perhaps in some cosmological applications. c) either attractive or **REPULSIVE**. d) neither attractive nor **REPULSIVE**. e) neither fish nor fowl.

   SUGGESTED ANSWER: (a) Our current cosmological model (called the concordance model) has something like a repulsive gravity in it. But at present, no one likes to call that antigravity.

   Wrong answers:
   e) A nonsense answer.

   Redaction: Jeffery, 2001jan01

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014 qmult 00130 1 1 4 easy memory: Newton's law of gravity

4. Newton's law of universal gravitation for the force exerted by point mass 1 on point mass 2 (where from 1 to 2 is indicated by subscript 12) is:

   a) $\vec{F}_{12} = -Gm_1m_2r^2\hat{r}_{12}$. b) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r^2\hat{r}_{12}$. c) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r\hat{r}_{12}$. d) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^2}\hat{r}_{12}$. e) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^3}\hat{r}_{12}$.

   SUGGESTED ANSWER: (d)

   Wrong answers:

a) Can the force of gravity get stronger with separation?

Redaction: Jeffery, 2001jan01

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014 qmult 00140 1 1 2 easy memory: 3rd law of motion and gravity

5. Newton's law of gravity is:

 a) inconsistent with Newton's 3rd law of motion.
 b) consistent with Newton's 3rd law of motion. c) violates Newton's 3rd law of motion.
 d) Newton's 3rd law of motion. e) Newton's 2nd law of motion.

SUGGESTED ANSWER: (b)

Wrong answers:
a) Exactly wrong.

Redaction: Jeffery, 2008jan01

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014 qmult 00150 1 4 5 easy deducto-memory: gravitational constant

6. "Let's play *Jeopardy*! For \$100, the answer is: It is the gravitational constant with MKS units $\mathrm{N\,m^2/kg^2}$ It is actually the poorest known of the fundamental constants because gravity is such a weak force between laboratory size objects which are used to measure it."

 What is ____________, Alex?

 a) $1.000\ldots$ b) $2.99792458 \times 10^{-8}$ c) 2.99792458×10^{8} d) $6.67384(80) \times 10^{11}$
 e) $6.67384(80) \times 10^{-11}$

SUGGESTED ANSWER: (e) This is the Wikipedia value (Wikipedia: Gravitational constant, 2012mar30).

Wrong answers:
c) This is the speed of light.

Redaction: Jeffery, 2008jan01

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014 qmult 00160 1 1 4 easy memory: point-like gravity law

7. Newton's classical gravity law is written for point masses. But this presents a paradox since point masses, at least classical point masses (and maybe not even quantum mechanical point masses, but who knows) do not really exist. The paradox is resolved by regarding the law as an ideal limiting form from which actual gravitational forces between masses can be calculated. Four cases of the application of Newton's gravity law come to mind for the situation of two objects.

 1. The general case: Here one makes the classical continuum assumption for matter and integrates up over the matter of the attractor and matter of the attractee in order to find the net force of the attractor on the attractee. As an equation for attractor (object 1) and attractee (object 2), one has

$$\vec{F}_{12} = -\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_2 - \vec{r}_1|^2}\,\frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}\,dV_1\,dV_2 \ ,$$

 where $\vec{F}_{12}$ is the force of object 1 on object 2, $G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2012mar30) is the gravitational constant, ρ designates density, $\vec{r}_1$ is the position vector for object 1, $\vec{r}_2$ is the position vector for object 1, and the integrals are over the volumes V_1 and V_2 of objects 1 and 2. If one can't make the continuum approximation for some reason, a double summation over point particles can be done.

 2. The case for general objects that are relatively far apart: Here the size scales of the two objects are both much smaller than the distance between them. In this case, Newton's gravity law applies directly as an approximation taking some fiducial points in the objects as being their equivalent point locations. The approximation becomes better, the greater the relative distance and the more cleverly the fiducial points are chosen (e.g., their centers of mass, but there may be even better choices), and approaches being exact in the limit that the relative distance goes to infinity. We will not prove this approximation.

3. The case of spherically symmetric objects that are not overlapping: It can be proven that the gravitational field of a spherically symmetric object outside of the object is exactly the field that a point mass would give according to Newton's gravity law if the point mass had the same mass as the object and was located where the object has its center. (The gravitational field is the cause of the gravitational force. It is a real thing in modern physics and not just a calculational device. Among other things, the gravitational field is force per unit mass on a point mass.) The proof is by Gauss's law for gravitation which itself is proven from Newton's law. Then using logic and Newton's 3rd law, one can prove that non-overlapping spherically symmetric objects attract each other exactly as if they were point masses of the sort specified. Newton himself first proved this result—but not with modern calculus techniques and therefore with some difficulty. The result is an essential result in understanding the motions of the larger solar system bodies which are nearly spherically symmetric.

4. The case of a relatively large, spherically symmetric object and a relatively small general object outside of the spherically symmetric object. In this case, Newton's gravity law approximately applies as if both objects were point masses: the large object's equivalent point mass is at its center and has its mass and the small object's equivalent point is located at some fiducial point inside of it (its center of mass usually being the best choice) and its mass. For planet and human size objects, the application of gravitation law is virtually exact. This case follows directly from:

   a) Newton's 1st law. b) Newton's 2nd law. c) Newton's 3rd law. d) cases 2 and 3.
   e) case 3.

SUGGESTED ANSWER: (d) From case 3, one sees that the relatively large, spherically symmetric object is equivalent to a point mass of the same mass at its center. Then this point mass and the relatively small objects are relatively far apart objects. Then case 2, tells us, they attract approximately like point masses.

Wrong answers:
a) One needs case 2 as an ingredient.

Redaction: Jeffery, 2008jan01

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014 qmult 00200 1 1 5 easy memory: fiducial case gravity law force calculation 1

8. A fiducial gravitational force is the force between two non-overlapping spherically symmetric objects each of mass 1 kg with the center-to-center distance one meter. The magnitude of this force is:

   a) 1. b) 1/2. c) 6.67428×10^{11}. d) 1.67×10^{-11}. e) 6.67384×10^{-11}.

SUGGESTED ANSWER: (e)

Behold:

$$F = \frac{Gm_1m_2}{r_{12}} = \frac{6.67384 \times 10^{-11} \times 1 \times 1}{1^2} = 6.67384 \times 10^{-11}\,\mathrm{N} .$$

This is a very small force. It is undetectable by human senses.

In fact, the gravitational force between laboratory sized objects can be measured, but not to wonderful accuracy. This is why the gravitational constant is the poorest known of the fundamental constants. It's current recommended value is $G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2012mar30) is uncertain in the 5th digit place—and some people think the given uncertainty in the brackets is too small.

Wrong answers:
a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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014 qmult 00204 1 3 3 easy math: gravity force law calculation 3

9. Using Newton's gravitation law

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

($G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$: e.g., Wikipedia: Gravitational constant, 2012mar30) calculate the magnitude of the force between two 3 kg objects 3 m apart. The answer is:

a) $60.1 \times 10^{-11} \approx 12 \times 10^{-11}$ lb. b) $20.0 \times 10^{-11} \approx 4 \times 10^{-11}$ lb.
c) $6.674 \times 10^{-11}\,\text{N} \approx 1.5 \times 10^{-11}$ lb. d) $2.224 \times 10^{-11}\,\text{N} \approx 0.5 \times 10^{-11}$ lb.
e) $0.741 \times 10^{-11}\,\text{N} \approx 0.15 \times 10^{-11}$ lb.

SUGGESTED ANSWER: (c)

Wrong answers:
a) You have to watch numerator and denominator both.

Fortran-95 Code
```
        conv=.2248d0
        f=6.674d-11
        fp=f*conv
        print*,'f,fp'
        print*,f,fp
!   6.674E-11 1.5003151999999997E-11
```

Redaction: Jeffery, 2001jan01

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014 qmult 00210 1 1 5 easy memory: ratio of gravity and electric forces

10. Two of the basic constituents of ordinary matter are the electron (mass $m = 9.10938215(45) \times 10^{-31}$ kg, charge $q = -1.602176487(40) \times 10^{-19}$ C) and the proton (mass $m = 1.672621636(83) \times 10^{-27}$ kg, charge $q = 1.602176487(40) \times 10^{-19}$ C). The magnitude of the electric force between these particles is given by Coulomb's law

$$F = \frac{kq_1q_2}{r_{12}^2} \ ,$$

where q_1 the charge on particle 1, q_2 the charge on particle 2, r_{12} is the distance between the particles, and $k = 8.987551787\ldots \times 10^9$ in MKS units is Coulomb's constant. What is the ratio of the magnitude of the gravitational force between the particles to the Coulomb force between?

a) $1.000\ldots$. b) 2.3×10^{-39}. c) 2.3×10^{39}. d) 4.4×10^{40}. e) 4.4×10^{-40}.

SUGGESTED ANSWER: (e)

Behold:

$$\frac{F_{\text{grav}}}{F_{\text{coul}}} = \frac{Gm_1m_2}{kq_1q_2} = 4.40787 \times 10^{-40} \ .$$

We see that in an important sense the gravitational force is much weaker than the Coulomb force. But the Coulomb force of matter is largely canceled over large distances by the fact that macroscopic matter is usually close neutral because of the cancellation of positive and negative charge.

But gravity has only one charge: i.e., mass. And mass is never canceled. Like attracts like for gravity. Thus, the gravitational force over long distances between massive bodies can be an immensely important force. It is key ingredient in the large-scale structure of the universe. But other forces are still important then, and so one can't say gravity dominates large-scale structure without some qualification of what one means.

Wrong answers:
a) Oh, c'mon.

Fortran-95 Code
```
        print*
        coulo=8.987551787d9
        grava=6.67428d-11
        xm1=9.10938215d-31
        q1=1.602176487d-19
        xm2=1.672621636d-27
        q2=1.602176487d-19
        ratio=(grava*xm1*xm2)/(coulo*q1*q2)
        print*,'ratio,1.d0/ratio'
        print*,ratio,1.d0/ratio
! 4.407871654758713E-40 2.2686685963744103E+39
```

Redaction: Jeffery, 2008jan01

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014 qmult 00212 1 4 5 easy deducto-memory: gravity and large-scale structure
Extra keywords: Sort of naturally follows the ratio of forces problem.

11. "Let's play *Jeopardy*! For $100, the answer is: This force is essential in making the large-scale structure of objects (of the kind we know) from middling asteroids to super-clusters of galaxies and perhaps to the universe as whole."

    a) What is ____________, Alex?

    a) a contact force b) friction c) a tension force d) a normal force e) gravity

SUGGESTED ANSWER: (e)

This is kind of a tricky problem to phrase. One often glibly says that gravity determines large scale structure. But for planets and stars pressure forces are important to sustain against gravitational collapse. Even for galaxies and clusters, pressure and radiation forces play a role in determining the interstellar medium. On the other hand gravity also plays a role in small scale structure. True, a table is not held together by gravity, but a lot of flimsy small structures are precluded by gravity: e.g., a skyscraper with cardboard girders. The key point is that other forces are important ingredients in large-scale structure and cannot be ignored, but since they are effectively short-range forces, there would be no large-scale structure as we know it without gravity. I suppose there could be other large-scale structure without gravity. A galaxy-size crystal could exist if gravity were turned off.

Wrong answers:

a) Contact force: no way.

Redaction: Jeffery, 2001jan01

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014 qmult 00220 1 1 4 easy memory: ratio of gravity forces 1

12. Consider two spherically symmetric objects and the gravitational force between them. What is force for separation r relative to the force for separation r_0 (i.e., $F(r)/F(r_0)$?

    a) $(r/r_0)^2$. b) r/r_0. c) r_0/r. d) $(r_0/r)^2$. e) $(r/r_0)^4$.

SUGGESTED ANSWER: (d)

Wrong answers:

a) Exactly wrong.
e) Bad guess.

Redaction: Jeffery, 2008jan01

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014 qmult 00222 1 1 1 easy memory: ratio of gravity forces 2
Extra keywords: physci

13. Given two spherically symmetric objects, if their separation is doubled, the force of gravity between them ____________ and, alternatively, if one object's mass is doubled, the force of gravity between them ____________.

    a) decreases by 1/4; is doubled b) decreases by 1/2; is doubled
    c) is doubled; decreases by 1/2 d) decreases by 1/4; increases by 4
    e) stays the same; stays the same

SUGGESTED ANSWER: (a)

Wrong answers:

e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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014 qmult 00230 2 1 3 mod. memory: mass and weight above Earth
Extra keywords: physci KB-59-25

14. An object has mass x and weight y on the Earth's surface. What is its mass and weight at 2 Earth radii above the Earth's surface? Note **ABOVE** the Earth's surface, not **FROM** the Earth's center.

a) x and $y/2$. b) $x/2$ and $y/2$. c) x and $y/9$. d) x and $y/4$. e) $x/9$ and $y/9$.

SUGGESTED ANSWER: (c)

Remember mass is an intrinsic property of a body and doesn't vary with location. It does vary with velocity according to special relativity, but that effect is small for everyday speeds. Weight is the force of gravity on an object. At 2 Earth radii above the surface, the object is at 3 Earth radii from the Earth's center. By the inverse-square law, the weight must be decreased by a factor of 9.

Wrong answers:
d) You may be forgetting to reference the distance to the Earth's center and using the distance to the Earth's surface instead.

Redaction: Jeffery, 2001jan01

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014 qmult 00300 1 4 4 easy deducto-memory: gravitational field 1

15. "Let's play *Jeopardy*! For \$100, the answer is: It can be defined as the gravitational force per unit mass at any point in space. But more fundamentally, it is a vector field that is the cause of the gravitational force: the conventional symbol for this vector field is $\vec{g}$. In modern, physics force fields are real things that are spread through space and cause forces."

What is ____________, Alex?

a) the gravitational force b) gravity c) levity d) the gravitational field
e) Gauss's law

SUGGESTED ANSWER: (d)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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014 qmult 00310 1 1 3 easy memory: gravitational field 2

16. The usual symbol for the gravitational field is $\vec{g}$ which has magnitude g. A special case of the gravitational field is the one near the Earth's surface. For this case, the gravitational field has the fiducial magnitude $g = 9.8\,\mathrm{N/kg}$ and is often called little g—big G is the gravitational constant. Context must decide if g means the general gravitational field magnitude or the special case of the gravitational field magnitude near the Earth's surface. The gravitational force on a point mass m in a general gravitational field $\vec{g}$ is given by ____________. The same formula applies to an extended object provided the field is uniform over the extent of the object.

a) $\vec{F} = \vec{g}/m$ b) $\vec{F} = m/\vec{g}$ c) $\vec{F} = m\vec{g}$ d) $\vec{F} = m^2\vec{g}$ e) $\vec{F} = \vec{g}/m^2$

SUGGESTED ANSWER: (c)

Wrong answers:
b) Now what does division by a vector mean.

Redaction: Jeffery, 2008jan01

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014 qmult 00320 1 1 1 easy memory: point mass grav field

17. The gravitational field

$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$

is that of a:

a) point mass at the origin. b) cubical mass centered on the origin.
c) tetrahedral mass centered on the origin. d) dodecahedral mass centered on the origin.
e) point mass **NOT** at the origin.

SUGGESTED ANSWER: (a)

Wrong answers:
a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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014 qmult 00322 1 1 4 easy memory: spherically-symmetric mass grav field

18. The gravitational field

$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$

is that of spherically symmetric mass distribution centered on the origin where m is all the mass within the sphere of radius ____________. This field result can be easily derived from Gauss's law for gravitation.

a) $2r$. b) r^2. c) $3r$. d) r. e) $1/r^2$.

SUGGESTED ANSWER: (d)

Wrong answers:

b) Dimensionally incorrect.

Redaction: Jeffery, 2008jan01

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014 qmult 00330 2 3 1 mod. math: fiducial-value grav field 1 (fg1)

19. Given the Earth's mass $M_{\rm Ea} = 5.9736\times10^{24}$ kg and mean radius $R_{\rm Ea} = 6.37101\times10^6$ m (e.g., Wikipedia 2007sep25), compute the magnitude of the Earth's gravitational field (force per unit mass) at the mean Earth's surface assuming spherical symmetry. The formula for the magnitude of gravitational field outside of a spherically-symmetric body of mass M is

$$g = \frac{GM}{r^2} \ ,$$

where r the distance from the body center and the current favored value for the gravitational constant $G = 6.67384(80)\times10^{-11}\,{\rm N\,m^2/kg^2}$ (e.g., Wikipedia: Gravitational constant 2012mar30). By desimplifying the expression for g, one can create a fiducial-value (i.e., reference-value) expression for the magnitude of the gravitational field g in terms of the Earth's values used as fiducial values. Identify this expression.

a) $9.822\,{\rm N/kg}\times(m/M_{\rm Ea})(R_{\rm Ea}/r)^2$ b) $6.822\,{\rm N/kg}\times(m/M_{\rm Ea})(R_{\rm Ea}/r)$
c) $6.822\,{\rm N/kg}\times(M_{\rm Ea}/m)(R_{\rm Ea}/r)$ d) $9.822\,{\rm N/kg}\times(M_{\rm Ea}/m)(r/R_{\rm Ea})$
e) $9.700\,{\rm N/kg}\times(m/M_{\rm Ea})(r/R_{\rm Ea})$

SUGGESTED ANSWER: (a)

Behold:

$$g = \frac{GM}{r^2} = \frac{GM_{\rm Ea}}{R_{\rm Ea}^2}\left(\frac{M}{M_{\rm Ea}}\right)\left(\frac{R_{\rm Ea}}{r}\right)^2 = 9.822\times\left(\frac{M}{M_{\rm Ea}}\right)\left(\frac{R_{\rm Ea}}{r}\right)^2 \ ,$$

where five digits of precision is clearly an upper limit on the precision one can claim.

Fortran-95 Code

```
      gravcon=6.67384d-11
      radius_earth_mean=6.37101d6
      xmass_earth=5.9736d24
      gg=gravcon*xmass_earth/radius_earth_mean**2
      print*,'gravcon,gg'
      print*,gravcon,gg
!   6.67384000000000009E-011   9.8218965457950205
```

Wrong answers:

b) Well no. Gravity is an inverse-square law force.

Redaction: Jeffery, 2008jan01

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014 qmult 00340 1 3 4 easy math: fiducial-value grav field 2 (fg2)

Extra keywords: Hold for tests with fg1

20. Given that the Moon's mass $M_{\rm Mo} = 0.0123\times M_{\rm Ea} \approx (1/80)M_{\rm Ea}$ and mean radius $R_{\rm Ea} = 0.273\times R_{\rm Ea} \approx (1/4)R_{\rm Ea}$ (e.g., Wikipedia 2007sep25) (where $M_{\rm Ea}$ is the Earth's mass and $R_{\rm Ea}$ is the Earth's mean

radius), compute to **3 SIGNIFICANT FIGURES** the gravitational field at the Moon's surface and give it in MKS and in Earth g-force (i.e., in units of $g_{\rm Ea} = 9.822\,{\rm N/kg}$, the Earth's surface mean gravitational field here with a subscript to differentiate it from the more general meaning of g).

a) $2\,{\rm N/kg}$; 1/5. b) $9.8225\,{\rm N/kg}$; 1.00. c) $1.62\,{\rm N/kg}$; 1/3. d) $1.62\,{\rm N/kg}$; 0.165.
e) $9.8225\,{\rm N/kg}$; 1/5.

SUGGESTED ANSWER: (d)

Behold:

$$g = 9.822 \times \left(\frac{m}{M_{\rm Ea}}\right)\left(\frac{R_{\rm Ea}}{r}\right)^2 = 1.62\,{\rm N/kg} = 0.165 g_{\rm Ea}\ ,$$

where three digits of precision is clearly an upper limit on the precision one can claim.

Fortran-95 Code

```
        gg=9.822           ! Mean Earth surface gravitational field.
        xmass_moon=.0123   ! Moon mass in Earth masses.
        radius_moon_mean=.273
        gg_moon=gg*xmass_moon*(1./radius_moon_mean)**2
        print*,'gg_moon,gg_moon/gg'
        print*,gg_moon,gg_moon/gg
!     1.6209876530513170       0.16503642102829028
```

Wrong answers:

a) A nonsense answer.

Redaction: Jeffery, 2008jan01

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014 qfull 00100 1 3 0 easy math: calculate g gravitational acceleration

21. Calculate the gravitational acceleration g for the Earth's surface given $G = 6.67384(80)\times10^{-11}\,{\rm N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2012mar30), Earth mass $M_{\rm E} = 5.9736\times10^{24}\,{\rm kg}$, and mean Earth radius $R_{\rm E} = 6.37101\times10^6\,{\rm m}$ (Wikipedia: 2007sep25). What is the percentage difference from the fiducial $g_{\rm fid} = 9.8\,{\rm m/s^2}$ that is commonly used.

SUGGESTED ANSWER:

We find

$$g = 9.822\,{\rm m/s^2} \qquad \text{and} \qquad \frac{g - g_{\rm fid}}{g_{\rm fid}} \times 100\,\% = 0.2\,\%\ .$$

Thus, the difference is rather slight.

Actually g varies due to deviations from spherical symmetry (mainly the slight oblate ellipsoidal shape of the Earth), the varying centrifugal force (which customarily included as part of g), and altitude. The major variation is the monotonic pole-to-equator variation due mainly to the centrifugal force variation, I think. The g value ranges from about $9.83\,{\rm m/s^2}$ at the poles to bout $9.78\,{\rm m/s^2}$ on the equator.

Fortran-95 Code

```
        print*
        gravcon=6.67384d-11
        xmasse=5.9742d+24
        rade=6.37123d+6
        g_fid=9.8
        gg=gravcon*xmasse/rade**2
        print*,'gg,(gg-g_fid)/g_fid'
        print*,gg,(gg-g_fid)/g_fid
!     9.8222047151866843       2.26576775710817056E-003
```

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1 - x} \approx 1 + x \;:\; (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 **Very Basic Newtonian Physics**

$$\vec{r}_{\rm cm} = \frac{\sum_i m_i \vec{r}_i}{m_{\rm total}} = \frac{\sum_{\rm sub} m_{\rm sub} \vec{r}_{\rm cm\ sub}}{m_{\rm total}} \qquad \vec{v}_{\rm cm} = \frac{\sum_i m_i \vec{v}_i}{m_{\rm total}} \qquad \vec{a}_{\rm cm} = \frac{\sum_i m_i \vec{a}_i}{m_{\rm total}}$$

$$\vec{r}_{\rm cm} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\rm total}}$$

$$\vec{F}_{\rm net} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,{\rm m/s^2}$$

$$\vec{F}_{\rm normal} = -\vec{F}_{\rm applied} \qquad F_{\rm linear} = -kx$$

$$f_{\rm normal} = \frac{T}{r} \qquad T = T_0 - F_{\rm parallel}(s) \qquad T = T_0$$

$$F_{\rm f\ static} = \min(F_{\rm applied}, F_{\rm f\ static\ max}) \qquad F_{\rm f\ static\ max} = \mu_{\rm static} F_{\rm N} \qquad F_{\rm f\ kinetic} = \mu_{\rm kinetic} F_{\rm N}$$

$$v_{\rm tangential} = r\omega = r\frac{d\theta}{dt} \qquad a_{\rm tangential} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\rm centripetal} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\rm centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\rm drag,lin} = bv \qquad v_{\rm T} = \frac{mg}{b} \qquad \tau = \frac{v_{\rm T}}{g} = \frac{m}{b} \qquad v = v_{\rm T}(1 - e^{-t/\tau})$$

$$F_{\rm drag,quad} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\rm T} = \sqrt{\frac{mg}{b}}$$

14 **Energy and Work**

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\rm mechanical} = KE + PE$$

$$P_{\rm avg} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\rm net} \quad \Delta PE_{\rm of\ a\ conservative\ force} = -W_{\rm by\ a\ conservative\ force} \quad \Delta E = W_{\rm nonconservative}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

---

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total}\ f} = KE_{\text{total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

---

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \qquad \tau_z = RF_{xy} \sin \gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

---

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net }x} = \sum F_x \qquad 0 = F_{\text{net }y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

---

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1 m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$

21 Fluids

$$\rho = \frac{\Delta m}{\Delta V} \qquad p = \frac{F}{A} \qquad p = p_0 + \rho g d_{\text{depth}}$$

Pascal's principle	$p = p_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta p = \Delta p_{\text{ext}}$
Archimedes principle	$F_{\text{buoy}} = m_{\text{fluid dis}} g = V_{\text{fluid dis}} \rho_{\text{fluid}} g$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$p + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

22 Oscillation

$$P = f^{-1} \qquad \omega = 2\pi f \qquad F = -kx \qquad PE = \frac{1}{2}kx^2 \qquad a(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$E_{\text{mec total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$P = 2\pi\sqrt{\frac{I}{mgr}} \qquad P = 2\pi\sqrt{\frac{r}{g}}$$

23 Waves

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad v = \sqrt{\frac{F_{\text{T}}}{\mu}} \qquad y = f(x \mp vt)$$

$$y = y_{\text{max}}\sin[k(x \mp vt)] = y_{\text{max}}\sin(kx \mp \omega t)$$

$$\text{Period} = \frac{1}{f} \qquad k = \frac{2\pi}{\lambda} \qquad v = f\lambda = \frac{\omega}{k} \qquad P \propto y_{\text{max}}^2$$

$$y = 2y_{\max} \sin(kx)\cos(\omega t) \qquad n = \frac{L}{\lambda/2} \qquad L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = n\frac{v}{2L}$$

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \qquad n\lambda = d\sin(\theta) \qquad \left(n + \frac{1}{2}\right)\lambda = d\sin(\theta)$$

$$I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_0}\right)$$

$$f = n\frac{v}{4L} \;:\; n = 1, 3, 5, \ldots \qquad f_{\mathrm{medium}} = \frac{f_0}{1 - v_0/v_{\mathrm{medium}}}$$

$$f' = f\left(1 - \frac{v'}{v}\right) \qquad f = \frac{f'}{1 - v'/v}$$